

Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "7 Inverse hyperbolic functions\7.3 Inverse hyperbolic tangent"

Test results for the 243 problems in "7.3.2 (d x)^m (a+b arctanh(c x^n))^p.m"

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x} dx$$

Optimal (type 4, 117 leaves, 6 steps):

$$2 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x}\right] - b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] + b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x}\right] + \frac{1}{2} b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x}\right] - \frac{1}{2} b^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x}\right]$$

Result (type 4, 151 leaves):

$$a^2 \operatorname{Log}[c x] + a b \left(-\operatorname{PolyLog}[2, -c x] + \operatorname{PolyLog}[2, c x]\right) + b^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c x]^3 - \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c x]}\right] + \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x]}\right] + \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[c x]}\right] + \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] + \frac{1}{2} \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}[c x]}\right] - \frac{1}{2} \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right]\right)$$

Problem 30: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^3}{x} dx$$

Optimal (type 4, 184 leaves, 8 steps):

$$2 \left(a + b \operatorname{ArcTanh}[c x] \right)^3 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x}\right] - \frac{3}{2} b \left(a + b \operatorname{ArcTanh}[c x] \right)^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] +$$

$$\frac{3}{2} b \left(a + b \operatorname{ArcTanh}[c x] \right)^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x}\right] + \frac{3}{2} b^2 \left(a + b \operatorname{ArcTanh}[c x] \right) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x}\right] -$$

$$\frac{3}{2} b^2 \left(a + b \operatorname{ArcTanh}[c x] \right) \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x}\right] - \frac{3}{4} b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2}{1 - c x}\right] + \frac{3}{4} b^3 \operatorname{PolyLog}\left[4, -1 + \frac{2}{1 - c x}\right]$$

Result (type 4, 315 leaves):

$$a^3 \operatorname{Log}[c x] + \frac{3}{2} a^2 b \left(-\operatorname{PolyLog}[2, -c x] + \operatorname{PolyLog}[2, c x] \right) + 3 a b^2$$

$$\left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c x]^3 - \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c x]}\right] + \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x]}\right] + \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[c x]}\right] + \right.$$

$$\left. \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] + \frac{1}{2} \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}[c x]}\right] - \frac{1}{2} \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right] \right) +$$

$$\frac{1}{64} b^3 \left(\pi^4 - 32 \operatorname{ArcTanh}[c x]^4 - 64 \operatorname{ArcTanh}[c x]^3 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c x]}\right] + 64 \operatorname{ArcTanh}[c x]^3 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x]}\right] + \right.$$

$$96 \operatorname{ArcTanh}[c x]^2 \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[c x]}\right] + 96 \operatorname{ArcTanh}[c x]^2 \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] + 96 \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}[c x]}\right] -$$

$$\left. 96 \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right] + 48 \operatorname{PolyLog}\left[4, -e^{-2 \operatorname{ArcTanh}[c x]}\right] + 48 \operatorname{PolyLog}\left[4, e^{2 \operatorname{ArcTanh}[c x]}\right] \right)$$

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^3}{x^2} dx$$

Optimal (type 4, 102 leaves, 5 steps):

$$c \left(a + b \operatorname{ArcTanh}[c x] \right)^3 - \frac{(a + b \operatorname{ArcTanh}[c x])^3}{x} + 3 b c \left(a + b \operatorname{ArcTanh}[c x] \right)^2 \operatorname{Log}\left[2 - \frac{2}{1 + c x}\right] -$$

$$3 b^2 c \left(a + b \operatorname{ArcTanh}[c x] \right) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x}\right] - \frac{3}{2} b^3 c \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 + c x}\right]$$

Result (type 4, 196 leaves):

$$-\frac{a^3}{x} - \frac{3 a^2 b \operatorname{ArcTanh}[c x]}{x} + 3 a^2 b c \operatorname{Log}[x] - \frac{3}{2} a^2 b c \operatorname{Log}\left[1 - c^2 x^2\right] +$$

$$3 a b^2 c \left(\operatorname{ArcTanh}[c x] \left(\operatorname{ArcTanh}[c x] - \frac{\operatorname{ArcTanh}[c x]}{c x} + 2 \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] \right) - \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] \right) +$$

$$b^3 c \left(\frac{i \pi^3}{8} - \operatorname{ArcTanh}[c x]^3 - \frac{\operatorname{ArcTanh}[c x]^3}{c x} + 3 \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x]}\right] + \right.$$

$$\left. 3 \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] - \frac{3}{2} \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right] \right)$$

Problem 33: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^3}{x^4} dx$$

Optimal (type 4, 200 leaves, 14 steps):

$$\begin{aligned} & -\frac{b^2 c^2 (a + b \operatorname{ArcTanh}[c x])}{x} + \frac{1}{2} b c^3 (a + b \operatorname{ArcTanh}[c x])^2 - \frac{b c (a + b \operatorname{ArcTanh}[c x])^2}{2 x^2} + \\ & \frac{1}{3} c^3 (a + b \operatorname{ArcTanh}[c x])^3 - \frac{(a + b \operatorname{ArcTanh}[c x])^3}{3 x^3} + b^3 c^3 \operatorname{Log}[x] - \frac{1}{2} b^3 c^3 \operatorname{Log}[1 - c^2 x^2] + \\ & b c^3 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[2 - \frac{2}{1 + c x}\right] - b^2 c^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x}\right] - \frac{1}{2} b^3 c^3 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 + c x}\right] \end{aligned}$$

Result (type 4, 323 leaves):

$$\begin{aligned} & -\frac{1}{24 x^3} \left(8 a^3 + 12 a^2 b c x + 24 a^2 b \operatorname{ArcTanh}[c x] - 24 a^2 b c^3 x^3 \operatorname{Log}[x] + 12 a^2 b c^3 x^3 \operatorname{Log}[1 - c^2 x^2] + \right. \\ & \quad 24 a b^2 (c^2 x^2 + (1 - c^3 x^3) \operatorname{ArcTanh}[c x])^2 - c x \operatorname{ArcTanh}[c x] (-1 + c^2 x^2 + 2 c^2 x^2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcTanh}[c x]}]) + c^3 x^3 \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcTanh}[c x]}] \left. \right) + \\ & \quad b^3 \left(-i c^3 \pi^3 x^3 + 24 c^2 x^2 \operatorname{ArcTanh}[c x] + 12 c x \operatorname{ArcTanh}[c x]^2 - 12 c^3 x^3 \operatorname{ArcTanh}[c x]^2 + 8 \operatorname{ArcTanh}[c x]^3 + \right. \\ & \quad \quad 8 c^3 x^3 \operatorname{ArcTanh}[c x]^3 - 24 c^3 x^3 \operatorname{ArcTanh}[c x]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c x]}] - 24 c^3 x^3 \operatorname{Log}\left[\frac{c x}{\sqrt{1 - c^2 x^2}}\right] - \\ & \quad \quad \left. \left. 24 c^3 x^3 \operatorname{ArcTanh}[c x] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x]}] + 12 c^3 x^3 \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x]}] \right) \right) \end{aligned}$$

Problem 68: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{x} dx$$

Optimal (type 4, 137 leaves, 7 steps):

$$\begin{aligned} & (a + b \operatorname{ArcTanh}[c x^2])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x^2}\right] - \frac{1}{2} b (a + b \operatorname{ArcTanh}[c x^2]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x^2}\right] + \\ & \frac{1}{2} b (a + b \operatorname{ArcTanh}[c x^2]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x^2}\right] + \frac{1}{4} b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x^2}\right] - \frac{1}{4} b^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x^2}\right] \end{aligned}$$

Result (type 4, 181 leaves):

$$\begin{aligned}
& a^2 \operatorname{Log}[x] + \frac{1}{2} a b \left(-\operatorname{PolyLog}[2, -c x^2] + \operatorname{PolyLog}[2, c x^2] \right) + \\
& \frac{1}{2} b^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c x^2]^3 - \operatorname{ArcTanh}[c x^2]^2 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c x^2]}] + \operatorname{ArcTanh}[c x^2]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c x^2]}] + \operatorname{ArcTanh}[c x^2] \right. \\
& \left. \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x^2]}] + \operatorname{ArcTanh}[c x^2] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x^2]}] + \frac{1}{2} \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c x^2]}] - \frac{1}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x^2]}] \right)
\end{aligned}$$

Problem 71: Unable to integrate problem.

$$\int x^4 (a + b \operatorname{ArcTanh}[c x^2])^2 dx$$

Optimal (type 4, 1173 leaves, 102 steps):

$$\begin{aligned}
& \frac{8 b^2 x}{15 c^2} + \frac{2 a b x^3}{15 c} - \frac{2}{25} a b x^5 + \frac{2 a b \operatorname{ArcTan}[\sqrt{c} x]}{5 c^{5/2}} - \frac{4 b^2 \operatorname{ArcTan}[\sqrt{c} x]}{15 c^{5/2}} + \frac{i b^2 \operatorname{ArcTan}[\sqrt{c} x]^2}{5 c^{5/2}} - \frac{4 b^2 \operatorname{ArcTanh}[\sqrt{c} x]}{15 c^{5/2}} - \\
& \frac{b^2 \operatorname{ArcTanh}[\sqrt{c} x]^2}{5 c^{5/2}} + \frac{2 b^2 \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1-\sqrt{c} x}\right]}{5 c^{5/2}} - \frac{2 b^2 \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1-i \sqrt{c} x}\right]}{5 c^{5/2}} + \frac{b^2 \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{(1+i)(1-\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{5 c^{5/2}} + \\
& \frac{2 b^2 \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1+i \sqrt{c} x}\right]}{5 c^{5/2}} - \frac{2 b^2 \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1+\sqrt{c} x}\right]}{5 c^{5/2}} + \frac{b^2 \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[-\frac{2 \sqrt{c}(1-\sqrt{c} x)}{(\sqrt{c}-\sqrt{c})(1+\sqrt{c} x)}\right]}{5 c^{5/2}} + \\
& \frac{b^2 \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[\frac{2 \sqrt{c}(1+\sqrt{c} x)}{(\sqrt{c}+\sqrt{c})(1+\sqrt{c} x)}\right]}{5 c^{5/2}} + \frac{b^2 \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{(1-i)(1+\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{5 c^{5/2}} - \frac{b^2 x^3 \operatorname{Log}[1-c x^2]}{15 c} + \frac{1}{25} b^2 x^5 \operatorname{Log}[1-c x^2] - \\
& \frac{b^2 \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}[1-c x^2]}{5 c^{5/2}} + \frac{b x^3 (2 a - b \operatorname{Log}[1-c x^2])}{15 c} + \frac{1}{25} b x^5 (2 a - b \operatorname{Log}[1-c x^2]) - \frac{b \operatorname{ArcTanh}[\sqrt{c} x] (2 a - b \operatorname{Log}[1-c x^2])}{5 c^{5/2}} + \\
& \frac{1}{20} x^5 (2 a - b \operatorname{Log}[1-c x^2])^2 + \frac{2 b^2 x^3 \operatorname{Log}[1+c x^2]}{15 c} + \frac{1}{5} a b x^5 \operatorname{Log}[1+c x^2] + \frac{b^2 \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}[1+c x^2]}{5 c^{5/2}} - \\
& \frac{b^2 \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}[1+c x^2]}{5 c^{5/2}} - \frac{1}{10} b^2 x^5 \operatorname{Log}[1-c x^2] \operatorname{Log}[1+c x^2] + \frac{1}{20} b^2 x^5 \operatorname{Log}[1+c x^2]^2 + \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-\sqrt{c} x}\right]}{5 c^{5/2}} + \\
& \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-i \sqrt{c} x}\right]}{5 c^{5/2}} - \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{(1+i)(1-\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{10 c^{5/2}} + \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i \sqrt{c} x}\right]}{5 c^{5/2}} + \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+\sqrt{c} x}\right]}{5 c^{5/2}} - \\
& \frac{b^2 \operatorname{PolyLog}\left[2, 1 + \frac{2 \sqrt{c}(1-\sqrt{c} x)}{(\sqrt{c}-\sqrt{c})(1+\sqrt{c} x)}\right]}{10 c^{5/2}} - \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{c}(1+\sqrt{c} x)}{(\sqrt{c}+\sqrt{c})(1+\sqrt{c} x)}\right]}{10 c^{5/2}} - \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{(1-i)(1+\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{10 c^{5/2}}
\end{aligned}$$

Result (type 8, 18 leaves):

$$\int x^4 (a + b \operatorname{ArcTanh}[c x^2])^2 dx$$

Problem 72: Unable to integrate problem.

$$\int x^2 (a + b \operatorname{ArcTanh}[c x^2])^2 dx$$

Optimal (type 4, 1129 leaves, 86 steps):

$$\begin{aligned} & \frac{4 a b x}{3 c} - \frac{2}{9} a b x^3 - \frac{2 a b \operatorname{ArcTan}[\sqrt{c} x]}{3 c^{3/2}} + \frac{4 b^2 \operatorname{ArcTan}[\sqrt{c} x]}{3 c^{3/2}} - \frac{i b^2 \operatorname{ArcTan}[\sqrt{c} x]^2}{3 c^{3/2}} - \frac{4 b^2 \operatorname{ArcTanh}[\sqrt{c} x]}{3 c^{3/2}} - \frac{b^2 \operatorname{ArcTanh}[\sqrt{c} x]^2}{3 c^{3/2}} + \\ & \frac{2 b^2 \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1-\sqrt{c} x}\right]}{3 c^{3/2}} + \frac{2 b^2 \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1-i \sqrt{c} x}\right]}{3 c^{3/2}} - \frac{b^2 \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{(1+i)(1-\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{3 c^{3/2}} - \frac{2 b^2 \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1+i \sqrt{c} x}\right]}{3 c^{3/2}} - \\ & \frac{2 b^2 \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[\frac{2}{1+\sqrt{c} x}\right]}{3 c^{3/2}} + \frac{b^2 \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[-\frac{2 \sqrt{c}(1-\sqrt{c} x)}{(\sqrt{c}-\sqrt{c})(1+\sqrt{c} x)}\right]}{3 c^{3/2}} + \frac{b^2 \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}\left[\frac{2 \sqrt{c}(1+\sqrt{c} x)}{(\sqrt{c}+\sqrt{c})(1+\sqrt{c} x)}\right]}{3 c^{3/2}} - \\ & \frac{b^2 \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}\left[\frac{(1-i)(1+\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{3 c^{3/2}} - \frac{2 b^2 x \operatorname{Log}[1-c x^2]}{3 c} + \frac{1}{9} b^2 x^3 \operatorname{Log}[1-c x^2] + \frac{b^2 \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}[1-c x^2]}{3 c^{3/2}} + \frac{1}{9} b x^3 (2 a - b \operatorname{Log}[1-c x^2]) - \\ & \frac{b \operatorname{ArcTanh}[\sqrt{c} x] (2 a - b \operatorname{Log}[1-c x^2])}{3 c^{3/2}} + \frac{1}{12} x^3 (2 a - b \operatorname{Log}[1-c x^2])^2 + \frac{2 b^2 x \operatorname{Log}[1+c x^2]}{3 c} + \frac{1}{3} a b x^3 \operatorname{Log}[1+c x^2] - \\ & \frac{b^2 \operatorname{ArcTan}[\sqrt{c} x] \operatorname{Log}[1+c x^2]}{3 c^{3/2}} - \frac{b^2 \operatorname{ArcTanh}[\sqrt{c} x] \operatorname{Log}[1+c x^2]}{3 c^{3/2}} - \frac{1}{6} b^2 x^3 \operatorname{Log}[1-c x^2] \operatorname{Log}[1+c x^2] + \frac{1}{12} b^2 x^3 \operatorname{Log}[1+c x^2]^2 + \\ & \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-\sqrt{c} x}\right]}{3 c^{3/2}} - \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-i \sqrt{c} x}\right]}{3 c^{3/2}} + \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{(1+i)(1-\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{6 c^{3/2}} - \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i \sqrt{c} x}\right]}{3 c^{3/2}} + \\ & \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+\sqrt{c} x}\right]}{3 c^{3/2}} - \frac{b^2 \operatorname{PolyLog}\left[2, 1 + \frac{2 \sqrt{c}(1-\sqrt{c} x)}{(\sqrt{c}-\sqrt{c})(1+\sqrt{c} x)}\right]}{6 c^{3/2}} - \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 \sqrt{c}(1+\sqrt{c} x)}{(\sqrt{c}+\sqrt{c})(1+\sqrt{c} x)}\right]}{6 c^{3/2}} + \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{(1-i)(1+\sqrt{c} x)}{1-i \sqrt{c} x}\right]}{6 c^{3/2}} \end{aligned}$$

Result (type 8, 18 leaves):

$$\int x^2 (a + b \operatorname{ArcTanh}[c x^2])^2 dx$$

Problem 75: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{x^4} dx$$

Optimal (type 4, 1102 leaves, 64 steps):

$$\begin{aligned} & -\frac{2abc}{3x} - \frac{2}{3}abc^{3/2}\operatorname{ArcTan}[\sqrt{c}x] + \frac{4}{3}b^2c^{3/2}\operatorname{ArcTan}[\sqrt{c}x] - \frac{1}{3}ib^2c^{3/2}\operatorname{ArcTan}[\sqrt{c}x]^2 + \frac{4}{3}b^2c^{3/2}\operatorname{ArcTanh}[\sqrt{c}x] + \\ & \frac{1}{3}b^2c^{3/2}\operatorname{ArcTanh}[\sqrt{c}x]^2 - \frac{2}{3}b^2c^{3/2}\operatorname{ArcTanh}[\sqrt{c}x]\operatorname{Log}\left[\frac{2}{1-\sqrt{c}x}\right] + \frac{2}{3}b^2c^{3/2}\operatorname{ArcTan}[\sqrt{c}x]\operatorname{Log}\left[\frac{2}{1-i\sqrt{c}x}\right] - \\ & \frac{1}{3}b^2c^{3/2}\operatorname{ArcTan}[\sqrt{c}x]\operatorname{Log}\left[\frac{(1+i)(1-\sqrt{c}x)}{1-i\sqrt{c}x}\right] - \frac{2}{3}b^2c^{3/2}\operatorname{ArcTan}[\sqrt{c}x]\operatorname{Log}\left[\frac{2}{1+i\sqrt{c}x}\right] + \frac{2}{3}b^2c^{3/2}\operatorname{ArcTanh}[\sqrt{c}x]\operatorname{Log}\left[\frac{2}{1+\sqrt{c}x}\right] - \\ & \frac{1}{3}b^2c^{3/2}\operatorname{ArcTanh}[\sqrt{c}x]\operatorname{Log}\left[-\frac{2\sqrt{c}(1-\sqrt{c}x)}{(\sqrt{-c}-\sqrt{c})(1+\sqrt{c}x)}\right] - \frac{1}{3}b^2c^{3/2}\operatorname{ArcTanh}[\sqrt{c}x]\operatorname{Log}\left[\frac{2\sqrt{c}(1+\sqrt{c}x)}{(\sqrt{-c}+\sqrt{c})(1+\sqrt{c}x)}\right] - \\ & \frac{1}{3}b^2c^{3/2}\operatorname{ArcTan}[\sqrt{c}x]\operatorname{Log}\left[\frac{(1-i)(1+\sqrt{c}x)}{1-i\sqrt{c}x}\right] + \frac{b^2c\operatorname{Log}[1-cx^2]}{3x} + \frac{1}{3}b^2c^{3/2}\operatorname{ArcTan}[\sqrt{c}x]\operatorname{Log}[1-cx^2] - \frac{bc(2a-b\operatorname{Log}[1-cx^2])}{3x} + \\ & \frac{1}{3}bc^{3/2}\operatorname{ArcTanh}[\sqrt{c}x](2a-b\operatorname{Log}[1-cx^2]) - \frac{(2a-b\operatorname{Log}[1-cx^2])^2}{12x^3} - \frac{ab\operatorname{Log}[1+cx^2]}{3x^3} - \frac{2b^2c\operatorname{Log}[1+cx^2]}{3x} - \\ & \frac{1}{3}b^2c^{3/2}\operatorname{ArcTan}[\sqrt{c}x]\operatorname{Log}[1+cx^2] + \frac{1}{3}b^2c^{3/2}\operatorname{ArcTanh}[\sqrt{c}x]\operatorname{Log}[1+cx^2] + \frac{b^2\operatorname{Log}[1-cx^2]\operatorname{Log}[1+cx^2]}{6x^3} - \frac{b^2\operatorname{Log}[1+cx^2]^2}{12x^3} - \\ & \frac{1}{3}b^2c^{3/2}\operatorname{PolyLog}\left[2, 1 - \frac{2}{1-\sqrt{c}x}\right] - \frac{1}{3}ib^2c^{3/2}\operatorname{PolyLog}\left[2, 1 - \frac{2}{1-i\sqrt{c}x}\right] + \frac{1}{6}ib^2c^{3/2}\operatorname{PolyLog}\left[2, 1 - \frac{(1+i)(1-\sqrt{c}x)}{1-i\sqrt{c}x}\right] - \\ & \frac{1}{3}ib^2c^{3/2}\operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i\sqrt{c}x}\right] - \frac{1}{3}b^2c^{3/2}\operatorname{PolyLog}\left[2, 1 - \frac{2}{1+\sqrt{c}x}\right] + \frac{1}{6}b^2c^{3/2}\operatorname{PolyLog}\left[2, 1 + \frac{2\sqrt{c}(1-\sqrt{c}x)}{(\sqrt{-c}-\sqrt{c})(1+\sqrt{c}x)}\right] + \\ & \frac{1}{6}b^2c^{3/2}\operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}(1+\sqrt{c}x)}{(\sqrt{-c}+\sqrt{c})(1+\sqrt{c}x)}\right] + \frac{1}{6}ib^2c^{3/2}\operatorname{PolyLog}\left[2, 1 - \frac{(1-i)(1+\sqrt{c}x)}{1-i\sqrt{c}x}\right] \end{aligned}$$

Result (type 8, 18 leaves):

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{x^4} dx$$

Problem 76: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{x^6} dx$$

Optimal (type 4, 1176 leaves, 77 steps):

$$\begin{aligned} & -\frac{2abc}{15x^3} + \frac{2abc^2}{5x} - \frac{8b^2c^2}{15x} + \frac{2}{5}abc^{5/2}\operatorname{ArcTan}[\sqrt{c}x] - \frac{4}{15}b^2c^{5/2}\operatorname{ArcTan}[\sqrt{c}x] + \frac{1}{5}ib^2c^{5/2}\operatorname{ArcTan}[\sqrt{c}x]^2 + \\ & \frac{4}{15}b^2c^{5/2}\operatorname{ArcTanh}[\sqrt{c}x] + \frac{1}{5}b^2c^{5/2}\operatorname{ArcTanh}[\sqrt{c}x]^2 - \frac{2}{5}b^2c^{5/2}\operatorname{ArcTanh}[\sqrt{c}x]\operatorname{Log}\left[\frac{2}{1-\sqrt{c}x}\right] - \\ & \frac{2}{5}b^2c^{5/2}\operatorname{ArcTan}[\sqrt{c}x]\operatorname{Log}\left[\frac{2}{1-i\sqrt{c}x}\right] + \frac{1}{5}b^2c^{5/2}\operatorname{ArcTan}[\sqrt{c}x]\operatorname{Log}\left[\frac{(1+i)(1-\sqrt{c}x)}{1-i\sqrt{c}x}\right] + \frac{2}{5}b^2c^{5/2}\operatorname{ArcTan}[\sqrt{c}x]\operatorname{Log}\left[\frac{2}{1+i\sqrt{c}x}\right] + \\ & \frac{2}{5}b^2c^{5/2}\operatorname{ArcTanh}[\sqrt{c}x]\operatorname{Log}\left[\frac{2}{1+\sqrt{c}x}\right] - \frac{1}{5}b^2c^{5/2}\operatorname{ArcTanh}[\sqrt{c}x]\operatorname{Log}\left[-\frac{2\sqrt{c}(1-\sqrt{c}x)}{(\sqrt{-c}-\sqrt{c})(1+\sqrt{c}x)}\right] - \\ & \frac{1}{5}b^2c^{5/2}\operatorname{ArcTanh}[\sqrt{c}x]\operatorname{Log}\left[\frac{2\sqrt{c}(1+\sqrt{c}x)}{(\sqrt{-c}+\sqrt{c})(1+\sqrt{c}x)}\right] + \frac{1}{5}b^2c^{5/2}\operatorname{ArcTan}[\sqrt{c}x]\operatorname{Log}\left[\frac{(1-i)(1+\sqrt{c}x)}{1-i\sqrt{c}x}\right] + \frac{b^2c\operatorname{Log}[1-cx^2]}{15x^3} - \\ & \frac{b^2c^2\operatorname{Log}[1-cx^2]}{5x} - \frac{1}{5}b^2c^{5/2}\operatorname{ArcTan}[\sqrt{c}x]\operatorname{Log}[1-cx^2] - \frac{bc(2a-b\operatorname{Log}[1-cx^2])}{15x^3} - \frac{bc^2(2a-b\operatorname{Log}[1-cx^2])}{5x} + \\ & \frac{1}{5}b^2c^{5/2}\operatorname{ArcTanh}[\sqrt{c}x](2a-b\operatorname{Log}[1-cx^2]) - \frac{(2a-b\operatorname{Log}[1-cx^2])^2}{20x^5} - \frac{ab\operatorname{Log}[1+cx^2]}{5x^5} - \frac{2b^2c\operatorname{Log}[1+cx^2]}{15x^3} + \\ & \frac{1}{5}b^2c^{5/2}\operatorname{ArcTan}[\sqrt{c}x]\operatorname{Log}[1+cx^2] + \frac{1}{5}b^2c^{5/2}\operatorname{ArcTanh}[\sqrt{c}x]\operatorname{Log}[1+cx^2] + \frac{b^2\operatorname{Log}[1-cx^2]\operatorname{Log}[1+cx^2]}{10x^5} - \frac{b^2\operatorname{Log}[1+cx^2]^2}{20x^5} - \\ & \frac{1}{5}b^2c^{5/2}\operatorname{PolyLog}\left[2, 1 - \frac{2}{1-\sqrt{c}x}\right] + \frac{1}{5}ib^2c^{5/2}\operatorname{PolyLog}\left[2, 1 - \frac{2}{1-i\sqrt{c}x}\right] - \frac{1}{10}ib^2c^{5/2}\operatorname{PolyLog}\left[2, 1 - \frac{(1+i)(1-\sqrt{c}x)}{1-i\sqrt{c}x}\right] + \\ & \frac{1}{5}ib^2c^{5/2}\operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i\sqrt{c}x}\right] - \frac{1}{5}b^2c^{5/2}\operatorname{PolyLog}\left[2, 1 - \frac{2}{1+\sqrt{c}x}\right] + \frac{1}{10}b^2c^{5/2}\operatorname{PolyLog}\left[2, 1 + \frac{2\sqrt{c}(1-\sqrt{c}x)}{(\sqrt{-c}-\sqrt{c})(1+\sqrt{c}x)}\right] + \\ & \frac{1}{10}b^2c^{5/2}\operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}(1+\sqrt{c}x)}{(\sqrt{-c}+\sqrt{c})(1+\sqrt{c}x)}\right] - \frac{1}{10}ib^2c^{5/2}\operatorname{PolyLog}\left[2, 1 - \frac{(1-i)(1+\sqrt{c}x)}{1-i\sqrt{c}x}\right] \end{aligned}$$

Result (type 8, 18 leaves):

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{x^6} dx$$

Problem 79: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^3}{x} dx$$

Optimal (type 4, 207 leaves, 9 steps):

$$\begin{aligned} & (a + b \operatorname{ArcTanh}[c x^2])^3 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x^2}\right] - \frac{3}{4} b (a + b \operatorname{ArcTanh}[c x^2])^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x^2}\right] + \\ & \frac{3}{4} b (a + b \operatorname{ArcTanh}[c x^2])^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x^2}\right] + \frac{3}{4} b^2 (a + b \operatorname{ArcTanh}[c x^2]) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x^2}\right] - \\ & \frac{3}{4} b^2 (a + b \operatorname{ArcTanh}[c x^2]) \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x^2}\right] - \frac{3}{8} b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2}{1 - c x^2}\right] + \frac{3}{8} b^3 \operatorname{PolyLog}\left[4, -1 + \frac{2}{1 - c x^2}\right] \end{aligned}$$

Result (type 4, 371 leaves):

$$\begin{aligned} & a^3 \operatorname{Log}[x] + \frac{3}{4} a^2 b \left(-\operatorname{PolyLog}[2, -c x^2] + \operatorname{PolyLog}[2, c x^2]\right) + \\ & \frac{3}{2} a b^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c x^2]^3 - \operatorname{ArcTanh}[c x^2]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c x^2]}\right] + \operatorname{ArcTanh}[c x^2]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x^2]}\right] + \operatorname{ArcTanh}[c x^2]\right. \\ & \quad \left. \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x^2]}] + \operatorname{ArcTanh}[c x^2] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x^2]}] + \frac{1}{2} \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c x^2]}] - \frac{1}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x^2]}]\right) + \\ & \frac{1}{128} b^3 \left(\pi^4 - 32 \operatorname{ArcTanh}[c x^2]^4 - 64 \operatorname{ArcTanh}[c x^2]^3 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c x^2]}\right] + 64 \operatorname{ArcTanh}[c x^2]^3 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x^2]}\right] + \right. \\ & \quad \left. 96 \operatorname{ArcTanh}[c x^2]^2 \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x^2]}] + 96 \operatorname{ArcTanh}[c x^2]^2 \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x^2]}] + 96 \operatorname{ArcTanh}[c x^2] \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c x^2]}] - \right. \\ & \quad \left. 96 \operatorname{ArcTanh}[c x^2] \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x^2]}] + 48 \operatorname{PolyLog}[4, -e^{-2 \operatorname{ArcTanh}[c x^2]}] + 48 \operatorname{PolyLog}[4, e^{2 \operatorname{ArcTanh}[c x^2]}]\right) \end{aligned}$$

Problem 80: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^3}{x^3} dx$$

Optimal (type 4, 125 leaves, 6 steps):

$$\begin{aligned} & \frac{1}{2} c (a + b \operatorname{ArcTanh}[c x^2])^3 - \frac{(a + b \operatorname{ArcTanh}[c x^2])^3}{2 x^2} + \frac{3}{2} b c (a + b \operatorname{ArcTanh}[c x^2])^2 \operatorname{Log}\left[2 - \frac{2}{1 + c x^2}\right] - \\ & \frac{3}{2} b^2 c (a + b \operatorname{ArcTanh}[c x^2]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x^2}\right] - \frac{3}{4} b^3 c \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 + c x^2}\right] \end{aligned}$$

Result (type 4, 222 leaves):

$$\frac{1}{4} \left(-\frac{2a^3}{x^2} - \frac{6a^2 b \operatorname{ArcTanh}[c x^2]}{x^2} + 12a^2 b c \operatorname{Log}[x] - 3a^2 b c \operatorname{Log}[1 - c^2 x^4] + \right. \\ \left. 6a b^2 c \left(\operatorname{ArcTanh}[c x^2] \left(\left(1 - \frac{1}{c x^2} \right) \operatorname{ArcTanh}[c x^2] + 2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcTanh}[c x^2]}] \right) - \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcTanh}[c x^2]}] \right) + 2b^3 c \left(\frac{i \pi^3}{8} - \operatorname{ArcTanh}[c x^2]^3 - \right. \right. \\ \left. \left. \frac{\operatorname{ArcTanh}[c x^2]^3}{c x^2} + 3 \operatorname{ArcTanh}[c x^2]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c x^2]}] + 3 \operatorname{ArcTanh}[c x^2] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x^2]}] - \frac{3}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x^2]}] \right) \right)$$

Problem 90: Attempted integration timed out after 120 seconds.

$$\int \sqrt{d x} (a + b \operatorname{ArcTanh}[c x^2])^2 dx$$

Optimal (type 4, 6327 leaves, 238 steps):

$$-\frac{8}{9} a b x \sqrt{d x} - \frac{2 \sqrt{2} a b \sqrt{d x} \operatorname{ArcTan}[1 - \sqrt{2} c^{1/4} \sqrt{x}]}{3 c^{3/4} \sqrt{x}} + \frac{2 \sqrt{2} a b \sqrt{d x} \operatorname{ArcTan}[1 + \sqrt{2} c^{1/4} \sqrt{x}]}{3 c^{3/4} \sqrt{x}} - \\ \frac{2 i b^2 \sqrt{d x} \operatorname{ArcTan}[(-c)^{1/4} \sqrt{x}]^2}{3 (-c)^{3/4} \sqrt{x}} - \frac{2 i b^2 \sqrt{d x} \operatorname{ArcTan}[c^{1/4} \sqrt{x}]^2}{3 c^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}[(-c)^{1/4} \sqrt{x}]^2}{3 (-c)^{3/4} \sqrt{x}} - \\ \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}[c^{1/4} \sqrt{x}]^2}{3 c^{3/4} \sqrt{x}} + \frac{4 b^2 \sqrt{d x} \operatorname{ArcTanh}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[\frac{2}{1 - (-c)^{1/4} \sqrt{x}}\right]}{3 (-c)^{3/4} \sqrt{x}} + \frac{4 b^2 \sqrt{d x} \operatorname{ArcTan}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[\frac{2}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{3 (-c)^{3/4} \sqrt{x}} - \\ \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[-\frac{2 (-c)^{1/4} (1 - \sqrt{-\sqrt{-c}} \sqrt{x})}{(i \sqrt{-\sqrt{-c}} - (-c)^{1/4}) (1 - i (-c)^{1/4} \sqrt{x})}\right]}{3 (-c)^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[\frac{2 (-c)^{1/4} (1 + \sqrt{-\sqrt{-c}} \sqrt{x})}{(i \sqrt{-\sqrt{-c}} + (-c)^{1/4}) (1 - i (-c)^{1/4} \sqrt{x})}\right]}{3 (-c)^{3/4} \sqrt{x}} + \\ \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[\frac{(1+i) (1 - (-c)^{1/4} \sqrt{x})}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{3 (-c)^{3/4} \sqrt{x}} - \frac{4 b^2 \sqrt{d x} \operatorname{ArcTan}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[\frac{2}{1 + i (-c)^{1/4} \sqrt{x}}\right]}{3 (-c)^{3/4} \sqrt{x}} - \\ \frac{4 b^2 \sqrt{d x} \operatorname{ArcTanh}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[\frac{2}{1 + (-c)^{1/4} \sqrt{x}}\right]}{3 (-c)^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[-\frac{2 (-c)^{1/4} (1 - \sqrt{-\sqrt{-c}} \sqrt{x})}{(\sqrt{-\sqrt{-c}} - (-c)^{1/4}) (1 + (-c)^{1/4} \sqrt{x})}\right]}{3 (-c)^{3/4} \sqrt{x}}$$

$$\begin{aligned}
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1-i)\left(1+(-c)^{1/4} \sqrt{x}\right)}{1-i(-c)^{1/4} \sqrt{x}}\right]}{3(-c)^{3/4} \sqrt{x}} + \\
& \frac{4 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-i c^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{4 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} - \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{i \sqrt{-\sqrt{-c}}-c^{1/4}}\right]}{3 c^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{i \sqrt{-\sqrt{-c}}+c^{1/4}}\right]}{3 c^{3/4} \sqrt{x}} - \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-(-c)^{1/4} \sqrt{x}\right)}{i(-c)^{1/4}-c^{1/4}}\right]}{3 c^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+(-c)^{1/4} \sqrt{x}\right)}{i(-c)^{1/4}+c^{1/4}}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1+i)\left(1-c^{1/4} \sqrt{x}\right)}{1+i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} - \frac{4 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} - \\
& \frac{4 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{c}}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+i c^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1-i)\left(1+c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{\sqrt{2} a b \sqrt{d x} \operatorname{Log}\left[1-\sqrt{2} c^{1/4} \sqrt{x}+\sqrt{c} x\right]}{3 c^{3/4} \sqrt{x}} - \frac{\sqrt{2} a b \sqrt{d x} \operatorname{Log}\left[1+\sqrt{2} c^{1/4} \sqrt{x}+\sqrt{c} x\right]}{3 c^{3/4} \sqrt{x}} + \frac{4}{9} b^2 x \sqrt{d x} \operatorname{Log}\left[1-c x^2\right] + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{3(-c)^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{4}{9} b x \sqrt{d x}\left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right) + \\
& \frac{2 b \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right]\left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)}{3 c^{3/4} \sqrt{x}} - \frac{2 b \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right]\left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)}{3 c^{3/4} \sqrt{x}} + \frac{1}{6} x \sqrt{d x}\left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^2 + \\
& \frac{2}{3} a b x \sqrt{d x} \operatorname{Log}\left[1+c x^2\right] - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{3(-c)^{3/4} \sqrt{x}} - \frac{2 b^2 \sqrt{d x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{3 c^{3/4} \sqrt{x}} - \frac{1}{3} b^2 x \sqrt{d x} \operatorname{Log}\left[1-c x^2\right] \operatorname{Log}\left[1+c x^2\right] + \\
& \frac{1}{6} b^2 x \sqrt{d x} \operatorname{Log}\left[1+c x^2\right]^2 + \frac{2 b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1-(-c)^{1/4} \sqrt{x}}\right]}{3(-c)^{3/4} \sqrt{x}} - \frac{2 i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1-i(-c)^{1/4} \sqrt{x}}\right]}{3(-c)^{3/4} \sqrt{x}} + \\
& \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1+\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}-(-c)^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1-\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{c}}+(-c)^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} - \\
& \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1-\frac{(1+i)\left(1-(-c)^{1/4} \sqrt{x}\right)}{1-i(-c)^{1/4} \sqrt{x}}\right]}{3(-c)^{3/4} \sqrt{x}} - \frac{2 i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1+i(-c)^{1/4} \sqrt{x}}\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1+(-c)^{1/4} \sqrt{x}}\right]}{3(-c)^{3/4} \sqrt{x}} +
\end{aligned}$$

$$\begin{aligned}
& \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 + \frac{2(-c)^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} \\
& \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 + \frac{2(-c)^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} - \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} \\
& \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i) \left(1 + (-c)^{1/4} \sqrt{x}\right)}{1-i(-c)^{1/4} \sqrt{x}}\right]}{3(-c)^{3/4} \sqrt{x}} + \frac{2 b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} + \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1 - c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4} - i c^{1/4}\right) \left(1 - i(-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} \\
& \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1 - c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4} - c^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{3(-c)^{3/4} \sqrt{x}} - \frac{2 i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} + \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1 - i c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1 - i c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - (-c)^{1/4} \sqrt{x}\right)}{\left(i(-c)^{1/4} - c^{1/4}\right) \left(1 - i c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + (-c)^{1/4} \sqrt{x}\right)}{\left(i(-c)^{1/4} + c^{1/4}\right) \left(1 - i c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} - \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{(1+i) \left(1 - c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} - \frac{2 i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{2 b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}} - \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} - \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \\
& \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} + \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}} - \frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - (-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4} - c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{3 c^{3/4} \sqrt{x}}
\end{aligned}$$

$$\frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} (1 + (-c)^{1/4} \sqrt{x})}{((-c)^{1/4} + c^{1/4}) (1 + c^{1/4} \sqrt{x})}\right]}{3 c^{3/4} \sqrt{x}} + \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} (1 + c^{1/4} \sqrt{x})}{((-c)^{1/4} + i c^{1/4}) (1 - i (-c)^{1/4} \sqrt{x})}\right]}{3 (-c)^{3/4} \sqrt{x}} -$$

$$\frac{b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} (1 + c^{1/4} \sqrt{x})}{((-c)^{1/4} + c^{1/4}) (1 + (-c)^{1/4} \sqrt{x})}\right]}{3 (-c)^{3/4} \sqrt{x}} - \frac{i b^2 \sqrt{d x} \operatorname{PolyLog}\left[2, 1 - \frac{(1 - i) (1 + c^{1/4} \sqrt{x})}{1 - i c^{1/4} \sqrt{x}}\right]}{3 c^{3/4} \sqrt{x}}$$

Result (type 1, 1 leaves):

???

Problem 91: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{\sqrt{d x}} dx$$

Optimal (type 4, 6177 leaves, 241 steps):

$$\frac{2 a^2 x}{\sqrt{d x}} - \frac{2 \sqrt{2} a b \sqrt{x} \operatorname{ArcTan}\left[1 - \sqrt{2} c^{1/4} \sqrt{x}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 \sqrt{2} a b \sqrt{x} \operatorname{ArcTan}\left[1 + \sqrt{2} c^{1/4} \sqrt{x}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 i b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right]^2}{(-c)^{1/4} \sqrt{d x}} -$$

$$\frac{4 a b \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 i b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right]^2}{c^{1/4} \sqrt{d x}} - \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right]^2}{(-c)^{1/4} \sqrt{d x}} - \frac{4 a b \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right]}{c^{1/4} \sqrt{d x}} -$$

$$\frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right]^2}{c^{1/4} \sqrt{d x}} + \frac{4 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1 - (-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} - \frac{4 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} +$$

$$\frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 (-c)^{1/4} (1 - \sqrt{-c} \sqrt{x})}{(i \sqrt{-c} - (-c)^{1/4}) (1 - i (-c)^{1/4} \sqrt{x})}\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} (1 + \sqrt{-c} \sqrt{x})}{(i \sqrt{-c} + (-c)^{1/4}) (1 - i (-c)^{1/4} \sqrt{x})}\right]}{(-c)^{1/4} \sqrt{d x}} -$$

$$\frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1 + i) (1 - (-c)^{1/4} \sqrt{x})}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{4 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1 + i (-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} -$$

$$\frac{4 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1 + (-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} - \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 (-c)^{1/4} (1 - \sqrt{-c} \sqrt{x})}{(\sqrt{-c} - (-c)^{1/4}) (1 + (-c)^{1/4} \sqrt{x})}\right]}{(-c)^{1/4} \sqrt{d x}} -$$

$$\begin{aligned}
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} + \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} - \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1-i)\left(1+(-c)^{1/4} \sqrt{x}\right)}{1-i(-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}} + \\
& \frac{4 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-i c^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} + \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}} - \frac{4 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} + \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} + \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-(-c)^{1/4} \sqrt{x}\right)}{\left(i(-c)^{1/4}-c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+(-c)^{1/4} \sqrt{x}\right)}{\left(i(-c)^{1/4}+c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} - \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1+i)\left(1-c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} + \frac{4 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} - \frac{4 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}} + \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} + \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} - \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} - \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{d x}} +
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-(-c)^{1/4} \sqrt{x}\right)}{(-c)^{1/4}-c^{1/4}}\left(1+c^{1/4} \sqrt{x}\right)\right]}{c^{1/4} \sqrt{d x}}+\frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+(-c)^{1/4} \sqrt{x}\right)}{(-c)^{1/4}+c^{1/4}}\left(1+c^{1/4} \sqrt{x}\right)\right]}{c^{1/4} \sqrt{d x}}+ \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+c^{1/4} \sqrt{x}\right)}{(-c)^{1/4}+i c^{1/4}}\left(1-i(-c)^{1/4} \sqrt{x}\right)\right]}{(-c)^{1/4} \sqrt{d x}}+\frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+c^{1/4} \sqrt{x}\right)}{(-c)^{1/4}+c^{1/4}}\left(1+(-c)^{1/4} \sqrt{x}\right)\right]}{(-c)^{1/4} \sqrt{d x}}- \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1-i)\left(1+c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{d x}}-\frac{\sqrt{2} a b \sqrt{x} \operatorname{Log}\left[1-\sqrt{2} c^{1/4} \sqrt{x}+\sqrt{c} x\right]}{c^{1/4} \sqrt{d x}}+\frac{\sqrt{2} a b \sqrt{x} \operatorname{Log}\left[1+\sqrt{2} c^{1/4} \sqrt{x}+\sqrt{c} x\right]}{c^{1/4} \sqrt{d x}}- \\
& \frac{2 a b x \operatorname{Log}\left[1-c x^2\right]}{\sqrt{d x}}-\frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{(-c)^{1/4} \sqrt{d x}}+\frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{c^{1/4} \sqrt{d x}}- \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{(-c)^{1/4} \sqrt{d x}}+\frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{c^{1/4} \sqrt{d x}}+\frac{b^2 x \operatorname{Log}\left[1-c x^2\right]^2}{2 \sqrt{d x}}+\frac{2 a b x \operatorname{Log}\left[1+c x^2\right]}{\sqrt{d x}}+ \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{(-c)^{1/4} \sqrt{d x}}-\frac{2 b^2 \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{c^{1/4} \sqrt{d x}}+\frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{(-c)^{1/4} \sqrt{d x}}- \\
& \frac{2 b^2 \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{c^{1/4} \sqrt{d x}}-\frac{b^2 x \operatorname{Log}\left[1-c x^2\right] \operatorname{Log}\left[1+c x^2\right]}{\sqrt{d x}}+\frac{b^2 x \operatorname{Log}\left[1+c x^2\right]^2}{2 \sqrt{d x}}+\frac{2 b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1-(-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}}+ \\
& \frac{2 i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1-i(-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}}-\frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1+\frac{2(-c)^{1/4}\left(1-\sqrt{-c} \sqrt{x}\right)}{i \sqrt{-c}-(-c)^{1/4}}\left(1-i(-c)^{1/4} \sqrt{x}\right)\right]}{(-c)^{1/4} \sqrt{d x}}- \\
& \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2(-c)^{1/4}\left(1+\sqrt{-c} \sqrt{x}\right)}{i \sqrt{-c}+(-c)^{1/4}}\left(1-i(-c)^{1/4} \sqrt{x}\right)\right]}{(-c)^{1/4} \sqrt{d x}}+\frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{(1+i)\left(1-(-c)^{1/4} \sqrt{x}\right)}{1-i(-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}}+ \\
& \frac{2 i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1+i(-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}}+\frac{2 b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1+(-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{d x}}+\frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1+\frac{2(-c)^{1/4}\left(1-\sqrt{-c} \sqrt{x}\right)}{\left(\sqrt{-c}-(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{d x}}+
\end{aligned}$$

$$\begin{aligned}
& \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{dx}} - \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2(-c)^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{dx}} - \\
& \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{dx}} + \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i) \left(1 + (-c)^{1/4} \sqrt{x}\right)}{1-i(-c)^{1/4} \sqrt{x}}\right]}{(-c)^{1/4} \sqrt{dx}} + \frac{2 b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{dx}} - \\
& \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1 - c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4} - i c^{1/4}\right) \left(1 - i(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{dx}} - \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1 - c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4} - c^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{dx}} + \frac{2 i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{dx}} - \\
& \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1 - i c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{dx}} - \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1 - i c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{dx}} - \\
& \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - (-c)^{1/4} \sqrt{x}\right)}{\left(i(-c)^{1/4} - c^{1/4}\right) \left(1 - i c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{dx}} - \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + (-c)^{1/4} \sqrt{x}\right)}{\left(i(-c)^{1/4} + c^{1/4}\right) \left(1 - i c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{dx}} + \\
& \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1+i) \left(1 - c^{1/4} \sqrt{x}\right)}{1+i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{dx}} + \frac{2 i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{dx}} + \frac{2 b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{dx}} - \\
& \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{dx}} - \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{dx}} + \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{dx}} + \\
& \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{dx}} - \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - (-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4} - c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{dx}} - \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + (-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4} + c^{1/4}\right) \left(1 + c^{1/4} \sqrt{x}\right)}\right]}{c^{1/4} \sqrt{dx}} - \\
& \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1 + c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4} + i c^{1/4}\right) \left(1 - i(-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{dx}} - \frac{b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1 + c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4} + c^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{(-c)^{1/4} \sqrt{dx}} + \frac{i b^2 \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i) \left(1 + c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{c^{1/4} \sqrt{dx}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 92: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{(d x)^{3/2}} dx$$

Optimal (type 4, 6334 leaves, 197 steps):

$$\begin{aligned} & - \frac{2 \sqrt{2} a b c^{1/4} \sqrt{x} \operatorname{ArcTan}[1 - \sqrt{2} c^{1/4} \sqrt{x}]}{d \sqrt{d x}} + \frac{2 \sqrt{2} a b c^{1/4} \sqrt{x} \operatorname{ArcTan}[1 + \sqrt{2} c^{1/4} \sqrt{x}]}{d \sqrt{d x}} + \\ & \frac{2 i b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}[(-c)^{1/4} \sqrt{x}]^2}{d \sqrt{d x}} + \frac{2 i b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}[c^{1/4} \sqrt{x}]^2}{d \sqrt{d x}} + \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}[(-c)^{1/4} \sqrt{x}]^2}{d \sqrt{d x}} + \\ & \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}[c^{1/4} \sqrt{x}]^2}{d \sqrt{d x}} - \frac{4 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[\frac{2}{1 - (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \\ & \frac{4 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[\frac{2}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[-\frac{2 (-c)^{1/4} (1 - \sqrt{-\sqrt{-c}} \sqrt{x})}{(i \sqrt{-\sqrt{-c}} - (-c)^{1/4}) (1 - i (-c)^{1/4} \sqrt{x})}\right]}{d \sqrt{d x}} + \\ & \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[\frac{2 (-c)^{1/4} (1 + \sqrt{-\sqrt{-c}} \sqrt{x})}{(i \sqrt{-\sqrt{-c}} + (-c)^{1/4}) (1 - i (-c)^{1/4} \sqrt{x})}\right]}{d \sqrt{d x}} - \\ & \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[\frac{(1+i) (1 - (-c)^{1/4} \sqrt{x})}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \frac{4 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[\frac{2}{1 + i (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \\ & \frac{4 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[\frac{2}{1 + (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}[(-c)^{1/4} \sqrt{x}] \operatorname{Log}\left[-\frac{2 (-c)^{1/4} (1 - \sqrt{-\sqrt{-c}} \sqrt{x})}{(\sqrt{-\sqrt{-c}} - (-c)^{1/4}) (1 + (-c)^{1/4} \sqrt{x})}\right]}{d \sqrt{d x}} + \end{aligned}$$

$$\frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} \left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}$$

$$\frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 (-c)^{1/4} \left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}$$

$$\frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} \left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1-i) \left(1+(-c)^{1/4} \sqrt{x}\right)}{1-i (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}}$$

$$\frac{4 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} \left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-i c^{1/4}\right) \left(1-i (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}$$

$$\frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 (-c)^{1/4} \left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right) \left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \frac{4 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} +$$

$$\frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}-c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}+c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} +$$

$$\frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left(i (-c)^{1/4}-c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{\left(i (-c)^{1/4}+c^{1/4}\right) \left(1-i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} -$$

$$\frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1+i) \left(1-c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \frac{4 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} +$$

$$\frac{4 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right) \left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}}$$

$$\begin{aligned}
& \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \\
& \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{2 b^2(-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+i c^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \\
& \frac{2 b^2(-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1-i)\left(1+c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \\
& \frac{\sqrt{2} a b c^{1/4} \sqrt{x} \operatorname{Log}\left[1-\sqrt{2} c^{1/4} \sqrt{x}+\sqrt{c} x\right]}{d \sqrt{d x}} - \frac{\sqrt{2} a b c^{1/4} \sqrt{x} \operatorname{Log}\left[1+\sqrt{2} c^{1/4} \sqrt{x}+\sqrt{c} x\right]}{d \sqrt{d x}} - \\
& \frac{2 b^2(-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{d \sqrt{d x}} + \frac{2 b^2(-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1-c x^2\right]}{d \sqrt{d x}} - \\
& \frac{2 b c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right]\left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)}{d \sqrt{d x}} + \frac{2 b c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right]\left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)}{d \sqrt{d x}} - \frac{\left(2 a-b \operatorname{Log}\left[1-c x^2\right]\right)^2}{2 d \sqrt{d x}} - \\
& \frac{2 a b \operatorname{Log}\left[1+c x^2\right]}{d \sqrt{d x}} + \frac{2 b^2(-c)^{1/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{d \sqrt{d x}} - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{d \sqrt{d x}} - \\
& \frac{2 b^2(-c)^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{d \sqrt{d x}} + \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1+c x^2\right]}{d \sqrt{d x}} + \\
& \frac{b^2 \operatorname{Log}\left[1-c x^2\right] \operatorname{Log}\left[1+c x^2\right]}{d \sqrt{d x}} - \frac{b^2 \operatorname{Log}\left[1+c x^2\right]^2}{2 d \sqrt{d x}} - \frac{2 b^2(-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1-(-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \\
& \frac{2 i b^2(-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1-\frac{2}{1-i(-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \frac{i b^2(-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1+\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{i b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} + (-c)^{1/4}\right) \left(1 - i (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{i b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1+i) \left(1 - (-c)^{1/4} \sqrt{x}\right)}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \\
& \frac{2 i b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \frac{2 b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+(-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \\
& \frac{b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2(-c)^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \frac{b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& \frac{b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2(-c)^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} - (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}} + (-c)^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& \frac{i b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i) \left(1 + (-c)^{1/4} \sqrt{x}\right)}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \\
& \frac{i b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1 - c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4} - i c^{1/4}\right) \left(1 - i (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} \left(1 - c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4} - c^{1/4}\right) \left(1 + (-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& \frac{2 i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \frac{i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} - c^{1/4}\right) \left(1 - i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \\
& \frac{i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + \sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}} + c^{1/4}\right) \left(1 - i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \frac{i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1 - (-c)^{1/4} \sqrt{x}\right)}{\left(i (-c)^{1/4} - c^{1/4}\right) \left(1 - i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \\
& \frac{i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1 + (-c)^{1/4} \sqrt{x}\right)}{\left(i (-c)^{1/4} + c^{1/4}\right) \left(1 - i c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1+i) \left(1 - c^{1/4} \sqrt{x}\right)}{1 - i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} +
\end{aligned}$$

$$\begin{aligned}
& \frac{2 i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} - \frac{2 b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}} + \frac{b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& \frac{b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \frac{b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \\
& \frac{b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4} \left(1-(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& \frac{b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4} \left(1+(-c)^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} - \frac{i b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+i c^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \\
& \frac{b^2 (-c)^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} \left(1+c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}+c^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{d \sqrt{d x}} + \frac{i b^2 c^{1/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i)\left(1+c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{d \sqrt{d x}}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 93: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{(d x)^{5/2}} dx$$

Optimal (type 4, 6520 leaves, 197 steps):

$$\begin{aligned}
& - \frac{2 \sqrt{2} a b c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[1 - \sqrt{2} c^{1/4} \sqrt{x}\right]}{3 d^2 \sqrt{d x}} + \frac{2 \sqrt{2} a b c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[1 + \sqrt{2} c^{1/4} \sqrt{x}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 i b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}} - \frac{2 i b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}} + \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}} +
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right]^2}{3 d^2 \sqrt{d x}} - \frac{4 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-(-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{4 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-i(-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1+i)\left(1-(-c)^{1/4} \sqrt{x}\right)}{1-i(-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{4 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+i(-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{4 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+(-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2(-c)^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+(-c)^{1/4}\right)\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} + \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1-i)\left(1+(-c)^{1/4} \sqrt{x}\right)}{1-i(-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{4 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1-c^{1/4} \sqrt{x}\right)}{\left((-c)^{1/4}-i c^{1/4}\right)\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1-c^{1/4} \sqrt{x}\right)}{(-c)^{1/4}-c^{1/4}}\left(1+(-c)^{1/4} \sqrt{x}\right)\right]}{3 d^2 \sqrt{d x}} + \frac{4 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1-i c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(i \sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-(-c)^{1/4} \sqrt{x}\right)}{\left(i(-c)^{1/4}-c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+(-c)^{1/4} \sqrt{x}\right)}{\left(i(-c)^{1/4}+c^{1/4}\right)\left(1-i c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1+i)\left(1-c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{4 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+i c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{4 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2}{1+c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} + \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}-c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+\sqrt{-\sqrt{-c}} \sqrt{x}\right)}{\left(\sqrt{-\sqrt{-c}}+c^{1/4}\right)\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[-\frac{2 c^{1/4}\left(1-(-c)^{1/4} \sqrt{x}\right)}{\left(-c)^{1/4}-c^{1/4}}\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2 c^{1/4}\left(1+(-c)^{1/4} \sqrt{x}\right)}{\left(-c)^{1/4}+c^{1/4}}\left(1+c^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+c^{1/4} \sqrt{x}\right)}{\left(-c)^{1/4}+i c^{1/4}}\left(1-i(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{2(-c)^{1/4}\left(1+c^{1/4} \sqrt{x}\right)}{\left(-c)^{1/4}+c^{1/4}}\left(1+(-c)^{1/4} \sqrt{x}\right)}\right]}{3 d^2 \sqrt{d x}} + \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[\frac{(1-i)\left(1+c^{1/4} \sqrt{x}\right)}{1-i c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{2} a b c^{3/4} \sqrt{x} \operatorname{Log}\left[1 - \sqrt{2} c^{1/4} \sqrt{x} + \sqrt{c} x\right]}{3 d^2 \sqrt{d x}} + \frac{\sqrt{2} a b c^{3/4} \sqrt{x} \operatorname{Log}\left[1 + \sqrt{2} c^{1/4} \sqrt{x} + \sqrt{c} x\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1 - c x^2\right]}{3 d^2 \sqrt{d x}} + \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1 - c x^2\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{2 b c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] (2 a - b \operatorname{Log}\left[1 - c x^2\right])}{3 d^2 \sqrt{d x}} + \frac{2 b c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] (2 a - b \operatorname{Log}\left[1 - c x^2\right])}{3 d^2 \sqrt{d x}} - \frac{(2 a - b \operatorname{Log}\left[1 - c x^2\right])^2}{6 d^2 x \sqrt{d x}} - \\
& \frac{2 a b \operatorname{Log}\left[1 + c x^2\right]}{3 d^2 x \sqrt{d x}} - \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTan}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1 + c x^2\right]}{3 d^2 \sqrt{d x}} + \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTan}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1 + c x^2\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[(-c)^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1 + c x^2\right]}{3 d^2 \sqrt{d x}} + \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{ArcTanh}\left[c^{1/4} \sqrt{x}\right] \operatorname{Log}\left[1 + c x^2\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{b^2 \operatorname{Log}\left[1 - c x^2\right] \operatorname{Log}\left[1 + c x^2\right]}{3 d^2 x \sqrt{d x}} - \frac{b^2 \operatorname{Log}\left[1 + c x^2\right]^2}{6 d^2 x \sqrt{d x}} - \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \frac{i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 (-c)^{1/4} (1 - \sqrt{-c} \sqrt{x})}{(i \sqrt{-c} - (-c)^{1/4}) (1 - i (-c)^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} (1 + \sqrt{-c} \sqrt{x})}{(i \sqrt{-c} + (-c)^{1/4}) (1 - i (-c)^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \frac{i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1+i) (1 - (-c)^{1/4} \sqrt{x})}{1 - i (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + i (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + (-c)^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 (-c)^{1/4} (1 - \sqrt{-c} \sqrt{x})}{(\sqrt{-c} - (-c)^{1/4}) (1 + (-c)^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \frac{b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} (1 + \sqrt{-c} \sqrt{x})}{(\sqrt{-c} + (-c)^{1/4}) (1 + (-c)^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 (-c)^{1/4} (1 - \sqrt{-c} \sqrt{x})}{(\sqrt{-c} - (-c)^{1/4}) (1 + (-c)^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \frac{b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 (-c)^{1/4} (1 + \sqrt{-c} \sqrt{x})}{(\sqrt{-c} + (-c)^{1/4}) (1 + (-c)^{1/4} \sqrt{x})}\right]}{3 d^2 \sqrt{d x}} -
\end{aligned}$$

$$\begin{aligned}
& \frac{i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i)(1+(-c)^{1/4}\sqrt{x})}{1-i(-c)^{1/4}\sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c^{1/4}\sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4}(1-c^{1/4}\sqrt{x})}{((-c)^{1/4}-i c^{1/4})(1-i(-c)^{1/4}\sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \frac{b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4}(1-c^{1/4}\sqrt{x})}{((-c)^{1/4}-c^{1/4})(1+(-c)^{1/4}\sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-i c^{1/4}\sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \frac{i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4}(1-\sqrt{-\sqrt{-c}}\sqrt{x})}{(i\sqrt{-\sqrt{-c}}-c^{1/4})(1-i c^{1/4}\sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4}(1+\sqrt{-\sqrt{-c}}\sqrt{x})}{(i\sqrt{-\sqrt{-c}}+c^{1/4})(1-i c^{1/4}\sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \frac{i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4}(1-(-c)^{1/4}\sqrt{x})}{(i(-c)^{1/4}-c^{1/4})(1-i c^{1/4}\sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4}(1+(-c)^{1/4}\sqrt{x})}{(i(-c)^{1/4}+c^{1/4})(1-i c^{1/4}\sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \frac{i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1+i)(1-c^{1/4}\sqrt{x})}{1-i c^{1/4}\sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{2 i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+i c^{1/4}\sqrt{x}}\right]}{3 d^2 \sqrt{d x}} - \frac{2 b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c^{1/4}\sqrt{x}}\right]}{3 d^2 \sqrt{d x}} + \frac{b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4}(1-\sqrt{-\sqrt{-c}}\sqrt{x})}{(\sqrt{-\sqrt{-c}}-c^{1/4})(1+c^{1/4}\sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4}(1+\sqrt{-\sqrt{-c}}\sqrt{x})}{(\sqrt{-\sqrt{-c}}+c^{1/4})(1+c^{1/4}\sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \frac{b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4}(1-\sqrt{-\sqrt{-c}}\sqrt{x})}{(\sqrt{-\sqrt{-c}}-c^{1/4})(1+c^{1/4}\sqrt{x})}\right]}{3 d^2 \sqrt{d x}} - \\
& \frac{b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4}(1+\sqrt{-\sqrt{-c}}\sqrt{x})}{(\sqrt{-\sqrt{-c}}+c^{1/4})(1+c^{1/4}\sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \frac{b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 + \frac{2 c^{1/4}(1-(-c)^{1/4}\sqrt{x})}{((-c)^{1/4}-c^{1/4})(1+c^{1/4}\sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \\
& \frac{b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2 c^{1/4}(1+(-c)^{1/4}\sqrt{x})}{((-c)^{1/4}+c^{1/4})(1+c^{1/4}\sqrt{x})}\right]}{3 d^2 \sqrt{d x}} + \frac{i b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4}(1+c^{1/4}\sqrt{x})}{((-c)^{1/4}+i c^{1/4})(1-i(-c)^{1/4}\sqrt{x})}\right]}{3 d^2 \sqrt{d x}} +
\end{aligned}$$

$$\frac{b^2 (-c)^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{2(-c)^{1/4} (1+c^{1/4} \sqrt{x})}{(-c)^{1/4} + c^{1/4}} (1+(-c)^{1/4} \sqrt{x})\right]}{3 d^2 \sqrt{d x}} - \frac{i b^2 c^{3/4} \sqrt{x} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i) (1+c^{1/4} \sqrt{x})}{1-i c^{1/4} \sqrt{x}}\right]}{3 d^2 \sqrt{d x}}$$

Result (type 1, 1 leaves):

???

Problem 120: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^3])^2}{x} dx$$

Optimal (type 4, 140 leaves, 7 steps):

$$\begin{aligned} & \frac{2}{3} (a + b \operatorname{ArcTanh}[c x^3])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x^3}\right] - \frac{1}{3} b (a + b \operatorname{ArcTanh}[c x^3]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x^3}\right] + \\ & \frac{1}{3} b (a + b \operatorname{ArcTanh}[c x^3]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x^3}\right] + \frac{1}{6} b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x^3}\right] - \frac{1}{6} b^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x^3}\right] \end{aligned}$$

Result (type 4, 181 leaves):

$$\begin{aligned} & a^2 \operatorname{Log}[x] + \frac{1}{3} a b (-\operatorname{PolyLog}[2, -c x^3] + \operatorname{PolyLog}[2, c x^3]) + \\ & \frac{1}{3} b^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c x^3]^3 - \operatorname{ArcTanh}[c x^3]^2 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c x^3]}] + \operatorname{ArcTanh}[c x^3]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c x^3]}] + \operatorname{ArcTanh}[c x^3] \right. \\ & \left. \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x^3]}] + \operatorname{ArcTanh}[c x^3] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x^3]}] + \frac{1}{2} \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c x^3]}] - \frac{1}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x^3]}] \right) \end{aligned}$$

Problem 127: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^3])^3}{x} dx$$

Optimal (type 4, 210 leaves, 9 steps):

$$\begin{aligned} & \frac{2}{3} (a + b \operatorname{ArcTanh}[c x^3])^3 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x^3}\right] - \frac{1}{2} b (a + b \operatorname{ArcTanh}[c x^3])^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x^3}\right] + \\ & \frac{1}{2} b (a + b \operatorname{ArcTanh}[c x^3])^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x^3}\right] + \frac{1}{2} b^2 (a + b \operatorname{ArcTanh}[c x^3]) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x^3}\right] - \\ & \frac{1}{2} b^2 (a + b \operatorname{ArcTanh}[c x^3]) \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x^3}\right] - \frac{1}{4} b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2}{1 - c x^3}\right] + \frac{1}{4} b^3 \operatorname{PolyLog}\left[4, -1 + \frac{2}{1 - c x^3}\right] \end{aligned}$$

Result (type 4, 368 leaves):

$$\begin{aligned}
& a^3 \operatorname{Log}[x] + \frac{1}{2} a^2 b \left(-\operatorname{PolyLog}[2, -c x^3] + \operatorname{PolyLog}[2, c x^3] \right) + \\
& a b^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c x^3]^3 - \operatorname{ArcTanh}[c x^3]^2 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c x^3]}] + \operatorname{ArcTanh}[c x^3]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c x^3]}] + \operatorname{ArcTanh}[c x^3] \right. \\
& \quad \left. \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x^3]}] + \operatorname{ArcTanh}[c x^3] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x^3]}] + \frac{1}{2} \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c x^3]}] - \frac{1}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x^3]}] \right) + \\
& \frac{1}{192} b^3 \left(\pi^4 - 32 \operatorname{ArcTanh}[c x^3]^4 - 64 \operatorname{ArcTanh}[c x^3]^3 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c x^3]}] + 64 \operatorname{ArcTanh}[c x^3]^3 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c x^3]}] + \right. \\
& \quad 96 \operatorname{ArcTanh}[c x^3]^2 \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x^3]}] + 96 \operatorname{ArcTanh}[c x^3]^2 \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x^3]}] + 96 \operatorname{ArcTanh}[c x^3] \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c x^3]}] - \\
& \quad \left. 96 \operatorname{ArcTanh}[c x^3] \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x^3]}] + 48 \operatorname{PolyLog}[4, -e^{-2 \operatorname{ArcTanh}[c x^3]}] + 48 \operatorname{PolyLog}[4, e^{2 \operatorname{ArcTanh}[c x^3]}] \right)
\end{aligned}$$

Problem 128: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^3])^3}{x^4} dx$$

Optimal (type 4, 120 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{3} c (a + b \operatorname{ArcTanh}[c x^3])^3 - \frac{(a + b \operatorname{ArcTanh}[c x^3])^3}{3 x^3} + b c (a + b \operatorname{ArcTanh}[c x^3])^2 \operatorname{Log}\left[2 - \frac{2}{1 + c x^3}\right] - \\
& b^2 c (a + b \operatorname{ArcTanh}[c x^3]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x^3}\right] - \frac{1}{2} b^3 c \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 + c x^3}\right]
\end{aligned}$$

Result (type 4, 223 leaves):

$$\begin{aligned}
& -\frac{a^3}{3 x^3} - \frac{a^2 b \operatorname{ArcTanh}[c x^3]}{x^3} + 3 a^2 b c \operatorname{Log}[x] - \frac{1}{2} a^2 b c \operatorname{Log}[1 - c^2 x^6] + \\
& a b^2 c \left(\operatorname{ArcTanh}[c x^3] \left(\left(1 - \frac{1}{c x^3}\right) \operatorname{ArcTanh}[c x^3] + 2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcTanh}[c x^3]}] \right) - \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcTanh}[c x^3]}] \right) + \\
& \frac{1}{3} b^3 c \left(\frac{i \pi^3}{8} - \operatorname{ArcTanh}[c x^3]^3 - \frac{\operatorname{ArcTanh}[c x^3]^3}{c x^3} + 3 \operatorname{ArcTanh}[c x^3]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c x^3]}] + \right. \\
& \quad \left. 3 \operatorname{ArcTanh}[c x^3] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x^3]}] - \frac{3}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x^3]}] \right)
\end{aligned}$$

Problem 147: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{c}{x}\right]\right)^2}{x} dx$$

Optimal (type 4, 133 leaves, 7 steps):

$$\begin{aligned} & -2 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - \frac{c}{x}}\right] + b \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - \frac{c}{x}}\right] - \\ & b \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - \frac{c}{x}}\right] - \frac{1}{2} b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - \frac{c}{x}}\right] + \frac{1}{2} b^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - \frac{c}{x}}\right] \end{aligned}$$

Result (type 4, 177 leaves):

$$\begin{aligned} & a^2 \operatorname{Log}[x] + a b \left(\operatorname{PolyLog}\left[2, -\frac{c}{x}\right] - \operatorname{PolyLog}\left[2, \frac{c}{x}\right]\right) + \\ & b^2 \left(-\frac{i \pi^3}{24} + \frac{2}{3} \operatorname{ArcTanh}\left[\frac{c}{x}\right]^3 + \operatorname{ArcTanh}\left[\frac{c}{x}\right]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - \operatorname{ArcTanh}\left[\frac{c}{x}\right]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - \right. \\ & \left. \operatorname{ArcTanh}\left[\frac{c}{x}\right] \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - \operatorname{ArcTanh}\left[\frac{c}{x}\right] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - \frac{1}{2} \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] + \frac{1}{2} \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right]\right) \end{aligned}$$

Problem 151: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \left(a + b \operatorname{ArcTanh}\left[\frac{c}{x}\right]\right)^3 dx$$

Optimal (type 4, 217 leaves, 15 steps):

$$\begin{aligned} & b^2 c^2 x \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right) - \frac{1}{2} b c^3 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 + \frac{1}{2} b c x^2 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 - \\ & \frac{1}{3} c^3 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^3 + \frac{1}{3} x^3 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^3 - b c^3 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 \operatorname{Log}\left[2 - \frac{2}{1 + \frac{c}{x}}\right] + \\ & \frac{1}{2} b^3 c^3 \operatorname{Log}\left[1 - \frac{c^2}{x^2}\right] + b^3 c^3 \operatorname{Log}[x] + b^2 c^3 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + \frac{c}{x}}\right] + \frac{1}{2} b^3 c^3 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 + \frac{c}{x}}\right] \end{aligned}$$

Result (type 4, 316 leaves):

$$\frac{1}{6} \left(3 a^2 b c x^2 + 2 a^3 x^3 + 6 a^2 b x^3 \operatorname{ArcTanh}\left[\frac{c}{x}\right] + 3 a^2 b c^3 \operatorname{Log}\left[-c^2 + x^2\right] + \right.$$

$$6 a b^2 \left(c^2 x + (-c^3 + x^3) \operatorname{ArcTanh}\left[\frac{c}{x}\right]^2 + c \operatorname{ArcTanh}\left[\frac{c}{x}\right] \left(-c^2 + x^2 - 2 c^2 \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] \right) + c^3 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] \right) +$$

$$\frac{1}{4} b^3 \left(-i c^3 \pi^3 + 24 c^2 x \operatorname{ArcTanh}\left[\frac{c}{x}\right] - 12 c^3 \operatorname{ArcTanh}\left[\frac{c}{x}\right]^2 + 12 c x^2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]^2 + 8 c^3 \operatorname{ArcTanh}\left[\frac{c}{x}\right]^3 + 8 x^3 \operatorname{ArcTanh}\left[\frac{c}{x}\right]^3 - \right.$$

$$\left. \left. 24 c^3 \operatorname{ArcTanh}\left[\frac{c}{x}\right]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - 24 c^3 \operatorname{Log}\left[\frac{c}{\sqrt{1 - \frac{c^2}{x^2}} x}\right] - 24 c^3 \operatorname{ArcTanh}\left[\frac{c}{x}\right] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] + 12 c^3 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] \right) \right)$$

Problem 153: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(a + b \operatorname{ArcTanh}\left[\frac{c}{x}\right] \right)^3 dx$$

Optimal (type 4, 108 leaves, 6 steps):

$$c \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right] \right)^3 + x \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right] \right)^3 - 3 b c \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right] \right)^2 \operatorname{Log}\left[\frac{2 c}{c - x}\right] -$$

$$3 b^2 c \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right] \right) \operatorname{PolyLog}\left[2, 1 - \frac{2 c}{c - x}\right] + \frac{3}{2} b^3 c \operatorname{PolyLog}\left[3, 1 - \frac{2 c}{c - x}\right]$$

Result (type 4, 198 leaves):

$$a^3 x + 3 a^2 b x \operatorname{ArcTanh}\left[\frac{c}{x}\right] + \frac{3}{2} a^2 b c \operatorname{Log}\left[-c^2 + x^2\right] -$$

$$3 a b^2 \left(\operatorname{ArcTanh}\left[\frac{c}{x}\right] \left((c - x) \operatorname{ArcTanh}\left[\frac{c}{x}\right] + 2 c \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] \right) - c \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] \right) +$$

$$\frac{1}{8} b^3 \left(-i c \pi^3 + 8 c \operatorname{ArcTanh}\left[\frac{c}{x}\right]^3 + 8 x \operatorname{ArcTanh}\left[\frac{c}{x}\right]^3 - \right.$$

$$\left. 24 c \operatorname{ArcTanh}\left[\frac{c}{x}\right]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - 24 c \operatorname{ArcTanh}\left[\frac{c}{x}\right] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] + 12 c \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] \right)$$

Problem 154: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{c}{x}\right]\right)^3}{x} dx$$

Optimal (type 4, 208 leaves, 9 steps):

$$\begin{aligned} & -2 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^3 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - \frac{c}{x}}\right] + \frac{3}{2} b \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - \frac{c}{x}}\right] - \\ & \frac{3}{2} b \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right)^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - \frac{c}{x}}\right] - \frac{3}{2} b^2 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - \frac{c}{x}}\right] + \\ & \frac{3}{2} b^2 \left(a + b \operatorname{ArcCoth}\left[\frac{x}{c}\right]\right) \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - \frac{c}{x}}\right] + \frac{3}{4} b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2}{1 - \frac{c}{x}}\right] - \frac{3}{4} b^3 \operatorname{PolyLog}\left[4, -1 + \frac{2}{1 - \frac{c}{x}}\right] \end{aligned}$$

Result (type 4, 373 leaves):

$$\begin{aligned} & a^3 \operatorname{Log}[x] + \frac{3}{2} a^2 b \left(\operatorname{PolyLog}\left[2, -\frac{c}{x}\right] - \operatorname{PolyLog}\left[2, \frac{c}{x}\right]\right) + \\ & 3 a b^2 \left(-\frac{i \pi^3}{24} + \frac{2}{3} \operatorname{ArcTanh}\left[\frac{c}{x}\right]^3 + \operatorname{ArcTanh}\left[\frac{c}{x}\right]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - \operatorname{ArcTanh}\left[\frac{c}{x}\right]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - \right. \\ & \left. \operatorname{ArcTanh}\left[\frac{c}{x}\right] \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - \operatorname{ArcTanh}\left[\frac{c}{x}\right] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - \frac{1}{2} \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] + \frac{1}{2} \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right]\right) + \\ & \frac{1}{64} b^3 \left(-\pi^4 + 32 \operatorname{ArcTanh}\left[\frac{c}{x}\right]^4 + 64 \operatorname{ArcTanh}\left[\frac{c}{x}\right]^3 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - 64 \operatorname{ArcTanh}\left[\frac{c}{x}\right]^3 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - \right. \\ & \left. 96 \operatorname{ArcTanh}\left[\frac{c}{x}\right]^2 \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - 96 \operatorname{ArcTanh}\left[\frac{c}{x}\right]^2 \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - 96 \operatorname{ArcTanh}\left[\frac{c}{x}\right] \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] + \right. \\ & \left. 96 \operatorname{ArcTanh}\left[\frac{c}{x}\right] \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - 48 \operatorname{PolyLog}\left[4, -e^{-2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right] - 48 \operatorname{PolyLog}\left[4, e^{2 \operatorname{ArcTanh}\left[\frac{c}{x}\right]}\right]\right) \end{aligned}$$

Problem 173: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2}{x} dx$$

Optimal (type 4, 144 leaves, 7 steps):

$$\begin{aligned}
& - \left(a + b \operatorname{ArcCoth} \left[\frac{x^2}{c} \right] \right)^2 \operatorname{ArcTanh} \left[1 - \frac{2}{1 - \frac{c}{x^2}} \right] + \frac{1}{2} b \left(a + b \operatorname{ArcCoth} \left[\frac{x^2}{c} \right] \right) \operatorname{PolyLog} \left[2, 1 - \frac{2}{1 - \frac{c}{x^2}} \right] - \\
& \frac{1}{2} b \left(a + b \operatorname{ArcCoth} \left[\frac{x^2}{c} \right] \right) \operatorname{PolyLog} \left[2, -1 + \frac{2}{1 - \frac{c}{x^2}} \right] - \frac{1}{4} b^2 \operatorname{PolyLog} \left[3, 1 - \frac{2}{1 - \frac{c}{x^2}} \right] + \frac{1}{4} b^2 \operatorname{PolyLog} \left[3, -1 + \frac{2}{1 - \frac{c}{x^2}} \right]
\end{aligned}$$

Result (type 4, 183 leaves):

$$\begin{aligned}
& a^2 \operatorname{Log}[x] + \frac{1}{2} a b \left(\operatorname{PolyLog} \left[2, -\frac{c}{x^2} \right] - \operatorname{PolyLog} \left[2, \frac{c}{x^2} \right] \right) + \\
& \frac{1}{2} b^2 \left(-\frac{i \pi^3}{24} + \frac{2}{3} \operatorname{ArcTanh} \left[\frac{c}{x^2} \right]^3 + \operatorname{ArcTanh} \left[\frac{c}{x^2} \right]^2 \operatorname{Log} \left[1 + e^{-2 \operatorname{ArcTanh} \left[\frac{c}{x^2} \right]} \right] - \operatorname{ArcTanh} \left[\frac{c}{x^2} \right]^2 \operatorname{Log} \left[1 - e^{2 \operatorname{ArcTanh} \left[\frac{c}{x^2} \right]} \right] - \right. \\
& \left. \operatorname{ArcTanh} \left[\frac{c}{x^2} \right] \operatorname{PolyLog} \left[2, -e^{-2 \operatorname{ArcTanh} \left[\frac{c}{x^2} \right]} \right] - \operatorname{ArcTanh} \left[\frac{c}{x^2} \right] \operatorname{PolyLog} \left[2, e^{2 \operatorname{ArcTanh} \left[\frac{c}{x^2} \right]} \right] - \frac{1}{2} \operatorname{PolyLog} \left[3, -e^{-2 \operatorname{ArcTanh} \left[\frac{c}{x^2} \right]} \right] + \frac{1}{2} \operatorname{PolyLog} \left[3, e^{2 \operatorname{ArcTanh} \left[\frac{c}{x^2} \right]} \right] \right)
\end{aligned}$$

Problem 176: Unable to integrate problem.

$$\int x^4 \left(a + b \operatorname{ArcTanh} \left[\frac{c}{x^2} \right] \right)^2 dx$$

Optimal (type 4, 1214 leaves, 98 steps):

$$\begin{aligned}
& \frac{8}{15} b^2 c^2 x + \frac{2}{15} a b c x^3 + \frac{2}{5} a b c^{5/2} \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] - \frac{4}{15} b^2 c^{5/2} \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] - \frac{1}{5} i b^2 c^{5/2} \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right]^2 - \\
& \frac{4}{15} b^2 c^{5/2} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] + \frac{1}{5} b^2 c^{5/2} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right]^2 + \frac{2}{5} b^2 c^{5/2} \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[2 - \frac{2\sqrt{c}}{\sqrt{c} - ix}\right] - \frac{1}{15} b^2 c x^3 \operatorname{Log}\left[1 - \frac{c}{x^2}\right] - \\
& \frac{1}{5} b^2 c^{5/2} \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[1 - \frac{c}{x^2}\right] + \frac{1}{15} b c x^3 \left(2a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right) - \frac{1}{5} b c^{5/2} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \left(2a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right) + \\
& \frac{1}{20} x^5 \left(2a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)^2 + \frac{2}{15} b^2 c x^3 \operatorname{Log}\left[1 + \frac{c}{x^2}\right] + \frac{1}{5} a b x^5 \operatorname{Log}\left[1 + \frac{c}{x^2}\right] + \frac{1}{5} b^2 c^{5/2} \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[1 + \frac{c}{x^2}\right] - \\
& \frac{1}{5} b^2 c^{5/2} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[1 + \frac{c}{x^2}\right] - \frac{1}{10} b^2 x^5 \operatorname{Log}\left[1 - \frac{c}{x^2}\right] \operatorname{Log}\left[1 + \frac{c}{x^2}\right] + \frac{1}{20} b^2 x^5 \operatorname{Log}\left[1 + \frac{c}{x^2}\right]^2 - \\
& \frac{2}{5} b^2 c^{5/2} \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{2\sqrt{c}}{\sqrt{c} - ix}\right] + \frac{1}{5} b^2 c^{5/2} \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{(1+i)(\sqrt{c}-x)}{\sqrt{c}-ix}\right] - \frac{2}{5} b^2 c^{5/2} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{2\sqrt{c}}{\sqrt{c}+x}\right] + \\
& \frac{1}{5} b^2 c^{5/2} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{2\sqrt{c}(\sqrt{c}-x)}{(\sqrt{c}-\sqrt{c})(\sqrt{c}+x)}\right] + \frac{1}{5} b^2 c^{5/2} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{2\sqrt{c}(\sqrt{c}+x)}{(\sqrt{c}+\sqrt{c})(\sqrt{c}+x)}\right] + \\
& \frac{1}{5} b^2 c^{5/2} \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{(1-i)(\sqrt{c}+x)}{\sqrt{c}-ix}\right] + \frac{2}{5} b^2 c^{5/2} \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[2 - \frac{2\sqrt{c}}{\sqrt{c}+x}\right] + \\
& \frac{1}{5} i b^2 c^{5/2} \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}}{\sqrt{c}-ix}\right] - \frac{1}{5} i b^2 c^{5/2} \operatorname{PolyLog}\left[2, -1 + \frac{2\sqrt{c}}{\sqrt{c}-ix}\right] - \frac{1}{10} i b^2 c^{5/2} \operatorname{PolyLog}\left[2, 1 - \frac{(1+i)(\sqrt{c}-x)}{\sqrt{c}-ix}\right] + \\
& \frac{1}{5} b^2 c^{5/2} \operatorname{PolyLog}\left[2, -\frac{x}{\sqrt{c}}\right] - \frac{1}{5} i b^2 c^{5/2} \operatorname{PolyLog}\left[2, -\frac{ix}{\sqrt{c}}\right] + \frac{1}{5} i b^2 c^{5/2} \operatorname{PolyLog}\left[2, \frac{ix}{\sqrt{c}}\right] - \frac{1}{5} b^2 c^{5/2} \operatorname{PolyLog}\left[2, \frac{x}{\sqrt{c}}\right] + \\
& \frac{1}{5} b^2 c^{5/2} \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}}{\sqrt{c}+x}\right] - \frac{1}{5} b^2 c^{5/2} \operatorname{PolyLog}\left[2, -1 + \frac{2\sqrt{c}}{\sqrt{c}+x}\right] - \frac{1}{10} b^2 c^{5/2} \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}(\sqrt{c}-x)}{(\sqrt{c}-\sqrt{c})(\sqrt{c}+x)}\right] - \\
& \frac{1}{10} b^2 c^{5/2} \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}(\sqrt{c}+x)}{(\sqrt{c}+\sqrt{c})(\sqrt{c}+x)}\right] - \frac{1}{10} i b^2 c^{5/2} \operatorname{PolyLog}\left[2, 1 - \frac{(1-i)(\sqrt{c}+x)}{\sqrt{c}-ix}\right]
\end{aligned}$$

Result (type 8, 18 leaves):

$$\int x^4 \left(a + b \operatorname{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2 dx$$

Problem 177: Unable to integrate problem.

$$\int x^2 \left(a + b \operatorname{ArcTanh} \left[\frac{c}{x^2} \right] \right)^2 dx$$

Optimal (type 4, 1172 leaves, 80 steps):

$$\begin{aligned} & \frac{4}{3} a b c x - \frac{2}{3} a b c^{3/2} \operatorname{ArcTan} \left[\frac{x}{\sqrt{c}} \right] + \frac{4}{3} b^2 c^{3/2} \operatorname{ArcTan} \left[\frac{x}{\sqrt{c}} \right] + \frac{1}{3} i b^2 c^{3/2} \operatorname{ArcTan} \left[\frac{x}{\sqrt{c}} \right]^2 - \frac{4}{3} b^2 c^{3/2} \operatorname{ArcTanh} \left[\frac{x}{\sqrt{c}} \right] + \\ & \frac{1}{3} b^2 c^{3/2} \operatorname{ArcTanh} \left[\frac{x}{\sqrt{c}} \right]^2 - \frac{2}{3} b^2 c^{3/2} \operatorname{ArcTan} \left[\frac{x}{\sqrt{c}} \right] \operatorname{Log} \left[2 - \frac{2\sqrt{c}}{\sqrt{c} - ix} \right] - \frac{2}{3} b^2 c x \operatorname{Log} \left[1 - \frac{c}{x^2} \right] + \frac{1}{3} b^2 c^{3/2} \operatorname{ArcTan} \left[\frac{x}{\sqrt{c}} \right] \operatorname{Log} \left[1 - \frac{c}{x^2} \right] - \\ & \frac{1}{3} b c^{3/2} \operatorname{ArcTanh} \left[\frac{x}{\sqrt{c}} \right] \left(2a - b \operatorname{Log} \left[1 - \frac{c}{x^2} \right] \right) + \frac{1}{12} x^3 \left(2a - b \operatorname{Log} \left[1 - \frac{c}{x^2} \right] \right)^2 + \frac{2}{3} b^2 c x \operatorname{Log} \left[1 + \frac{c}{x^2} \right] + \frac{1}{3} a b x^3 \operatorname{Log} \left[1 + \frac{c}{x^2} \right] - \\ & \frac{1}{3} b^2 c^{3/2} \operatorname{ArcTan} \left[\frac{x}{\sqrt{c}} \right] \operatorname{Log} \left[1 + \frac{c}{x^2} \right] - \frac{1}{3} b^2 c^{3/2} \operatorname{ArcTanh} \left[\frac{x}{\sqrt{c}} \right] \operatorname{Log} \left[1 + \frac{c}{x^2} \right] - \frac{1}{6} b^2 x^3 \operatorname{Log} \left[1 - \frac{c}{x^2} \right] \operatorname{Log} \left[1 + \frac{c}{x^2} \right] + \frac{1}{12} b^2 x^3 \operatorname{Log} \left[1 + \frac{c}{x^2} \right]^2 + \\ & \frac{2}{3} b^2 c^{3/2} \operatorname{ArcTan} \left[\frac{x}{\sqrt{c}} \right] \operatorname{Log} \left[\frac{2\sqrt{c}}{\sqrt{c} - ix} \right] - \frac{1}{3} b^2 c^{3/2} \operatorname{ArcTan} \left[\frac{x}{\sqrt{c}} \right] \operatorname{Log} \left[\frac{(1+i)(\sqrt{c}-x)}{\sqrt{c}-ix} \right] - \frac{2}{3} b^2 c^{3/2} \operatorname{ArcTanh} \left[\frac{x}{\sqrt{c}} \right] \operatorname{Log} \left[\frac{2\sqrt{c}}{\sqrt{c}+x} \right] + \\ & \frac{1}{3} b^2 c^{3/2} \operatorname{ArcTanh} \left[\frac{x}{\sqrt{c}} \right] \operatorname{Log} \left[\frac{2\sqrt{c}(\sqrt{c}-x)}{(\sqrt{c}-\sqrt{c})(\sqrt{c}+x)} \right] + \frac{1}{3} b^2 c^{3/2} \operatorname{ArcTanh} \left[\frac{x}{\sqrt{c}} \right] \operatorname{Log} \left[\frac{2\sqrt{c}(\sqrt{c}+x)}{(\sqrt{c}+\sqrt{c})(\sqrt{c}+x)} \right] - \\ & \frac{1}{3} b^2 c^{3/2} \operatorname{ArcTan} \left[\frac{x}{\sqrt{c}} \right] \operatorname{Log} \left[\frac{(1-i)(\sqrt{c}+x)}{\sqrt{c}-ix} \right] + \frac{2}{3} b^2 c^{3/2} \operatorname{ArcTanh} \left[\frac{x}{\sqrt{c}} \right] \operatorname{Log} \left[2 - \frac{2\sqrt{c}}{\sqrt{c}+x} \right] - \\ & \frac{1}{3} i b^2 c^{3/2} \operatorname{PolyLog} \left[2, 1 - \frac{2\sqrt{c}}{\sqrt{c}-ix} \right] + \frac{1}{3} i b^2 c^{3/2} \operatorname{PolyLog} \left[2, -1 + \frac{2\sqrt{c}}{\sqrt{c}-ix} \right] + \frac{1}{6} i b^2 c^{3/2} \operatorname{PolyLog} \left[2, 1 - \frac{(1+i)(\sqrt{c}-x)}{\sqrt{c}-ix} \right] + \\ & \frac{1}{3} b^2 c^{3/2} \operatorname{PolyLog} \left[2, -\frac{x}{\sqrt{c}} \right] + \frac{1}{3} i b^2 c^{3/2} \operatorname{PolyLog} \left[2, -\frac{ix}{\sqrt{c}} \right] - \frac{1}{3} i b^2 c^{3/2} \operatorname{PolyLog} \left[2, \frac{ix}{\sqrt{c}} \right] - \frac{1}{3} b^2 c^{3/2} \operatorname{PolyLog} \left[2, \frac{x}{\sqrt{c}} \right] + \\ & \frac{1}{3} b^2 c^{3/2} \operatorname{PolyLog} \left[2, 1 - \frac{2\sqrt{c}}{\sqrt{c}+x} \right] - \frac{1}{3} b^2 c^{3/2} \operatorname{PolyLog} \left[2, -1 + \frac{2\sqrt{c}}{\sqrt{c}+x} \right] - \frac{1}{6} b^2 c^{3/2} \operatorname{PolyLog} \left[2, 1 - \frac{2\sqrt{c}(\sqrt{c}-x)}{(\sqrt{c}-\sqrt{c})(\sqrt{c}+x)} \right] - \\ & \frac{1}{6} b^2 c^{3/2} \operatorname{PolyLog} \left[2, 1 - \frac{2\sqrt{c}(\sqrt{c}+x)}{(\sqrt{c}+\sqrt{c})(\sqrt{c}+x)} \right] + \frac{1}{6} i b^2 c^{3/2} \operatorname{PolyLog} \left[2, 1 - \frac{(1-i)(\sqrt{c}+x)}{\sqrt{c}-ix} \right] \end{aligned}$$

Result (type 8, 18 leaves):

$$\int x^2 \left(a + b \operatorname{ArcTanh} \left[\frac{c}{x^2} \right] \right)^2 dx$$

Problem 180: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2}{x^4} dx$$

Optimal (type 4, 1263 leaves, 105 steps):

$$\begin{aligned} & \frac{2 a b}{9 x^3} - \frac{2 a b}{3 c x} - \frac{2 a b \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right]}{3 c^{3/2}} + \frac{4 b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right]}{3 c^{3/2}} + \frac{i b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right]^2}{3 c^{3/2}} + \frac{4 b^2 \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right]}{3 c^{3/2}} - \frac{b^2 \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right]^2}{3 c^{3/2}} \\ & - \frac{2 b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[2 - \frac{2\sqrt{c}}{\sqrt{c}-ix}\right]}{3 c^{3/2}} - \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x^2}\right]}{9 x^3} + \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x^2}\right]}{3 c x} + \frac{b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[1 - \frac{c}{x^2}\right]}{3 c^{3/2}} - \frac{b\left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)}{9 x^3} \\ & - \frac{b\left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)}{3 c x} + \frac{b \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)}{3 c^{3/2}} - \frac{\left(2 a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)^2}{12 x^3} - \frac{a b \operatorname{Log}\left[1 + \frac{c}{x^2}\right]}{3 x^3} - \frac{2 b^2 \operatorname{Log}\left[1 + \frac{c}{x^2}\right]}{3 c x} \\ & - \frac{b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[1 + \frac{c}{x^2}\right]}{3 c^{3/2}} + \frac{b^2 \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[1 + \frac{c}{x^2}\right]}{3 c^{3/2}} + \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x^2}\right] \operatorname{Log}\left[1 + \frac{c}{x^2}\right]}{6 x^3} - \frac{b^2 \operatorname{Log}\left[1 + \frac{c}{x^2}\right]^2}{12 x^3} + \frac{2 b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{2\sqrt{c}}{\sqrt{c}-ix}\right]}{3 c^{3/2}} \\ & - \frac{b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{(1+i)(\sqrt{c}-x)}{\sqrt{c}-ix}\right]}{3 c^{3/2}} + \frac{2 b^2 \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{2\sqrt{c}}{\sqrt{c}+x}\right]}{3 c^{3/2}} - \frac{b^2 \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{2\sqrt{c}(\sqrt{c}-x)}{(\sqrt{c}-\sqrt{c})(\sqrt{c}+x)}\right]}{3 c^{3/2}} \\ & - \frac{b^2 \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{2\sqrt{c}(\sqrt{c}+x)}{(\sqrt{c}+\sqrt{c})(\sqrt{c}+x)}\right]}{3 c^{3/2}} - \frac{b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{(1-i)(\sqrt{c}+x)}{\sqrt{c}-ix}\right]}{3 c^{3/2}} - \frac{2 b^2 \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[2 - \frac{2\sqrt{c}}{\sqrt{c}+x}\right]}{3 c^{3/2}} \\ & - \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}}{\sqrt{c}-ix}\right]}{3 c^{3/2}} + \frac{i b^2 \operatorname{PolyLog}\left[2, -1 + \frac{2\sqrt{c}}{\sqrt{c}-ix}\right]}{3 c^{3/2}} + \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{(1+i)(\sqrt{c}-x)}{\sqrt{c}-ix}\right]}{6 c^{3/2}} - \frac{b^2 \operatorname{PolyLog}\left[2, -\frac{x}{\sqrt{c}}\right]}{3 c^{3/2}} + \\ & - \frac{i b^2 \operatorname{PolyLog}\left[2, -\frac{ix}{\sqrt{c}}\right]}{3 c^{3/2}} - \frac{i b^2 \operatorname{PolyLog}\left[2, \frac{ix}{\sqrt{c}}\right]}{3 c^{3/2}} + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{x}{\sqrt{c}}\right]}{3 c^{3/2}} - \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}}{\sqrt{c}+x}\right]}{3 c^{3/2}} + \frac{b^2 \operatorname{PolyLog}\left[2, -1 + \frac{2\sqrt{c}}{\sqrt{c}+x}\right]}{3 c^{3/2}} + \\ & - \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}(\sqrt{c}-x)}{(\sqrt{c}-\sqrt{c})(\sqrt{c}+x)}\right]}{6 c^{3/2}} + \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}(\sqrt{c}+x)}{(\sqrt{c}+\sqrt{c})(\sqrt{c}+x)}\right]}{6 c^{3/2}} + \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{(1-i)(\sqrt{c}+x)}{\sqrt{c}-ix}\right]}{6 c^{3/2}} \end{aligned}$$

Result (type 8, 18 leaves):

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2}{x^4} dx$$

Problem 181: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2}{x^6} dx$$

Optimal (type 4, 1337 leaves, 130 steps):

$$\begin{aligned} & \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{8b^2}{15c^2x} + \frac{2ab \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right]}{5c^{5/2}} - \frac{4b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right]}{15c^{5/2}} - \frac{i b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right]^2}{5c^{5/2}} + \frac{4b^2 \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right]}{15c^{5/2}} - \\ & \frac{b^2 \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right]^2}{5c^{5/2}} + \frac{2b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[2 - \frac{2\sqrt{c}}{\sqrt{c}-ix}\right]}{5c^{5/2}} - \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x^2}\right]}{25x^5} + \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x^2}\right]}{15cx^3} - \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x^2}\right]}{5c^2x} - \frac{b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[1 - \frac{c}{x^2}\right]}{5c^{5/2}} - \\ & \frac{b\left(2a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)}{25x^5} - \frac{b\left(2a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)}{15cx^3} - \frac{b\left(2a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)}{5c^2x} + \frac{b \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \left(2a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)}{5c^{5/2}} - \frac{\left(2a - b \operatorname{Log}\left[1 - \frac{c}{x^2}\right]\right)^2}{20x^5} - \\ & \frac{ab \operatorname{Log}\left[1 + \frac{c}{x^2}\right]}{5x^5} - \frac{2b^2 \operatorname{Log}\left[1 + \frac{c}{x^2}\right]}{15cx^3} + \frac{b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[1 + \frac{c}{x^2}\right]}{5c^{5/2}} + \frac{b^2 \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[1 + \frac{c}{x^2}\right]}{5c^{5/2}} + \frac{b^2 \operatorname{Log}\left[1 - \frac{c}{x^2}\right] \operatorname{Log}\left[1 + \frac{c}{x^2}\right]}{10x^5} - \\ & \frac{b^2 \operatorname{Log}\left[1 + \frac{c}{x^2}\right]^2}{20x^5} - \frac{2b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{2\sqrt{c}}{\sqrt{c}-ix}\right]}{5c^{5/2}} + \frac{b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{(1+i)(\sqrt{c}-x)}{\sqrt{c}-ix}\right]}{5c^{5/2}} + \frac{2b^2 \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{2\sqrt{c}}{\sqrt{c}+x}\right]}{5c^{5/2}} - \\ & \frac{b^2 \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{2\sqrt{c}(\sqrt{c}-x)}{(\sqrt{c}-\sqrt{c})(\sqrt{c}+x)}\right]}{5c^{5/2}} - \frac{b^2 \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{2\sqrt{c}(\sqrt{c}+x)}{(\sqrt{c}+\sqrt{c})(\sqrt{c}+x)}\right]}{5c^{5/2}} + \frac{b^2 \operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[\frac{(1-i)(\sqrt{c}+x)}{\sqrt{c}-ix}\right]}{5c^{5/2}} - \\ & \frac{2b^2 \operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] \operatorname{Log}\left[2 - \frac{2\sqrt{c}}{\sqrt{c}+x}\right]}{5c^{5/2}} + \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}}{\sqrt{c}-ix}\right]}{5c^{5/2}} - \frac{i b^2 \operatorname{PolyLog}\left[2, -1 + \frac{2\sqrt{c}}{\sqrt{c}-ix}\right]}{5c^{5/2}} - \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{(1+i)(\sqrt{c}-x)}{\sqrt{c}-ix}\right]}{10c^{5/2}} - \\ & \frac{b^2 \operatorname{PolyLog}\left[2, -\frac{x}{\sqrt{c}}\right]}{5c^{5/2}} - \frac{i b^2 \operatorname{PolyLog}\left[2, -\frac{ix}{\sqrt{c}}\right]}{5c^{5/2}} + \frac{i b^2 \operatorname{PolyLog}\left[2, \frac{ix}{\sqrt{c}}\right]}{5c^{5/2}} + \frac{b^2 \operatorname{PolyLog}\left[2, \frac{x}{\sqrt{c}}\right]}{5c^{5/2}} - \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}}{\sqrt{c}+x}\right]}{5c^{5/2}} + \\ & \frac{b^2 \operatorname{PolyLog}\left[2, -1 + \frac{2\sqrt{c}}{\sqrt{c}+x}\right]}{5c^{5/2}} + \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}(\sqrt{c}-x)}{(\sqrt{c}-\sqrt{c})(\sqrt{c}+x)}\right]}{10c^{5/2}} + \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}(\sqrt{c}+x)}{(\sqrt{c}+\sqrt{c})(\sqrt{c}+x)}\right]}{10c^{5/2}} - \frac{i b^2 \operatorname{PolyLog}\left[2, 1 - \frac{(1-i)(\sqrt{c}+x)}{\sqrt{c}-ix}\right]}{10c^{5/2}} \end{aligned}$$

Result (type 8, 18 leaves):

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{c}{x^2}\right]\right)^2}{x^6} dx$$

Problem 199: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c \sqrt{x}])^2}{x} dx$$

Optimal (type 4, 145 leaves, 7 steps):

$$4 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c \sqrt{x}}\right] (a + b \operatorname{ArcTanh}[c \sqrt{x}])^2 - 2 b (a + b \operatorname{ArcTanh}[c \sqrt{x}]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c \sqrt{x}}\right] +$$

$$2 b (a + b \operatorname{ArcTanh}[c \sqrt{x}]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c \sqrt{x}}\right] + b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c \sqrt{x}}\right] - b^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c \sqrt{x}}\right]$$

Result (type 4, 203 leaves):

$$a^2 \operatorname{Log}[x] + 2 a b \left(-\operatorname{PolyLog}\left[2, -c \sqrt{x}\right] + \operatorname{PolyLog}\left[2, c \sqrt{x}\right]\right) +$$

$$2 b^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c \sqrt{x}]^3 - \operatorname{ArcTanh}[c \sqrt{x}]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]}\right] + \operatorname{ArcTanh}[c \sqrt{x}]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c \sqrt{x}]}\right] + \operatorname{ArcTanh}[c \sqrt{x}]\right.$$

$$\left. \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]}\right] + \operatorname{ArcTanh}[c \sqrt{x}] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c \sqrt{x}]}\right] + \frac{1}{2} \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]}\right] - \frac{1}{2} \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c \sqrt{x}]}\right]\right)$$

Problem 206: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c \sqrt{x}])^3}{x} dx$$

Optimal (type 4, 224 leaves, 9 steps):

$$4 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c \sqrt{x}}\right] (a + b \operatorname{ArcTanh}[c \sqrt{x}])^3 - 3 b (a + b \operatorname{ArcTanh}[c \sqrt{x}])^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c \sqrt{x}}\right] +$$

$$3 b (a + b \operatorname{ArcTanh}[c \sqrt{x}])^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c \sqrt{x}}\right] + 3 b^2 (a + b \operatorname{ArcTanh}[c \sqrt{x}]) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c \sqrt{x}}\right] -$$

$$3 b^2 (a + b \operatorname{ArcTanh}[c \sqrt{x}]) \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c \sqrt{x}}\right] - \frac{3}{2} b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2}{1 - c \sqrt{x}}\right] + \frac{3}{2} b^3 \operatorname{PolyLog}\left[4, -1 + \frac{2}{1 - c \sqrt{x}}\right]$$

Result (type 4, 423 leaves):

$$\begin{aligned}
& a^3 \operatorname{Log}[x] + 3 a^2 b \left(-\operatorname{PolyLog}[2, -c \sqrt{x}] + \operatorname{PolyLog}[2, c \sqrt{x}] \right) + \\
& 6 a b^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c \sqrt{x}]^3 - \operatorname{ArcTanh}[c \sqrt{x}]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]}\right] + \operatorname{ArcTanh}[c \sqrt{x}]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c \sqrt{x}]}\right] + \right. \\
& \quad \operatorname{ArcTanh}[c \sqrt{x}] \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]}] + \operatorname{ArcTanh}[c \sqrt{x}] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c \sqrt{x}]}] + \\
& \quad \left. \frac{1}{2} \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]}] - \frac{1}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c \sqrt{x}]}] \right) + \\
& \frac{1}{32} b^3 \left(\pi^4 - 32 \operatorname{ArcTanh}[c \sqrt{x}]^4 - 64 \operatorname{ArcTanh}[c \sqrt{x}]^3 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]}\right] + 64 \operatorname{ArcTanh}[c \sqrt{x}]^3 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c \sqrt{x}]}\right] + 96 \operatorname{ArcTanh}[c \sqrt{x}]^2 \right. \\
& \quad \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]}] + 96 \operatorname{ArcTanh}[c \sqrt{x}]^2 \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c \sqrt{x}]}] + 96 \operatorname{ArcTanh}[c \sqrt{x}] \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]}] - \\
& \quad \left. 96 \operatorname{ArcTanh}[c \sqrt{x}] \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c \sqrt{x}]}] + 48 \operatorname{PolyLog}[4, -e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]}] + 48 \operatorname{PolyLog}[4, e^{2 \operatorname{ArcTanh}[c \sqrt{x}]}] \right)
\end{aligned}$$

Problem 222: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^{3/2}])^2}{x} dx$$

Optimal (type 4, 156 leaves, 7 steps):

$$\begin{aligned}
& \frac{4}{3} (a + b \operatorname{ArcTanh}[c x^{3/2}])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x^{3/2}}\right] - \frac{2}{3} b (a + b \operatorname{ArcTanh}[c x^{3/2}]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x^{3/2}}\right] + \\
& \frac{2}{3} b (a + b \operatorname{ArcTanh}[c x^{3/2}]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x^{3/2}}\right] + \frac{1}{3} b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x^{3/2}}\right] - \frac{1}{3} b^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x^{3/2}}\right]
\end{aligned}$$

Result (type 4, 207 leaves):

$$\begin{aligned}
& a^2 \operatorname{Log}[x] + \frac{2}{3} a b \left(-\operatorname{PolyLog}[2, -c x^{3/2}] + \operatorname{PolyLog}[2, c x^{3/2}] \right) + \\
& \frac{2}{3} b^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c x^{3/2}]^3 - \operatorname{ArcTanh}[c x^{3/2}]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c x^{3/2}]}\right] + \operatorname{ArcTanh}[c x^{3/2}]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x^{3/2}]}\right] + \operatorname{ArcTanh}[c x^{3/2}] \right. \\
& \quad \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x^{3/2}]}] + \operatorname{ArcTanh}[c x^{3/2}] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x^{3/2}]}] + \frac{1}{2} \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c x^{3/2}]}] - \frac{1}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x^{3/2}]}] \left. \right)
\end{aligned}$$

Problem 227: Result unnecessarily involves higher level functions.

$$\int \frac{a + b \operatorname{ArcTanh}[c x^n]}{x} dx$$

Optimal (type 4, 36 leaves, 2 steps):

$$a \operatorname{Log}[x] - \frac{b \operatorname{PolyLog}[2, -c x^n]}{2 n} + \frac{b \operatorname{PolyLog}[2, c x^n]}{2 n}$$

Result (type 5, 39 leaves):

$$\frac{b c x^n \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, c^2 x^{2n}\right]}{n} + a \text{Log}[x]$$

Problem 233: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \text{ArcTanh}[c x^n])^2}{x} dx$$

Optimal (type 4, 148 leaves, 7 steps):

$$\frac{2 (a + b \text{ArcTanh}[c x^n])^2 \text{ArcTanh}\left[1 - \frac{2}{1 - c x^n}\right]}{n} - \frac{b (a + b \text{ArcTanh}[c x^n]) \text{PolyLog}\left[2, 1 - \frac{2}{1 - c x^n}\right]}{n} +$$

$$\frac{b (a + b \text{ArcTanh}[c x^n]) \text{PolyLog}\left[2, -1 + \frac{2}{1 - c x^n}\right]}{n} + \frac{b^2 \text{PolyLog}\left[3, 1 - \frac{2}{1 - c x^n}\right]}{2 n} - \frac{b^2 \text{PolyLog}\left[3, -1 + \frac{2}{1 - c x^n}\right]}{2 n}$$

Result (type 4, 181 leaves):

$$a^2 \text{Log}[x] + \frac{a b (-\text{PolyLog}[2, -c x^n] + \text{PolyLog}[2, c x^n])}{n} + \frac{1}{n}$$

$$b^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \text{ArcTanh}[c x^n]^3 - \text{ArcTanh}[c x^n]^2 \text{Log}[1 + e^{-2 \text{ArcTanh}[c x^n]}] + \text{ArcTanh}[c x^n]^2 \text{Log}[1 - e^{2 \text{ArcTanh}[c x^n]}] + \text{ArcTanh}[c x^n] \right.$$

$$\left. \text{PolyLog}[2, -e^{-2 \text{ArcTanh}[c x^n]}] + \text{ArcTanh}[c x^n] \text{PolyLog}[2, e^{2 \text{ArcTanh}[c x^n]}] + \frac{1}{2} \text{PolyLog}[3, -e^{-2 \text{ArcTanh}[c x^n]}] - \frac{1}{2} \text{PolyLog}[3, e^{2 \text{ArcTanh}[c x^n]}] \right)$$

Problem 236: Result unnecessarily involves higher level functions.

$$\int \frac{\text{ArcTanh}[a x^n]}{x} dx$$

Optimal (type 4, 30 leaves, 2 steps):

$$-\frac{\text{PolyLog}[2, -a x^n]}{2 n} + \frac{\text{PolyLog}[2, a x^n]}{2 n}$$

Result (type 5, 33 leaves):

$$\frac{a x^n \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, a^2 x^{2n}\right]}{n}$$

Problem 5: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{ArcTanh}[c x]}{d + e x} dx$$

Optimal (type 4, 114 leaves, 4 steps):

$$-\frac{(a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{-2}{1+c x}\right]}{e} + \frac{(a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{-2 c (d+e x)}{(c d+e)(1+c x)}\right]}{e} + \frac{b \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{2 e} - \frac{b \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{2 e}$$

Result (type 4, 257 leaves):

$$\begin{aligned} & \frac{1}{e} \left(a \operatorname{Log}[d + e x] + b \operatorname{ArcTanh}[c x] \left(\frac{1}{2} \operatorname{Log}[1 - c^2 x^2] + \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right]\right] \right) \right) - \\ & \frac{1}{2} i b \left(-\frac{1}{4} i (\pi - 2 i \operatorname{ArcTanh}[c x])^2 + i \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x] \right)^2 + (\pi - 2 i \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[c x]}\right] + \right. \\ & \quad \left. 2 i \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x] \right) \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right)}\right] - (\pi - 2 i \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{\sqrt{1 - c^2 x^2}}\right] - \right. \\ & \quad \left. 2 i \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x] \right) \operatorname{Log}\left[2 i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right]\right] - \right. \\ & \quad \left. i \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcTanh}[c x]}\right] - i \operatorname{PolyLog}\left[2, e^{-2 \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right)}\right] \right) \end{aligned}$$

Problem 12: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{d + e x} dx$$

Optimal (type 4, 188 leaves, 1 step):

$$-\frac{(a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{-2}{1+c x}\right]}{e} + \frac{(a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{-2 c (d+e x)}{(c d+e)(1+c x)}\right]}{e} + \frac{b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{e} - \\ \frac{b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{e} + \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+c x}\right]}{2 e} - \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{2 e}$$

Result (type 8, 20 leaves):

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{d + e x} dx$$

Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{(d + e x)^2} dx$$

Optimal (type 4, 321 leaves, 12 steps):

$$\begin{aligned} & -\frac{(a + b \operatorname{ArcTanh}[c x])^2}{e (d + e x)} + \frac{b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1 - c x}\right]}{e (c d + e)} - \frac{b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1 + c x}\right]}{(c d - e) e} + \\ & \frac{2 b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1 + c x}\right]}{c^2 d^2 - e^2} - \frac{2 b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2 c (d + e x)}{(c d + e) (1 + c x)}\right]}{c^2 d^2 - e^2} + \frac{b^2 c \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right]}{2 e (c d + e)} + \\ & \frac{b^2 c \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c x}\right]}{2 (c d - e) e} - \frac{b^2 c \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c x}\right]}{c^2 d^2 - e^2} + \frac{b^2 c \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d + e x)}{(c d + e) (1 + c x)}\right]}{c^2 d^2 - e^2} \end{aligned}$$

Result (type 4, 317 leaves):

$$\begin{aligned} & -\frac{a^2}{e (d + e x)} + \frac{a b c \left(-\frac{2 \operatorname{ArcTanh}[c x]}{c d + c e x} + \frac{(-c d + e) \operatorname{Log}[1 - c x] + (c d + e) \operatorname{Log}[1 + c x] - 2 e \operatorname{Log}[c (d + e x)]}{(c d - e) (c d + e)} \right)}{e} + \\ & \frac{1}{d} b^2 \left(-\frac{e^{-\operatorname{ArcTanh}\left[\frac{c d}{e}\right]} \operatorname{ArcTanh}[c x]^2}{\sqrt{1 - \frac{c^2 d^2}{e^2}} e} + \frac{x \operatorname{ArcTanh}[c x]^2}{d + e x} + \frac{1}{c^2 d^2 - e^2} c d \left(i \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[c x]}\right] - \right. \right. \\ & \left. \left. 2 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right)}\right] - i \pi \left(\operatorname{ArcTanh}[c x] - \frac{1}{2} \operatorname{Log}\left[1 - c^2 x^2\right] \right) - 2 \operatorname{ArcTanh}\left[\frac{c d}{e}\right] \left(\operatorname{ArcTanh}[c x] + \right. \right. \right. \\ & \left. \left. \left. \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right)}\right] - \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right]\right] \right) + \operatorname{PolyLog}\left[2, e^{-2 \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right)}\right] \right) \right) \end{aligned}$$

Problem 14: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{(d + e x)^3} dx$$

Optimal (type 4, 480 leaves, 18 steps):

$$\begin{aligned} & \frac{b c (a + b \operatorname{ArcTanh}[c x])}{(c^2 d^2 - e^2) (d + e x)} - \frac{(a + b \operatorname{ArcTanh}[c x])^2}{2 e (d + e x)^2} + \frac{b c^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1 - c x}\right]}{2 e (c d + e)^2} + \frac{b^2 c^2 \operatorname{Log}[1 - c x]}{2 (c d - e) (c d + e)^2} - \frac{b c^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1 + c x}\right]}{2 (c d - e)^2 e} + \\ & \frac{2 b c^3 d (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1 + c x}\right]}{(c d - e)^2 (c d + e)^2} - \frac{b^2 c^2 \operatorname{Log}[1 + c x]}{2 (c d - e)^2 (c d + e)} + \frac{b^2 c^2 e \operatorname{Log}[d + e x]}{(c d - e)^2 (c d + e)^2} - \frac{2 b c^3 d (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2 c (d + e x)}{(c d + e) (1 + c x)}\right]}{(c d - e)^2 (c d + e)^2} + \\ & \frac{b^2 c^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right]}{4 e (c d + e)^2} + \frac{b^2 c^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c x}\right]}{4 (c d - e)^2 e} - \frac{b^2 c^3 d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c x}\right]}{(c d - e)^2 (c d + e)^2} + \frac{b^2 c^3 d \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d + e x)}{(c d + e) (1 + c x)}\right]}{(c d - e)^2 (c d + e)^2} \end{aligned}$$

Result (type 4, 467 leaves):

$$\begin{aligned} & -\frac{a^2}{2 e (d + e x)^2} - \frac{a b c^2 \left(\frac{2 \operatorname{ArcTanh}[c x]}{(c d + c e x)^2} + \frac{\operatorname{Log}[1 - c x]}{(c d + e)^2} + \frac{-\operatorname{Log}[1 + c x] + \frac{2 e (-c^2 d^2 + e^2 + 2 c^2 d (d + e x) \operatorname{Log}[c (d + e x)])}{c (c d + e)^2 (d + e x)}}{(-c d + e)^2} \right)}{2 e} + \\ & \frac{1}{2 (c d - e) (c d + e)} b^2 c^2 \left(\frac{e (1 - c^2 x^2) \operatorname{ArcTanh}[c x]^2}{(c d + c e x)^2} + \frac{2 x \operatorname{ArcTanh}[c x] (-e + c d \operatorname{ArcTanh}[c x])}{c d (d + e x)} + \right. \\ & \left. \frac{2 e \left(-e \operatorname{ArcTanh}[c x] + c d \operatorname{Log}\left[\frac{c (d + e x)}{\sqrt{1 - c^2 x^2}}\right] \right)}{c^3 d^3 - c d e^2} + \frac{1}{c^2 d^2 - e^2} 2 \left(\sqrt{1 - \frac{c^2 d^2}{e^2}} e e^{-\operatorname{ArcTanh}\left[\frac{c d}{e}\right]} \operatorname{ArcTanh}[c x]^2 + i c d \right. \right. \\ & \left. \left. \left(-\left(\pi - 2 i \operatorname{ArcTanh}\left[\frac{c d}{e}\right] \right) \operatorname{ArcTanh}[c x] + \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[c x]}\right] + 2 i \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x] \right) \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x] \right)}\right] + \right. \right. \\ & \left. \left. \frac{1}{2} \pi \operatorname{Log}[1 - c^2 x^2] - 2 i \operatorname{ArcTanh}\left[\frac{c d}{e}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right]\right] - i \operatorname{PolyLog}\left[2, e^{-2 \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x] \right)}\right] \right) \right) \right) \end{aligned}$$

Problem 18: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^3}{d + e x} dx$$

Optimal (type 4, 272 leaves, 1 step):

$$\begin{aligned}
& - \frac{(a + b \operatorname{ArcTanh}[c x])^3 \operatorname{Log}\left[\frac{2}{1+c x}\right]}{e} + \frac{(a + b \operatorname{ArcTanh}[c x])^3 \operatorname{Log}\left[\frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{e} + \frac{3 b (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{2 e} \\
& - \frac{3 b (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{2 e} + \frac{3 b^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+c x}\right]}{2 e} \\
& - \frac{3 b^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[3, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{2 e} + \frac{3 b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2}{1+c x}\right]}{4 e} - \frac{3 b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{4 e}
\end{aligned}$$

Result (type 8, 20 leaves):

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^3}{d + e x} dx$$

Problem 19: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^3}{(d + e x)^2} dx$$

Optimal (type 4, 517 leaves, 9 steps):

$$\begin{aligned}
& - \frac{(a + b \operatorname{ArcTanh}[c x])^3}{e (d + e x)} + \frac{3 b c (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2}{1-c x}\right]}{2 e (c d + e)} - \frac{3 b c (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2}{1+c x}\right]}{2 (c d - e) e} + \frac{3 b c (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2}{1+c x}\right]}{c^2 d^2 - e^2} \\
& - \frac{3 b c (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{c^2 d^2 - e^2} + \frac{3 b^2 c (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c x}\right]}{2 e (c d + e)} + \frac{3 b^2 c (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{2 (c d - e) e} \\
& - \frac{3 b^2 c (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{c^2 d^2 - e^2} + \frac{3 b^2 c (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{c^2 d^2 - e^2} \\
& - \frac{3 b^3 c \operatorname{PolyLog}\left[3, 1 - \frac{2}{1-c x}\right]}{4 e (c d + e)} + \frac{3 b^3 c \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+c x}\right]}{4 (c d - e) e} - \frac{3 b^3 c \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+c x}\right]}{2 (c^2 d^2 - e^2)} + \frac{3 b^3 c \operatorname{PolyLog}\left[3, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{2 (c^2 d^2 - e^2)}
\end{aligned}$$

Result (type 8, 20 leaves):

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^3}{(d + e x)^2} dx$$

Problem 20: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^3}{(d + e x)^3} dx$$

Optimal (type 4, 953 leaves, 21 steps):

$$\begin{aligned}
& \frac{3 b c (a + b \operatorname{ArcTanh}[c x])^2}{2 (c^2 d^2 - e^2) (d + e x)} - \frac{(a + b \operatorname{ArcTanh}[c x])^3}{2 e (d + e x)^2} - \frac{3 b^2 c^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{-2}{1 - c x}\right]}{2 (c d - e) (c d + e)^2} + \frac{3 b c^2 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{-2}{1 - c x}\right]}{4 e (c d + e)^2} \\
& \frac{3 b^2 c^2 e (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{-2}{1 + c x}\right]}{(c d - e)^2 (c d + e)^2} + \frac{3 b^2 c^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{-2}{1 + c x}\right]}{2 (c d - e)^2 (c d + e)} - \frac{3 b c^2 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{-2}{1 + c x}\right]}{4 (c d - e)^2 e} + \\
& \frac{3 b c^3 d (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{-2}{1 + c x}\right]}{(c d - e)^2 (c d + e)^2} + \frac{3 b^2 c^2 e (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{-2 c (d + e x)}{(c d + e) (1 + c x)}\right]}{(c d - e)^2 (c d + e)^2} - \frac{3 b c^3 d (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{-2 c (d + e x)}{(c d + e) (1 + c x)}\right]}{(c d - e)^2 (c d + e)^2} \\
& \frac{3 b^3 c^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right]}{4 (c d - e) (c d + e)^2} + \frac{3 b^2 c^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right]}{4 e (c d + e)^2} + \frac{3 b^3 c^2 e \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right]}{2 (c d - e)^2 (c d + e)^2} - \\
& \frac{3 b^3 c^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c x}\right]}{4 (c d - e)^2 (c d + e)} + \frac{3 b^2 c^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c x}\right]}{4 (c d - e)^2 e} - \frac{3 b^2 c^3 d (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c x}\right]}{(c d - e)^2 (c d + e)^2} \\
& \frac{3 b^3 c^2 e \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d + e x)}{(c d + e) (1 + c x)}\right]}{2 (c d - e)^2 (c d + e)^2} + \frac{3 b^2 c^3 d (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d + e x)}{(c d + e) (1 + c x)}\right]}{(c d - e)^2 (c d + e)^2} - \frac{3 b^3 c^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x}\right]}{8 e (c d + e)^2} + \\
& \frac{3 b^3 c^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 + c x}\right]}{8 (c d - e)^2 e} - \frac{3 b^3 c^3 d \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 + c x}\right]}{2 (c d - e)^2 (c d + e)^2} + \frac{3 b^3 c^3 d \operatorname{PolyLog}\left[3, 1 - \frac{2 c (d + e x)}{(c d + e) (1 + c x)}\right]}{2 (c d - e)^2 (c d + e)^2}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 21: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{ArcTanh}[c x]}{1 + 2 c x} dx$$

Optimal (type 4, 67 leaves, 4 steps):

$$\frac{\left(a - b \operatorname{ArcTanh}\left[\frac{1}{2}\right]\right) \operatorname{Log}\left[-\frac{1 + 2 c x}{2 d}\right]}{2 c} - \frac{b \operatorname{PolyLog}\left[2, -1 - 2 c x\right]}{4 c} + \frac{b \operatorname{PolyLog}\left[2, \frac{1}{3} (1 + 2 c x)\right]}{4 c}$$

Result (type 4, 240 leaves):

$$\begin{aligned} & \frac{1}{2c} \left(a \operatorname{Log}[1 + 2cx] + b \operatorname{ArcTanh}[cx] \left(\frac{1}{2} \operatorname{Log}[1 - c^2x^2] + \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{1}{2} \right] + \operatorname{ArcTanh}[cx] \right] \right] \right) - \right. \\ & \left. \frac{1}{2} i b \left(-\frac{1}{4} i (\pi - 2 i \operatorname{ArcTanh}[cx])^2 + i \left(\operatorname{ArcTanh}\left[\frac{1}{2} \right] + \operatorname{ArcTanh}[cx] \right)^2 + (\pi - 2 i \operatorname{ArcTanh}[cx]) \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[cx]} \right] + \right. \right. \\ & \left. \left. 2 i \left(\operatorname{ArcTanh}\left[\frac{1}{2} \right] + \operatorname{ArcTanh}[cx] \right) \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{1}{2} \right] + \operatorname{ArcTanh}[cx] \right)} \right] - (\pi - 2 i \operatorname{ArcTanh}[cx]) \operatorname{Log}\left[\frac{2}{\sqrt{1 - c^2x^2}} \right] - \right. \right. \\ & \left. \left. 2 i \left(\operatorname{ArcTanh}\left[\frac{1}{2} \right] + \operatorname{ArcTanh}[cx] \right) \operatorname{Log}\left[2 i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{1}{2} \right] + \operatorname{ArcTanh}[cx] \right] \right] - \right. \right. \\ & \left. \left. i \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcTanh}[cx]} \right] - i \operatorname{PolyLog}\left[2, e^{-2 \left(\operatorname{ArcTanh}\left[\frac{1}{2} \right] + \operatorname{ArcTanh}[cx] \right)} \right] \right) \right) \end{aligned}$$

Problem 22: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcTanh}[x]}{1 - \sqrt{2}x} dx$$

Optimal (type 4, 88 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] \operatorname{Log}[1 - \sqrt{2}x]}{\sqrt{2}} - \frac{\operatorname{PolyLog}\left[2, -\frac{\sqrt{2}-2x}{2-\sqrt{2}}\right]}{2\sqrt{2}} + \frac{\operatorname{PolyLog}\left[2, \frac{\sqrt{2}-2x}{2+\sqrt{2}}\right]}{2\sqrt{2}}$$

Result (type 4, 272 leaves):

$$\begin{aligned} & \frac{1}{8\sqrt{2}} \left(\pi^2 - 4 \operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right]^2 - 4 i \pi \operatorname{ArcTanh}[x] + 8 \operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] \operatorname{ArcTanh}[x] - 8 \operatorname{ArcTanh}[x]^2 + \right. \\ & \left. 8 \operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] - 2 \operatorname{ArcTanh}[x]} \right] - 8 \operatorname{ArcTanh}[x] \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] - 2 \operatorname{ArcTanh}[x]} \right] + 4 i \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[x]} \right] + \right. \\ & \left. 8 \operatorname{ArcTanh}[x] \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[x]} \right] - 4 i \pi \operatorname{Log}\left[\frac{2}{\sqrt{1-x^2}} \right] - 8 \operatorname{ArcTanh}[x] \operatorname{Log}\left[\frac{2}{\sqrt{1-x^2}} \right] - 4 \operatorname{ArcTanh}[x] \operatorname{Log}\left[1 - x^2 \right] - \right. \\ & \left. 8 \operatorname{ArcTanh}[x] \operatorname{Log}\left[-i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] - \operatorname{ArcTanh}[x] \right] \right] - 8 \operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] \operatorname{Log}\left[-2 i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] - \operatorname{ArcTanh}[x] \right] \right] + \right. \\ & \left. 8 \operatorname{ArcTanh}[x] \operatorname{Log}\left[-2 i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] - \operatorname{ArcTanh}[x] \right] \right] + 4 \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}\left[\frac{1}{\sqrt{2}}\right] - 2 \operatorname{ArcTanh}[x]} \right] + 4 \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcTanh}[x]} \right] \right) \end{aligned}$$

Problem 26: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcTanh}[c x^2]}{d + e x} dx$$

Optimal (type 4, 325 leaves, 19 steps):

$$\frac{(a + b \operatorname{ArcTanh}[c x^2]) \operatorname{Log}[d + e x]}{e} - \frac{b \operatorname{Log}\left[\frac{e(1 - \sqrt{-c} x)}{\sqrt{-c} d + e}\right] \operatorname{Log}[d + e x]}{2e} - \frac{b \operatorname{Log}\left[-\frac{e(1 + \sqrt{-c} x)}{\sqrt{-c} d - e}\right] \operatorname{Log}[d + e x]}{2e} + \frac{b \operatorname{Log}\left[\frac{e(1 - \sqrt{c} x)}{\sqrt{c} d + e}\right] \operatorname{Log}[d + e x]}{2e} +$$

$$\frac{b \operatorname{Log}\left[-\frac{e(1 + \sqrt{c} x)}{\sqrt{c} d - e}\right] \operatorname{Log}[d + e x]}{2e} - \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{-c}(d + e x)}{\sqrt{-c} d - e}\right]}{2e} + \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{c}(d + e x)}{\sqrt{c} d - e}\right]}{2e} - \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{-c}(d + e x)}{\sqrt{-c} d + e}\right]}{2e} + \frac{b \operatorname{PolyLog}\left[2, \frac{\sqrt{c}(d + e x)}{\sqrt{c} d + e}\right]}{2e}$$

Result (type 4, 285 leaves):

$$\frac{a \operatorname{Log}[d + e x]}{e} + \frac{1}{2e}$$

$$b \left(2 \operatorname{ArcTanh}[c x^2] \operatorname{Log}[d + e x] - \operatorname{Log}\left[\frac{e(i - \sqrt{c} x)}{\sqrt{c} d + i e}\right] \operatorname{Log}[d + e x] - \operatorname{Log}\left[-\frac{e(i + \sqrt{c} x)}{\sqrt{c} d - i e}\right] \operatorname{Log}[d + e x] + \operatorname{Log}\left[-\frac{e(1 + \sqrt{c} x)}{\sqrt{c} d - e}\right] \operatorname{Log}[d + e x] +$$

$$\operatorname{Log}[d + e x] \operatorname{Log}\left[\frac{e - \sqrt{c} e x}{\sqrt{c} d + e}\right] + \operatorname{PolyLog}\left[2, \frac{\sqrt{c}(d + e x)}{\sqrt{c} d - e}\right] - \operatorname{PolyLog}\left[2, \frac{\sqrt{c}(d + e x)}{\sqrt{c} d - i e}\right] -$$

$$\operatorname{PolyLog}\left[2, \frac{\sqrt{c}(d + e x)}{\sqrt{c} d + i e}\right] + \operatorname{PolyLog}\left[2, \frac{\sqrt{c}(d + e x)}{\sqrt{c} d + e}\right] \right)$$

Problem 30: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{d + e x} dx$$

Optimal (type 9, 22 leaves, 0 steps):

$$\operatorname{Unintegrable}\left[\frac{(a + b \operatorname{ArcTanh}[c x^2])^2}{d + e x}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 34: Attempted integration timed out after 120 seconds.

$$\int \frac{a + b \operatorname{ArcTanh}[c x^3]}{d + e x} dx$$

Optimal (type 4, 523 leaves, 25 steps):

$$\begin{aligned} & \frac{(a + b \operatorname{ArcTanh}[c x^3]) \operatorname{Log}[d + e x]}{e} + \frac{b \operatorname{Log}\left[\frac{e(1 - c^{1/3} x)}{c^{1/3} d + e}\right] \operatorname{Log}[d + e x]}{2 e} - \frac{b \operatorname{Log}\left[-\frac{e(1 + c^{1/3} x)}{c^{1/3} d - e}\right] \operatorname{Log}[d + e x]}{2 e} + \\ & \frac{b \operatorname{Log}\left[-\frac{e(-1)^{1/3} + c^{1/3} x}{c^{1/3} d - (-1)^{1/3} e}\right] \operatorname{Log}[d + e x]}{2 e} - \frac{b \operatorname{Log}\left[-\frac{e(-1)^{2/3} + c^{1/3} x}{c^{1/3} d - (-1)^{2/3} e}\right] \operatorname{Log}[d + e x]}{2 e} + \frac{b \operatorname{Log}\left[\frac{(-1)^{2/3} e(1 + (-1)^{1/3} c^{1/3} x)}{c^{1/3} d + (-1)^{2/3} e}\right] \operatorname{Log}[d + e x]}{2 e} - \\ & \frac{b \operatorname{Log}\left[\frac{(-1)^{1/3} e(1 + (-1)^{2/3} c^{1/3} x)}{c^{1/3} d + (-1)^{1/3} e}\right] \operatorname{Log}[d + e x]}{2 e} - \frac{b \operatorname{PolyLog}\left[2, \frac{c^{1/3}(d + e x)}{c^{1/3} d - e}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, \frac{c^{1/3}(d + e x)}{c^{1/3} d + e}\right]}{2 e} + \\ & \frac{b \operatorname{PolyLog}\left[2, \frac{c^{1/3}(d + e x)}{c^{1/3} d - (-1)^{1/3} e}\right]}{2 e} - \frac{b \operatorname{PolyLog}\left[2, \frac{c^{1/3}(d + e x)}{c^{1/3} d + (-1)^{1/3} e}\right]}{2 e} - \frac{b \operatorname{PolyLog}\left[2, \frac{c^{1/3}(d + e x)}{c^{1/3} d - (-1)^{2/3} e}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, \frac{c^{1/3}(d + e x)}{c^{1/3} d + (-1)^{2/3} e}\right]}{2 e} \end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 44: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 (a + b \operatorname{ArcTanh}[c \sqrt{x}])}{d + e x} dx$$

Optimal (type 4, 460 leaves, 20 steps):

$$\begin{aligned}
& -\frac{b d \sqrt{x}}{c e^2} + \frac{b \sqrt{x}}{2 c^3 e} + \frac{b x^{3/2}}{6 c e} + \frac{b d \operatorname{ArcTanh}[c \sqrt{x}]}{c^2 e^2} - \frac{b \operatorname{ArcTanh}[c \sqrt{x}]}{2 c^4 e} - \\
& \frac{d x (a + b \operatorname{ArcTanh}[c \sqrt{x}])}{e^2} + \frac{x^2 (a + b \operatorname{ArcTanh}[c \sqrt{x}])}{2 e} - \frac{2 d^2 (a + b \operatorname{ArcTanh}[c \sqrt{x}]) \operatorname{Log}\left[\frac{2}{1+c \sqrt{x}}\right]}{e^3} + \\
& \frac{d^2 (a + b \operatorname{ArcTanh}[c \sqrt{x}]) \operatorname{Log}\left[\frac{2 c (\sqrt{-d} - \sqrt{e} \sqrt{x})}{(c \sqrt{-d} - \sqrt{e}) (1+c \sqrt{x})}\right]}{e^3} + \frac{d^2 (a + b \operatorname{ArcTanh}[c \sqrt{x}]) \operatorname{Log}\left[\frac{2 c (\sqrt{-d} + \sqrt{e} \sqrt{x})}{(c \sqrt{-d} + \sqrt{e}) (1+c \sqrt{x})}\right]}{e^3} + \\
& \frac{b d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c \sqrt{x}}\right]}{e^3} - \frac{b d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 c (\sqrt{-d} - \sqrt{e} \sqrt{x})}{(c \sqrt{-d} - \sqrt{e}) (1+c \sqrt{x})}\right]}{2 e^3} - \frac{b d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 c (\sqrt{-d} + \sqrt{e} \sqrt{x})}{(c \sqrt{-d} + \sqrt{e}) (1+c \sqrt{x})}\right]}{2 e^3}
\end{aligned}$$

Result (type 4, 558 leaves):

$$\begin{aligned}
& \frac{1}{6 e^3} \left(-6 a d e x + 3 a e^2 x^2 + 6 a d^2 \operatorname{Log}[d + e x] + \right. \\
& \frac{1}{c^4} b \left(2 c e (-3 c^2 d + 2 e) \sqrt{x} + c e^2 \sqrt{x} (-1 + c^2 x) - 6 (c^2 d - e) e (-1 + c^2 x) \operatorname{ArcTanh}[c \sqrt{x}] + 3 e^2 (-1 + c^2 x)^2 \operatorname{ArcTanh}[c \sqrt{x}] - \right. \\
& 6 c^4 d^2 \left(\operatorname{ArcTanh}[c \sqrt{x}] \left(\operatorname{ArcTanh}[c \sqrt{x}] + 2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]}\right]\right) - \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]}\right] \right) + \\
& 3 c^4 d^2 \left(2 \operatorname{ArcTanh}[c \sqrt{x}]^2 - 4 i \operatorname{ArcSin}\left[\sqrt{\frac{c^2 d}{c^2 d + e}}\right] \operatorname{ArcTanh}\left[\frac{c e \sqrt{x}}{\sqrt{-c^2 d e}}\right] + \right. \\
& 2 \left(-i \operatorname{ArcSin}\left[\sqrt{\frac{c^2 d}{c^2 d + e}}\right] + \operatorname{ArcTanh}[c \sqrt{x}] \right) \operatorname{Log}\left[\frac{e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]} \left(-2 \sqrt{-c^2 d e} + e \left(-1 + e^{2 \operatorname{ArcTanh}[c \sqrt{x}]} \right) + c^2 d \left(1 + e^{2 \operatorname{ArcTanh}[c \sqrt{x}]} \right) \right)}{c^2 d + e}\right] + \\
& 2 \left(i \operatorname{ArcSin}\left[\sqrt{\frac{c^2 d}{c^2 d + e}}\right] + \operatorname{ArcTanh}[c \sqrt{x}] \right) \operatorname{Log}\left[\frac{e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]} \left(2 \sqrt{-c^2 d e} + e \left(-1 + e^{2 \operatorname{ArcTanh}[c \sqrt{x}]} \right) + c^2 d \left(1 + e^{2 \operatorname{ArcTanh}[c \sqrt{x}]} \right) \right)}{c^2 d + e}\right] - \\
& \left. \left. \operatorname{PolyLog}\left[2, \frac{(-c^2 d + e - 2 \sqrt{-c^2 d e}) e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]}}{c^2 d + e}\right] - \operatorname{PolyLog}\left[2, \frac{(-c^2 d + e + 2 \sqrt{-c^2 d e}) e^{-2 \operatorname{ArcTanh}[c \sqrt{x}]}}{c^2 d + e}\right] \right) \right)
\end{aligned}$$

Problem 45: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x \left(a + b \operatorname{ArcTanh} \left[c \sqrt{x} \right] \right)}{d + e x} dx$$

Optimal (type 4, 374 leaves, 15 steps):

$$\begin{aligned} & \frac{b \sqrt{x}}{c e} - \frac{b \operatorname{ArcTanh} \left[c \sqrt{x} \right]}{c^2 e} + \frac{x \left(a + b \operatorname{ArcTanh} \left[c \sqrt{x} \right] \right)}{e} + \frac{2 d \left(a + b \operatorname{ArcTanh} \left[c \sqrt{x} \right] \right) \operatorname{Log} \left[\frac{2}{1 + c \sqrt{x}} \right]}{e^2} - \\ & \frac{d \left(a + b \operatorname{ArcTanh} \left[c \sqrt{x} \right] \right) \operatorname{Log} \left[\frac{2 c \left(\sqrt{-d} - \sqrt{e} \sqrt{x} \right)}{\left(c \sqrt{-d} - \sqrt{e} \right) \left(1 + c \sqrt{x} \right)} \right]}{e^2} - \frac{d \left(a + b \operatorname{ArcTanh} \left[c \sqrt{x} \right] \right) \operatorname{Log} \left[\frac{2 c \left(\sqrt{-d} + \sqrt{e} \sqrt{x} \right)}{\left(c \sqrt{-d} + \sqrt{e} \right) \left(1 + c \sqrt{x} \right)} \right]}{e^2} - \\ & \frac{b d \operatorname{PolyLog} \left[2, 1 - \frac{2}{1 + c \sqrt{x}} \right]}{e^2} + \frac{b d \operatorname{PolyLog} \left[2, 1 - \frac{2 c \left(\sqrt{-d} - \sqrt{e} \sqrt{x} \right)}{\left(c \sqrt{-d} - \sqrt{e} \right) \left(1 + c \sqrt{x} \right)} \right]}{2 e^2} + \frac{b d \operatorname{PolyLog} \left[2, 1 - \frac{2 c \left(\sqrt{-d} + \sqrt{e} \sqrt{x} \right)}{\left(c \sqrt{-d} + \sqrt{e} \right) \left(1 + c \sqrt{x} \right)} \right]}{2 e^2} \end{aligned}$$

Result (type 4, 568 leaves):

$$\begin{aligned}
& \frac{1}{2e^2} \left(2ae^x - 2ad \operatorname{Log}[d+ex] + \frac{1}{c^2} \right. \\
& 2b \left(ce\sqrt{x} + c^2d \operatorname{ArcTanh}[c\sqrt{x}]^2 + \operatorname{ArcTanh}[c\sqrt{x}] \left(-e + c^2ex + 2c^2d \operatorname{Log}\left[1 + e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \right] \right) - c^2d \operatorname{PolyLog}\left[2, -e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \right] \right) + \\
& b d \left(-2 \left(\operatorname{ArcTanh}[c\sqrt{x}]^2 - \right. \right. \\
& \quad \left. \left. i \operatorname{ArcSin}\left[\sqrt{\frac{c^2d}{c^2d+e}}\right] \left(2 \operatorname{ArcTanh}\left[\frac{ce\sqrt{x}}{\sqrt{-c^2de}}\right] + \operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \left(-2\sqrt{-c^2de} + e \left(-1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) + c^2d \left(1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) \right)}{c^2d+e}\right] \right) - \right. \right. \\
& \quad \left. \left. \operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \left(2\sqrt{-c^2de} + e \left(-1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) + c^2d \left(1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) \right)}{c^2d+e}\right] \right) \right) + \\
& \quad \operatorname{ArcTanh}[c\sqrt{x}] \left(\operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \left(-2\sqrt{-c^2de} + e \left(-1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) + c^2d \left(1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) \right)}{c^2d+e}\right] \right) + \\
& \quad \left. \left. \operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \left(2\sqrt{-c^2de} + e \left(-1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) + c^2d \left(1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) \right)}{c^2d+e}\right] \right) \right) \right) + \\
& \quad \left. \left. \operatorname{PolyLog}\left[2, \frac{\left(-c^2d+e-2\sqrt{-c^2de} \right) e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} }{c^2d+e}\right] + \operatorname{PolyLog}\left[2, \frac{\left(-c^2d+e+2\sqrt{-c^2de} \right) e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} }{c^2d+e}\right] \right) \right)
\end{aligned}$$

Problem 46: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcTanh}[c\sqrt{x}]}{d + ex} dx$$

Optimal (type 4, 318 leaves, 11 steps):

$$\begin{aligned}
& - \frac{2 \left(a + b \operatorname{ArcTanh} [c \sqrt{x}] \right) \operatorname{Log} \left[\frac{2}{1+c \sqrt{x}} \right]}{e} + \frac{\left(a + b \operatorname{ArcTanh} [c \sqrt{x}] \right) \operatorname{Log} \left[\frac{2 c (\sqrt{-d} - \sqrt{e} \sqrt{x})}{(c \sqrt{-d} - \sqrt{e}) (1+c \sqrt{x})} \right]}{e} + \frac{\left(a + b \operatorname{ArcTanh} [c \sqrt{x}] \right) \operatorname{Log} \left[\frac{2 c (\sqrt{-d} + \sqrt{e} \sqrt{x})}{(c \sqrt{-d} + \sqrt{e}) (1+c \sqrt{x})} \right]}{e} + \\
& \frac{b \operatorname{PolyLog} \left[2, 1 - \frac{2}{1+c \sqrt{x}} \right]}{e} - \frac{b \operatorname{PolyLog} \left[2, 1 - \frac{2 c (\sqrt{-d} - \sqrt{e} \sqrt{x})}{(c \sqrt{-d} - \sqrt{e}) (1+c \sqrt{x})} \right]}{2 e} - \frac{b \operatorname{PolyLog} \left[2, 1 - \frac{2 c (\sqrt{-d} + \sqrt{e} \sqrt{x})}{(c \sqrt{-d} + \sqrt{e}) (1+c \sqrt{x})} \right]}{2 e}
\end{aligned}$$

Result (type 4, 551 leaves):

$$\begin{aligned}
& \frac{a \operatorname{Log} [d + e x]}{e} - \frac{1}{2 e} b \left(4 i \operatorname{ArcSin} \left[\sqrt{\frac{c^2 d}{c^2 d + e}} \right] \operatorname{ArcTanh} \left[\frac{c e \sqrt{x}}{\sqrt{-c^2 d e}} \right] + 4 \operatorname{ArcTanh} [c \sqrt{x}] \operatorname{Log} [1 + e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]}] + \right. \\
& \left. 2 i \operatorname{ArcSin} \left[\sqrt{\frac{c^2 d}{c^2 d + e}} \right] \operatorname{Log} \left[\frac{e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]} \left(-2 \sqrt{-c^2 d e} + e \left(-1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) + c^2 d \left(1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) \right)}{c^2 d + e} \right] - \right. \\
& \left. 2 \operatorname{ArcTanh} [c \sqrt{x}] \operatorname{Log} \left[\frac{e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]} \left(-2 \sqrt{-c^2 d e} + e \left(-1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) + c^2 d \left(1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) \right)}{c^2 d + e} \right] - \right. \\
& \left. 2 i \operatorname{ArcSin} \left[\sqrt{\frac{c^2 d}{c^2 d + e}} \right] \operatorname{Log} \left[\frac{e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]} \left(2 \sqrt{-c^2 d e} + e \left(-1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) + c^2 d \left(1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) \right)}{c^2 d + e} \right] - \right. \\
& \left. 2 \operatorname{ArcTanh} [c \sqrt{x}] \operatorname{Log} \left[\frac{e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]} \left(2 \sqrt{-c^2 d e} + e \left(-1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) + c^2 d \left(1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) \right)}{c^2 d + e} \right] - 2 \operatorname{PolyLog} [2, -e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]}] + \right. \\
& \left. \operatorname{PolyLog} \left[2, \frac{\left(-c^2 d + e - 2 \sqrt{-c^2 d e} \right) e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]} }{c^2 d + e} \right] + \operatorname{PolyLog} \left[2, \frac{\left(-c^2 d + e + 2 \sqrt{-c^2 d e} \right) e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]} }{c^2 d + e} \right] \right)
\end{aligned}$$

Problem 47: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcTanh} [c \sqrt{x}]}{x (d + e x)} dx$$

Optimal (type 4, 358 leaves, 15 steps):

$$\begin{aligned}
& \frac{2 \left(a + b \operatorname{ArcTanh} [c \sqrt{x}] \right) \operatorname{Log} \left[\frac{2}{1+c \sqrt{x}} \right]}{d} - \frac{\left(a + b \operatorname{ArcTanh} [c \sqrt{x}] \right) \operatorname{Log} \left[\frac{2 c \left(\sqrt{-d} - \sqrt{e} \sqrt{x} \right)}{\left(c \sqrt{-d} - \sqrt{e} \right) \left(1+c \sqrt{x} \right)} \right]}{d} - \\
& \frac{\left(a + b \operatorname{ArcTanh} [c \sqrt{x}] \right) \operatorname{Log} \left[\frac{2 c \left(\sqrt{-d} + \sqrt{e} \sqrt{x} \right)}{\left(c \sqrt{-d} + \sqrt{e} \right) \left(1+c \sqrt{x} \right)} \right]}{d} + \frac{a \operatorname{Log} [x]}{d} - \frac{b \operatorname{PolyLog} \left[2, 1 - \frac{2}{1+c \sqrt{x}} \right]}{d} + \\
& \frac{b \operatorname{PolyLog} \left[2, 1 - \frac{2 c \left(\sqrt{-d} - \sqrt{e} \sqrt{x} \right)}{\left(c \sqrt{-d} - \sqrt{e} \right) \left(1+c \sqrt{x} \right)} \right]}{2 d} + \frac{b \operatorname{PolyLog} \left[2, 1 - \frac{2 c \left(\sqrt{-d} + \sqrt{e} \sqrt{x} \right)}{\left(c \sqrt{-d} + \sqrt{e} \right) \left(1+c \sqrt{x} \right)} \right]}{2 d} - \frac{b \operatorname{PolyLog} \left[2, -c \sqrt{x} \right]}{d} + \frac{b \operatorname{PolyLog} \left[2, c \sqrt{x} \right]}{d}
\end{aligned}$$

Result (type 4, 563 leaves):

$$\begin{aligned}
& \frac{1}{2 d} \left(4 i b \operatorname{ArcSin} \left[\sqrt{\frac{c^2 d}{c^2 d + e}} \right] \operatorname{ArcTanh} \left[\frac{c e \sqrt{x}}{\sqrt{-c^2 d e}} \right] + 4 b \operatorname{ArcTanh} [c \sqrt{x}] \operatorname{Log} \left[1 - e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]} \right] + \right. \\
& 2 i b \operatorname{ArcSin} \left[\sqrt{\frac{c^2 d}{c^2 d + e}} \right] \operatorname{Log} \left[\frac{e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]} \left(-2 \sqrt{-c^2 d e} + e \left(-1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) + c^2 d \left(1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) \right)}{c^2 d + e} \right] - \\
& 2 b \operatorname{ArcTanh} [c \sqrt{x}] \operatorname{Log} \left[\frac{e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]} \left(-2 \sqrt{-c^2 d e} + e \left(-1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) + c^2 d \left(1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) \right)}{c^2 d + e} \right] - \\
& 2 i b \operatorname{ArcSin} \left[\sqrt{\frac{c^2 d}{c^2 d + e}} \right] \operatorname{Log} \left[\frac{e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]} \left(2 \sqrt{-c^2 d e} + e \left(-1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) + c^2 d \left(1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) \right)}{c^2 d + e} \right] - \\
& 2 b \operatorname{ArcTanh} [c \sqrt{x}] \operatorname{Log} \left[\frac{e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]} \left(2 \sqrt{-c^2 d e} + e \left(-1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) + c^2 d \left(1 + e^{2 \operatorname{ArcTanh} [c \sqrt{x}]} \right) \right)}{c^2 d + e} \right] + \\
& 2 a \operatorname{Log} [x] - 2 a \operatorname{Log} [d + e x] - 2 b \operatorname{PolyLog} \left[2, e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]} \right] + \\
& \left. b \operatorname{PolyLog} \left[2, \frac{\left(-c^2 d + e - 2 \sqrt{-c^2 d e} \right) e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]} }{c^2 d + e} \right] + b \operatorname{PolyLog} \left[2, \frac{\left(-c^2 d + e + 2 \sqrt{-c^2 d e} \right) e^{-2 \operatorname{ArcTanh} [c \sqrt{x}]} }{c^2 d + e} \right] \right)
\end{aligned}$$

Problem 48: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcTanh} [c \sqrt{x}]}{x^2 (d + e x)} dx$$

Optimal (type 4, 413 leaves, 19 steps):

$$\begin{aligned}
 & -\frac{bc}{d\sqrt{x}} + \frac{bc^2 \operatorname{ArcTanh}[c\sqrt{x}]}{d} - \frac{a + b \operatorname{ArcTanh}[c\sqrt{x}]}{dx} - \frac{2e \left(a + b \operatorname{ArcTanh}[c\sqrt{x}] \right) \operatorname{Log}\left[\frac{2}{1+c\sqrt{x}}\right]}{d^2} + \\
 & \frac{e \left(a + b \operatorname{ArcTanh}[c\sqrt{x}] \right) \operatorname{Log}\left[\frac{2c(\sqrt{-d}-\sqrt{e}\sqrt{x})}{(c\sqrt{-d}-\sqrt{e})(1+c\sqrt{x})}\right]}{d^2} + \frac{e \left(a + b \operatorname{ArcTanh}[c\sqrt{x}] \right) \operatorname{Log}\left[\frac{2c(\sqrt{-d}+\sqrt{e}\sqrt{x})}{(c\sqrt{-d}+\sqrt{e})(1+c\sqrt{x})}\right]}{d^2} - \frac{ae \operatorname{Log}[x]}{d^2} + \frac{be \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c\sqrt{x}}\right]}{d^2} - \\
 & \frac{be \operatorname{PolyLog}\left[2, 1 - \frac{2c(\sqrt{-d}-\sqrt{e}\sqrt{x})}{(c\sqrt{-d}-\sqrt{e})(1+c\sqrt{x})}\right]}{2d^2} - \frac{be \operatorname{PolyLog}\left[2, 1 - \frac{2c(\sqrt{-d}+\sqrt{e}\sqrt{x})}{(c\sqrt{-d}+\sqrt{e})(1+c\sqrt{x})}\right]}{2d^2} + \frac{be \operatorname{PolyLog}\left[2, -c\sqrt{x}\right]}{d^2} - \frac{be \operatorname{PolyLog}\left[2, c\sqrt{x}\right]}{d^2}
 \end{aligned}$$

Result (type 4, 567 leaves):

$$\begin{aligned}
& -\frac{1}{2d^2x} \left(2ad + 2aex \operatorname{Log}[x] - 2aex \operatorname{Log}[d+ex] + \right. \\
& 2b \left(cd\sqrt{x} + \operatorname{ArcTanh}[c\sqrt{x}] \left(d - c^2dx + ex \operatorname{ArcTanh}[c\sqrt{x}] + 2ex \operatorname{Log}[1 - e^{-2\operatorname{ArcTanh}[c\sqrt{x}]}] \right) - ex \operatorname{PolyLog}\left[2, e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \right] \right) + \\
& bex \left(-2 \left(\operatorname{ArcTanh}[c\sqrt{x}]^2 - \right. \right. \\
& \left. \left. i \operatorname{ArcSin}\left[\sqrt{\frac{c^2d}{c^2d+e}}\right] \left(2 \operatorname{ArcTanh}\left[\frac{ce\sqrt{x}}{\sqrt{-c^2de}}\right] + \operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \left(-2\sqrt{-c^2de} + e \left(-1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) + c^2d \left(1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) \right)}{c^2d+e}\right] \right) - \right. \\
& \left. \operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \left(2\sqrt{-c^2de} + e \left(-1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) + c^2d \left(1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) \right)}{c^2d+e}\right] \right) + \\
& \left. \operatorname{ArcTanh}[c\sqrt{x}] \left(\operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \left(-2\sqrt{-c^2de} + e \left(-1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) + c^2d \left(1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) \right)}{c^2d+e}\right] \right) + \right. \\
& \left. \operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \left(2\sqrt{-c^2de} + e \left(-1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) + c^2d \left(1 + e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) \right)}{c^2d+e}\right] \right) \right) + \\
& \left. \operatorname{PolyLog}\left[2, \frac{(-c^2d+e-2\sqrt{-c^2de})e^{-2\operatorname{ArcTanh}[c\sqrt{x}]}}{c^2d+e}\right] + \operatorname{PolyLog}\left[2, \frac{(-c^2d+e+2\sqrt{-c^2de})e^{-2\operatorname{ArcTanh}[c\sqrt{x}]}}{c^2d+e}\right] \right) \right)
\end{aligned}$$

Problem 49: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcTanh}[c\sqrt{x}]}{x^3(d+ex)} dx$$

Optimal (type 4, 506 leaves, 24 steps):

$$\begin{aligned}
& -\frac{bc}{6dx^{3/2}} - \frac{bc^3}{2d\sqrt{x}} + \frac{bce}{d^2\sqrt{x}} + \frac{bc^4 \operatorname{ArcTanh}[c\sqrt{x}]}{2d} - \frac{bc^2 e \operatorname{ArcTanh}[c\sqrt{x}]}{d^2} - \frac{a + b \operatorname{ArcTanh}[c\sqrt{x}]}{2dx^2} + \\
& \frac{e(a + b \operatorname{ArcTanh}[c\sqrt{x}])}{d^2 x} + \frac{2e^2(a + b \operatorname{ArcTanh}[c\sqrt{x}]) \operatorname{Log}\left[\frac{2}{1+c\sqrt{x}}\right]}{d^3} - \frac{e^2(a + b \operatorname{ArcTanh}[c\sqrt{x}]) \operatorname{Log}\left[\frac{2c(\sqrt{-d}-\sqrt{e}\sqrt{x})}{(c\sqrt{-d}-\sqrt{e})(1+c\sqrt{x})}\right]}{d^3} - \\
& \frac{e^2(a + b \operatorname{ArcTanh}[c\sqrt{x}]) \operatorname{Log}\left[\frac{2c(\sqrt{-d}+\sqrt{e}\sqrt{x})}{(c\sqrt{-d}+\sqrt{e})(1+c\sqrt{x})}\right]}{d^3} + \frac{ae^2 \operatorname{Log}[x]}{d^3} - \frac{be^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c\sqrt{x}}\right]}{d^3} + \\
& \frac{be^2 \operatorname{PolyLog}\left[2, 1 - \frac{2c(\sqrt{-d}-\sqrt{e}\sqrt{x})}{(c\sqrt{-d}-\sqrt{e})(1+c\sqrt{x})}\right]}{2d^3} + \frac{be^2 \operatorname{PolyLog}\left[2, 1 - \frac{2c(\sqrt{-d}+\sqrt{e}\sqrt{x})}{(c\sqrt{-d}+\sqrt{e})(1+c\sqrt{x})}\right]}{2d^3} - \frac{be^2 \operatorname{PolyLog}[2, -c\sqrt{x}]}{d^3} + \frac{be^2 \operatorname{PolyLog}[2, c\sqrt{x}]}{d^3}
\end{aligned}$$

Result (type 4, 626 leaves):

$$\begin{aligned}
& -\frac{1}{6d^3x^2} \left(3ad^2 - 6adex - 6ae^2x^2 \operatorname{Log}[x] + 6ae^2x^2 \operatorname{Log}[d+ex] + \right. \\
& \left. b \left(cd\sqrt{x} (d+3c^2dx-6ex) - 3\operatorname{ArcTanh}[c\sqrt{x}] \left(d(-1+c^2x)(d+c^2dx-2ex) + 2e^2x^2 \operatorname{ArcTanh}[c\sqrt{x}] + 4e^2x^2 \operatorname{Log}[1-e^{-2\operatorname{ArcTanh}[c\sqrt{x}}]] \right) \right) + \right. \\
& \left. 6e^2x^2 \operatorname{PolyLog}\left[2, e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \right] - 3e^2x^2 \left(-2 \left(\operatorname{ArcTanh}[c\sqrt{x}]^2 - i \operatorname{ArcSin}\left[\sqrt{\frac{c^2d}{c^2d+e}} \right] \right) \right. \right. \\
& \left. \left(2\operatorname{ArcTanh}\left[\frac{ce\sqrt{x}}{\sqrt{-c^2de}}\right] + \operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \left(-2\sqrt{-c^2de} + e(-1+e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) + c^2d(1+e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right)}{c^2d+e}\right] \right) - \right. \\
& \left. \operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \left(2\sqrt{-c^2de} + e(-1+e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) + c^2d(1+e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right)}{c^2d+e}\right] \right) + \\
& \left. \operatorname{ArcTanh}[c\sqrt{x}] \left(\operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \left(-2\sqrt{-c^2de} + e(-1+e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) + c^2d(1+e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right)}{c^2d+e}\right] \right) + \right. \\
& \left. \operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[c\sqrt{x}]} \left(2\sqrt{-c^2de} + e(-1+e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right) + c^2d(1+e^{2\operatorname{ArcTanh}[c\sqrt{x}]} \right)}{c^2d+e}\right] \right) \right) + \\
& \left. \operatorname{PolyLog}\left[2, \frac{(-c^2d+e-2\sqrt{-c^2de})e^{-2\operatorname{ArcTanh}[c\sqrt{x}]}}{c^2d+e}\right] + \operatorname{PolyLog}\left[2, \frac{(-c^2d+e+2\sqrt{-c^2de})e^{-2\operatorname{ArcTanh}[c\sqrt{x}]}}{c^2d+e}\right] \right) \right)
\end{aligned}$$

Test results for the 538 problems in "7.3.4 u (a+b arctanh(c x))^p.m"

Problem 4: Result more than twice size of optimal antiderivative.

$$\int (d+cdx)(a+b\operatorname{ArcTanh}[cx]) dx$$

Optimal (type 3, 44 leaves, 4 steps):

$$\frac{b d x}{2} + \frac{d (1 + c x)^2 (a + b \operatorname{ArcTanh}[c x])}{2 c} + \frac{b d \operatorname{Log}[1 - c x]}{c}$$

Result (type 3, 95 leaves):

$$a d x + \frac{b d x}{2} + \frac{1}{2} a c d x^2 + b d x \operatorname{ArcTanh}[c x] + \frac{1}{2} b c d x^2 \operatorname{ArcTanh}[c x] + \frac{b d \operatorname{Log}[1 - c x]}{4 c} - \frac{b d \operatorname{Log}[1 + c x]}{4 c} + \frac{b d \operatorname{Log}[1 - c^2 x^2]}{2 c}$$

Problem 72: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + c d x) (a + b \operatorname{ArcTanh}[c x])^2}{x} dx$$

Optimal (type 4, 191 leaves, 13 steps):

$$\begin{aligned} & d (a + b \operatorname{ArcTanh}[c x])^2 + c d x (a + b \operatorname{ArcTanh}[c x])^2 + 2 d (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x}\right] - \\ & 2 b d (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1 - c x}\right] - b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] - b d (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] + \\ & b d (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x}\right] + \frac{1}{2} b^2 d \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x}\right] - \frac{1}{2} b^2 d \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x}\right] \end{aligned}$$

Result (type 4, 228 leaves):

$$\begin{aligned} & d \left(a^2 c x + a^2 \operatorname{Log}[c x] + a b (2 c x \operatorname{ArcTanh}[c x] + \operatorname{Log}[1 - c^2 x^2]) + b^2 \right. \\ & \quad \left. (\operatorname{ArcTanh}[c x] ((-1 + c x) \operatorname{ArcTanh}[c x] - 2 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c x]}]) + \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x]}]) + a b (-\operatorname{PolyLog}[2, -c x] + \operatorname{PolyLog}[2, c x]) + \right. \\ & \quad \left. b^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c x]^3 - \operatorname{ArcTanh}[c x]^2 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x] \right. \right. \\ & \quad \left. \left. \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x]}] + \frac{1}{2} \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c x]}] - \frac{1}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x]}] \right) \right) \end{aligned}$$

Problem 73: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + c d x) (a + b \operatorname{ArcTanh}[c x])^2}{x^2} dx$$

Optimal (type 4, 201 leaves, 12 steps):

$$\begin{aligned}
& c d (a + b \operatorname{ArcTanh}[c x])^2 - \frac{d (a + b \operatorname{ArcTanh}[c x])^2}{x} + 2 c d (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x}\right] + 2 b c d (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[2 - \frac{2}{1 + c x}\right] - \\
& b c d (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] + b c d (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x}\right] - \\
& b^2 c d \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x}\right] + \frac{1}{2} b^2 c d \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x}\right] - \frac{1}{2} b^2 c d \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x}\right]
\end{aligned}$$

Result (type 4, 249 leaves):

$$\begin{aligned}
& -\frac{1}{x} d \left(a^2 - a^2 c x \operatorname{Log}[x] + a b (2 \operatorname{ArcTanh}[c x] + c x (-2 \operatorname{Log}[c x] + \operatorname{Log}[1 - c^2 x^2])) + \right. \\
& \quad b^2 (\operatorname{ArcTanh}[c x] ((1 - c x) \operatorname{ArcTanh}[c x] - 2 c x \operatorname{Log}[1 - e^{-2 \operatorname{ArcTanh}[c x]}]) + c x \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcTanh}[c x]}]) + \\
& \quad a b c x (\operatorname{PolyLog}[2, -c x] - \operatorname{PolyLog}[2, c x]) - \\
& \quad b^2 c x \left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c x]^3 - \operatorname{ArcTanh}[c x]^2 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x] \right. \\
& \quad \left. \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x]}] + \frac{1}{2} \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c x]}] - \frac{1}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x]}] \right) \Big)
\end{aligned}$$

Problem 80: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + c d x)^2 (a + b \operatorname{ArcTanh}[c x])^2}{x} dx$$

Optimal (type 4, 278 leaves, 19 steps):

$$\begin{aligned}
& a b c d^2 x + b^2 c d^2 x \operatorname{ArcTanh}[c x] + \frac{3}{2} d^2 (a + b \operatorname{ArcTanh}[c x])^2 + 2 c d^2 x (a + b \operatorname{ArcTanh}[c x])^2 + \\
& \frac{1}{2} c^2 d^2 x^2 (a + b \operatorname{ArcTanh}[c x])^2 + 2 d^2 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x}\right] - 4 b d^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1 - c x}\right] + \\
& \frac{1}{2} b^2 d^2 \operatorname{Log}[1 - c^2 x^2] - 2 b^2 d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] - b d^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] + \\
& b d^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x}\right] + \frac{1}{2} b^2 d^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x}\right] - \frac{1}{2} b^2 d^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x}\right]
\end{aligned}$$

Result (type 4, 324 leaves):

$$\begin{aligned} & \frac{1}{2} d^2 \left(4 a^2 c x + a^2 c^2 x^2 + 2 a^2 \operatorname{Log}[c x] + a b (2 c x + 2 c^2 x^2 \operatorname{ArcTanh}[c x] + \operatorname{Log}[1 - c x] - \operatorname{Log}[1 + c x]) + \right. \\ & 4 a b (2 c x \operatorname{ArcTanh}[c x] + \operatorname{Log}[1 - c^2 x^2]) + b^2 (2 c x \operatorname{ArcTanh}[c x] + (-1 + c^2 x^2) \operatorname{ArcTanh}[c x]^2 + \operatorname{Log}[1 - c^2 x^2]) + \\ & 4 b^2 (\operatorname{ArcTanh}[c x] ((-1 + c x) \operatorname{ArcTanh}[c x] - 2 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c x]}]) + \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x]}]) + \\ & 2 a b (-\operatorname{PolyLog}[2, -c x] + \operatorname{PolyLog}[2, c x]) + \\ & \left. 2 b^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c x]^3 - \operatorname{ArcTanh}[c x]^2 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x] \right. \right. \\ & \left. \left. \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x]}] + \frac{1}{2} \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c x]}] - \frac{1}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x]}] \right) \right) \end{aligned}$$

Problem 81: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + c d x)^2 (a + b \operatorname{ArcTanh}[c x])^2}{x^2} dx$$

Optimal (type 4, 283 leaves, 17 steps):

$$\begin{aligned} & 2 c d^2 (a + b \operatorname{ArcTanh}[c x])^2 - \frac{d^2 (a + b \operatorname{ArcTanh}[c x])^2}{x} + c^2 d^2 x (a + b \operatorname{ArcTanh}[c x])^2 + 4 c d^2 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x}\right] - \\ & 2 b c d^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1 - c x}\right] + 2 b c d^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[2 - \frac{2}{1 + c x}\right] - b^2 c d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] - \\ & 2 b c d^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] + 2 b c d^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x}\right] - \\ & b^2 c d^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x}\right] + b^2 c d^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x}\right] - b^2 c d^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x}\right] \end{aligned}$$

Result (type 4, 341 leaves):

$$\begin{aligned} & \frac{1}{12 x} d^2 (-12 a^2 + i b^2 c \pi^3 x + 12 a^2 c^2 x^2 - 24 a b \operatorname{ArcTanh}[c x] + 24 a b c^2 x^2 \operatorname{ArcTanh}[c x]) - \\ & 12 b^2 \operatorname{ArcTanh}[c x]^2 + 12 b^2 c^2 x^2 \operatorname{ArcTanh}[c x]^2 - 16 b^2 c x \operatorname{ArcTanh}[c x]^3 + 24 b^2 c x \operatorname{ArcTanh}[c x] \operatorname{Log}[1 - e^{-2 \operatorname{ArcTanh}[c x]}] - \\ & 24 b^2 c x \operatorname{ArcTanh}[c x] \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c x]}] - 24 b^2 c x \operatorname{ArcTanh}[c x]^2 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c x]}] + 24 b^2 c x \operatorname{ArcTanh}[c x]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c x]}] + \\ & 24 a^2 c x \operatorname{Log}[x] + 24 a b c x \operatorname{Log}[c x] + 12 b^2 c x (1 + 2 \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x]}] - \\ & 12 b^2 c x \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcTanh}[c x]}] + 24 b^2 c x \operatorname{ArcTanh}[c x] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x]}] - 24 a b c x \operatorname{PolyLog}[2, -c x] + \\ & 24 a b c x \operatorname{PolyLog}[2, c x] + 12 b^2 c x \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c x]}] - 12 b^2 c x \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x]}] \end{aligned}$$

Problem 82: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + c d x)^2 (a + b \operatorname{ArcTanh}[c x])^2}{x^3} dx$$

Optimal (type 4, 313 leaves, 20 steps):

$$\begin{aligned}
& -\frac{b c d^2 (a + b \operatorname{ArcTanh}[c x])}{x} + \frac{5}{2} c^2 d^2 (a + b \operatorname{ArcTanh}[c x])^2 - \frac{d^2 (a + b \operatorname{ArcTanh}[c x])^2}{2 x^2} - \frac{2 c d^2 (a + b \operatorname{ArcTanh}[c x])^2}{x} \\
& + 2 c^2 d^2 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x}\right] + b^2 c^2 d^2 \operatorname{Log}[x] - \frac{1}{2} b^2 c^2 d^2 \operatorname{Log}[1 - c^2 x^2] + 4 b c^2 d^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[2 - \frac{2}{1 + c x}\right] - \\
& b c^2 d^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] + b c^2 d^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x}\right] - \\
& 2 b^2 c^2 d^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x}\right] + \frac{1}{2} b^2 c^2 d^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x}\right] - \frac{1}{2} b^2 c^2 d^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x}\right]
\end{aligned}$$

Result (type 4, 370 leaves):

$$\begin{aligned}
& -\frac{1}{2 x^2} d^2 \left(a^2 + 4 a^2 c x - 2 a^2 c^2 x^2 \operatorname{Log}[x] + a b (2 \operatorname{ArcTanh}[c x] + c x (2 + c x \operatorname{Log}[1 - c x] - c x \operatorname{Log}[1 + c x])) + \right. \\
& b^2 \left(2 c x \operatorname{ArcTanh}[c x] + (1 - c^2 x^2) \operatorname{ArcTanh}[c x]^2 - 2 c^2 x^2 \operatorname{Log}\left[\frac{c x}{\sqrt{1 - c^2 x^2}}\right] \right) + 4 a b c x (2 \operatorname{ArcTanh}[c x] + c x (-2 \operatorname{Log}[c x] + \operatorname{Log}[1 - c^2 x^2])) + \\
& 4 b^2 c x (\operatorname{ArcTanh}[c x] ((1 - c x) \operatorname{ArcTanh}[c x] - 2 c x \operatorname{Log}[1 - e^{-2 \operatorname{ArcTanh}[c x]}]) + c x \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcTanh}[c x]}]) + \\
& 2 a b c^2 x^2 (\operatorname{PolyLog}[2, -c x] - \operatorname{PolyLog}[2, c x]) - \\
& 2 b^2 c^2 x^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c x]^3 - \operatorname{ArcTanh}[c x]^2 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x] \right. \\
& \left. \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x]}] + \frac{1}{2} \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c x]}] - \frac{1}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x]}] \right) \Big)
\end{aligned}$$

Problem 88: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + c d x)^3 (a + b \operatorname{ArcTanh}[c x])^2}{x} dx$$

Optimal (type 4, 355 leaves, 28 steps):

$$\begin{aligned}
& 3 a b c d^3 x + \frac{1}{3} b^2 c d^3 x - \frac{1}{3} b^2 d^3 \operatorname{ArcTanh}[c x] + 3 b^2 c d^3 x \operatorname{ArcTanh}[c x] + \frac{1}{3} b c^2 d^3 x^2 (a + b \operatorname{ArcTanh}[c x]) + \\
& \frac{11}{6} d^3 (a + b \operatorname{ArcTanh}[c x])^2 + 3 c d^3 x (a + b \operatorname{ArcTanh}[c x])^2 + \frac{3}{2} c^2 d^3 x^2 (a + b \operatorname{ArcTanh}[c x])^2 + \\
& \frac{1}{3} c^3 d^3 x^3 (a + b \operatorname{ArcTanh}[c x])^2 + 2 d^3 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x}\right] - \frac{20}{3} b d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1 - c x}\right] + \\
& \frac{3}{2} b^2 d^3 \operatorname{Log}[1 - c^2 x^2] - \frac{10}{3} b^2 d^3 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] - b d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] + \\
& b d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x}\right] + \frac{1}{2} b^2 d^3 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x}\right] - \frac{1}{2} b^2 d^3 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x}\right]
\end{aligned}$$

Result (type 4, 448 leaves):

$$\frac{1}{24} d^3 \left(i b^2 \pi^3 + 72 a^2 c x + 72 a b c x + 8 b^2 c x + 36 a^2 c^2 x^2 + 8 a b c^2 x^2 + 8 a^2 c^3 x^3 - 8 b^2 \operatorname{ArcTanh}[c x] + \right. \\ \left. 144 a b c x \operatorname{ArcTanh}[c x] + 72 b^2 c x \operatorname{ArcTanh}[c x] + 72 a b c^2 x^2 \operatorname{ArcTanh}[c x] + 8 b^2 c^2 x^2 \operatorname{ArcTanh}[c x] + 16 a b c^3 x^3 \operatorname{ArcTanh}[c x] - \right. \\ \left. 116 b^2 \operatorname{ArcTanh}[c x]^2 + 72 b^2 c x \operatorname{ArcTanh}[c x]^2 + 36 b^2 c^2 x^2 \operatorname{ArcTanh}[c x]^2 + 8 b^2 c^3 x^3 \operatorname{ArcTanh}[c x]^2 - 16 b^2 \operatorname{ArcTanh}[c x]^3 - \right. \\ \left. 160 b^2 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c x]}\right] - 24 b^2 \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c x]}\right] + 24 b^2 \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x]}\right] + \right. \\ \left. 24 a^2 \operatorname{Log}[c x] + 36 a b \operatorname{Log}[1 - c x] - 36 a b \operatorname{Log}[1 + c x] + 72 a b \operatorname{Log}[1 - c^2 x^2] + 36 b^2 \operatorname{Log}[1 - c^2 x^2] + 8 a b \operatorname{Log}[-1 + c^2 x^2] + \right. \\ \left. 8 b^2 (10 + 3 \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[c x]}\right] + 24 b^2 \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] - \right. \\ \left. 24 a b \operatorname{PolyLog}\left[2, -c x\right] + 24 a b \operatorname{PolyLog}\left[2, c x\right] + 12 b^2 \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}[c x]}\right] - 12 b^2 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right] \right)$$

Problem 89: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + c d x)^3 (a + b \operatorname{ArcTanh}[c x])^2}{x^2} dx$$

Optimal (type 4, 361 leaves, 23 steps):

$$a b c^2 d^3 x + b^2 c^2 d^3 x \operatorname{ArcTanh}[c x] + \frac{7}{2} c d^3 (a + b \operatorname{ArcTanh}[c x])^2 - \frac{d^3 (a + b \operatorname{ArcTanh}[c x])^2}{x} + 3 c^2 d^3 x (a + b \operatorname{ArcTanh}[c x])^2 + \\ \frac{1}{2} c^3 d^3 x^2 (a + b \operatorname{ArcTanh}[c x])^2 + 6 c d^3 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x}\right] - 6 b c d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1 - c x}\right] + \\ \frac{1}{2} b^2 c d^3 \operatorname{Log}[1 - c^2 x^2] + 2 b c d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[2 - \frac{2}{1 + c x}\right] - 3 b^2 c d^3 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] - \\ 3 b c d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] + 3 b c d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x}\right] - \\ b^2 c d^3 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x}\right] + \frac{3}{2} b^2 c d^3 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x}\right] - \frac{3}{2} b^2 c d^3 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x}\right]$$

Result (type 4, 479 leaves):

$$\frac{1}{8 x} d^3 \left(-8 a^2 + i b^2 c \pi^3 x + 24 a^2 c^2 x^2 + 8 a b c^2 x^2 + 4 a^2 c^3 x^3 - 16 a b \operatorname{ArcTanh}[c x] + 48 a b c^2 x^2 \operatorname{ArcTanh}[c x] + 8 b^2 c^2 x^2 \operatorname{ArcTanh}[c x] + \right. \\ \left. 8 a b c^3 x^3 \operatorname{ArcTanh}[c x] - 8 b^2 \operatorname{ArcTanh}[c x]^2 - 20 b^2 c x \operatorname{ArcTanh}[c x]^2 + 24 b^2 c^2 x^2 \operatorname{ArcTanh}[c x]^2 + 4 b^2 c^3 x^3 \operatorname{ArcTanh}[c x]^2 - \right. \\ \left. 16 b^2 c x \operatorname{ArcTanh}[c x]^3 + 16 b^2 c x \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] - 48 b^2 c x \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c x]}\right] - \right. \\ \left. 24 b^2 c x \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c x]}\right] + 24 b^2 c x \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x]}\right] + 24 a^2 c x \operatorname{Log}[x] + \right. \\ \left. 16 a b c x \operatorname{Log}[c x] + 4 a b c x \operatorname{Log}[1 - c x] - 4 a b c x \operatorname{Log}[1 + c x] + 16 a b c x \operatorname{Log}[1 - c^2 x^2] + 4 b^2 c x \operatorname{Log}[1 - c^2 x^2] + \right. \\ \left. 24 b^2 c x (1 + \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[c x]}\right] - 8 b^2 c x \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] + 24 b^2 c x \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] - \right. \\ \left. 24 a b c x \operatorname{PolyLog}\left[2, -c x\right] + 24 a b c x \operatorname{PolyLog}\left[2, c x\right] + 12 b^2 c x \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}[c x]}\right] - 12 b^2 c x \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right] \right)$$

Problem 90: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + c d x)^3 (a + b \operatorname{ArcTanh}[c x])^2}{x^3} dx$$

Optimal (type 4, 385 leaves, 25 steps):

$$\begin{aligned} & -\frac{b c d^3 (a + b \operatorname{ArcTanh}[c x])}{x} + \frac{9}{2} c^2 d^3 (a + b \operatorname{ArcTanh}[c x])^2 - \frac{d^3 (a + b \operatorname{ArcTanh}[c x])^2}{2 x^2} - \frac{3 c d^3 (a + b \operatorname{ArcTanh}[c x])^2}{x} + \\ & c^3 d^3 x (a + b \operatorname{ArcTanh}[c x])^2 + 6 c^2 d^3 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x}\right] + b^2 c^2 d^3 \operatorname{Log}[x] - 2 b c^2 d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1 - c x}\right] - \\ & \frac{1}{2} b^2 c^2 d^3 \operatorname{Log}[1 - c^2 x^2] + 6 b c^2 d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[2 - \frac{2}{1 + c x}\right] - b^2 c^2 d^3 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] - \\ & 3 b c^2 d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] + 3 b c^2 d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x}\right] - \\ & 3 b^2 c^2 d^3 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x}\right] + \frac{3}{2} b^2 c^2 d^3 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x}\right] - \frac{3}{2} b^2 c^2 d^3 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x}\right] \end{aligned}$$

Result (type 4, 461 leaves):

$$\begin{aligned} & \frac{1}{2} d^3 \left(-\frac{a^2}{x^2} - \frac{6 a^2 c}{x} + 2 a^2 c^3 x + 6 a^2 c^2 \operatorname{Log}[x] - \frac{a b (2 \operatorname{ArcTanh}[c x] + c x (2 + c x \operatorname{Log}[1 - c x] - c x \operatorname{Log}[1 + c x]))}{x^2} + \right. \\ & \left. \frac{b^2 \left(-2 c x \operatorname{ArcTanh}[c x] + (-1 + c^2 x^2) \operatorname{ArcTanh}[c x]^2 + 2 c^2 x^2 \operatorname{Log}\left[\frac{c x}{\sqrt{1 - c^2 x^2}}\right] \right)}{x^2} + \right. \\ & 2 a b c^2 (2 c x \operatorname{ArcTanh}[c x] + \operatorname{Log}[1 - c^2 x^2]) - \frac{6 a b c (2 \operatorname{ArcTanh}[c x] + c x (-2 \operatorname{Log}[c x] + \operatorname{Log}[1 - c^2 x^2]))}{x} + \\ & 2 b^2 c^2 (\operatorname{ArcTanh}[c x] ((-1 + c x) \operatorname{ArcTanh}[c x] - 2 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c x]}]) + \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x]}]) + \frac{1}{x} \\ & 6 b^2 c (\operatorname{ArcTanh}[c x] ((-1 + c x) \operatorname{ArcTanh}[c x] + 2 c x \operatorname{Log}[1 - e^{-2 \operatorname{ArcTanh}[c x]}]) - c x \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcTanh}[c x]}]) - \\ & 6 a b c^2 (\operatorname{PolyLog}[2, -c x] - \operatorname{PolyLog}[2, c x]) + \\ & 6 b^2 c^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c x]^3 - \operatorname{ArcTanh}[c x]^2 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x] \right. \\ & \left. \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c x]}] + \frac{1}{2} \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c x]}] - \frac{1}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c x]}] \right) \end{aligned}$$

Problem 91: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(d + c d x)^3 (a + b \operatorname{ArcTanh}[c x])^2}{x^4} dx$$

Optimal (type 4, 396 leaves, 28 steps):

$$\begin{aligned} & -\frac{b^2 c^2 d^3}{3 x} + \frac{1}{3} b^2 c^3 d^3 \operatorname{ArcTanh}[c x] - \frac{b c d^3 (a + b \operatorname{ArcTanh}[c x])}{3 x^2} - \frac{3 b c^2 d^3 (a + b \operatorname{ArcTanh}[c x])}{x} + \\ & \frac{29}{6} c^3 d^3 (a + b \operatorname{ArcTanh}[c x])^2 - \frac{d^3 (a + b \operatorname{ArcTanh}[c x])^2}{3 x^3} - \frac{3 c d^3 (a + b \operatorname{ArcTanh}[c x])^2}{2 x^2} - \frac{3 c^2 d^3 (a + b \operatorname{ArcTanh}[c x])^2}{x} + \\ & 2 c^3 d^3 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x}\right] + 3 b^2 c^3 d^3 \operatorname{Log}[x] - \frac{3}{2} b^2 c^3 d^3 \operatorname{Log}[1 - c^2 x^2] + \frac{20}{3} b c^3 d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[2 - \frac{2}{1 + c x}\right] - \\ & b c^3 d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right] + b c^3 d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x}\right] - \\ & \frac{10}{3} b^2 c^3 d^3 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x}\right] + \frac{1}{2} b^2 c^3 d^3 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x}\right] - \frac{1}{2} b^2 c^3 d^3 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x}\right] \end{aligned}$$

Result (type 4, 569 leaves):

$$\begin{aligned} & \frac{1}{24 x^3} \\ & d^3 \left(-8 a^2 - 36 a^2 c x - 8 a b c x - 72 a^2 c^2 x^2 - 72 a b c^2 x^2 - 8 b^2 c^2 x^2 + i b^2 c^3 \pi^3 x^3 - 16 a b \operatorname{ArcTanh}[c x] - 72 a b c x \operatorname{ArcTanh}[c x] - 8 b^2 c x \operatorname{ArcTanh}[c x] - \right. \\ & 144 a b c^2 x^2 \operatorname{ArcTanh}[c x] - 72 b^2 c^2 x^2 \operatorname{ArcTanh}[c x] + 8 b^2 c^3 x^3 \operatorname{ArcTanh}[c x] - 8 b^2 \operatorname{ArcTanh}[c x]^2 - 36 b^2 c x \operatorname{ArcTanh}[c x]^2 - \\ & 72 b^2 c^2 x^2 \operatorname{ArcTanh}[c x]^2 + 116 b^2 c^3 x^3 \operatorname{ArcTanh}[c x]^2 - 16 b^2 c^3 x^3 \operatorname{ArcTanh}[c x]^3 + 160 b^2 c^3 x^3 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] - \\ & 24 b^2 c^3 x^3 \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c x]}\right] + 24 b^2 c^3 x^3 \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x]}\right] + 24 a^2 c^3 x^3 \operatorname{Log}[x] + \\ & 160 a b c^3 x^3 \operatorname{Log}[c x] - 36 a b c^3 x^3 \operatorname{Log}[1 - c x] + 36 a b c^3 x^3 \operatorname{Log}[1 + c x] + 72 b^2 c^3 x^3 \operatorname{Log}\left[\frac{c x}{\sqrt{1 - c^2 x^2}}\right] - 80 a b c^3 x^3 \operatorname{Log}[1 - c^2 x^2] + \\ & 24 b^2 c^3 x^3 \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[c x]}\right] - 80 b^2 c^3 x^3 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] + 24 b^2 c^3 x^3 \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] - \\ & \left. 24 a b c^3 x^3 \operatorname{PolyLog}\left[2, -c x\right] + 24 a b c^3 x^3 \operatorname{PolyLog}\left[2, c x\right] + 12 b^2 c^3 x^3 \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}[c x]}\right] - 12 b^2 c^3 x^3 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right] \right) \end{aligned}$$

Problem 99: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x (d + c d x)} dx$$

Optimal (type 4, 77 leaves, 3 steps):

$$\frac{(a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[2 - \frac{2}{1+cx}\right]}{d} - \frac{b(a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+cx}\right]}{d} - \frac{b^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1+cx}\right]}{2d}$$

Result (type 4, 132 leaves):

$$\frac{1}{d} \left(a^2 \operatorname{Log}[c x] - a^2 \operatorname{Log}[1 + c x] + a b \left(2 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] - \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] \right) + b^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c x]^3 + \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x]}\right] + \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] - \frac{1}{2} \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right] \right) \right)$$

Problem 100: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x^2 (d + c d x)} dx$$

Optimal (type 4, 162 leaves, 8 steps):

$$\frac{c(a + b \operatorname{ArcTanh}[c x])^2}{d} - \frac{(a + b \operatorname{ArcTanh}[c x])^2}{d x} + \frac{2 b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[2 - \frac{2}{1+cx}\right]}{d} - \frac{c(a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[2 - \frac{2}{1+cx}\right]}{d} - \frac{b^2 c \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+cx}\right]}{d} + \frac{b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+cx}\right]}{d} + \frac{b^2 c \operatorname{PolyLog}\left[3, -1 + \frac{2}{1+cx}\right]}{2d}$$

Result (type 4, 225 leaves):

$$\frac{1}{d} \left(-\frac{a^2}{x} - a^2 c \operatorname{Log}[x] + a^2 c \operatorname{Log}[1 + c x] + \frac{1}{x} a b \left(-2 \operatorname{ArcTanh}[c x] (1 + c x \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right]) + 2 c x \operatorname{Log}\left[\frac{c x}{\sqrt{1 - c^2 x^2}}\right] + c x \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] \right) + b^2 c \left(-\frac{i \pi^3}{24} + \operatorname{ArcTanh}[c x]^2 - \frac{\operatorname{ArcTanh}[c x]^2}{c x} + \frac{2}{3} \operatorname{ArcTanh}[c x]^3 + 2 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] - \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x]}\right] - \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] - \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] + \frac{1}{2} \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right] \right) \right)$$

Problem 101: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x^3 (d + c d x)} dx$$

Optimal (type 4, 250 leaves, 17 steps):

$$\begin{aligned}
& - \frac{b c (a + b \operatorname{ArcTanh}[c x])}{d x} - \frac{c^2 (a + b \operatorname{ArcTanh}[c x])^2}{2 d} - \frac{(a + b \operatorname{ArcTanh}[c x])^2}{2 d x^2} + \frac{c (a + b \operatorname{ArcTanh}[c x])^2}{d x} \\
& + \frac{b^2 c^2 \operatorname{Log}[x]}{d} - \frac{b^2 c^2 \operatorname{Log}[1 - c^2 x^2]}{2 d} - \frac{2 b c^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[2 - \frac{2}{1+c x}\right]}{d} + \frac{c^2 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[2 - \frac{2}{1+c x}\right]}{d} \\
& + \frac{b^2 c^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+c x}\right]}{d} - \frac{b c^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+c x}\right]}{d} - \frac{b^2 c^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1+c x}\right]}{2 d}
\end{aligned}$$

Result (type 4, 317 leaves):

$$\begin{aligned}
& \frac{1}{2 d} \left(-\frac{a^2}{x^2} + \frac{2 a^2 c}{x} + 2 a^2 c^2 \operatorname{Log}[x] - 2 a^2 c^2 \operatorname{Log}[1 + c x] + \frac{1}{x^2} \right. \\
& \left. 2 a b \left(\operatorname{ArcTanh}[c x] (-1 + 2 c x + c^2 x^2 + 2 c^2 x^2 \operatorname{Log}[1 - e^{-2 \operatorname{ArcTanh}[c x]}]) - c x \left(1 + 2 c x \operatorname{Log}\left[\frac{c x}{\sqrt{1 - c^2 x^2}}\right]\right) - c^2 x^2 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] \right) + \right. \\
& \left. 2 b^2 c^2 \left(\frac{i \pi^3}{24} - \frac{\operatorname{ArcTanh}[c x]}{c x} - \frac{1}{2} \operatorname{ArcTanh}[c x]^2 - \frac{\operatorname{ArcTanh}[c x]^2}{2 c^2 x^2} + \frac{\operatorname{ArcTanh}[c x]^2}{c x} - \frac{2}{3} \operatorname{ArcTanh}[c x]^3 - \right. \right. \\
& \left. \left. 2 \operatorname{ArcTanh}[c x] \operatorname{Log}[1 - e^{-2 \operatorname{ArcTanh}[c x]}] + \operatorname{ArcTanh}[c x]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c x]}] + \operatorname{Log}\left[\frac{c x}{\sqrt{1 - c^2 x^2}}\right] + \right. \right. \\
& \left. \left. \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] + \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] - \frac{1}{2} \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right] \right) \right)
\end{aligned}$$

Problem 102: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x^4 (d + c d x)} dx$$

Optimal (type 4, 334 leaves, 26 steps):

$$\begin{aligned}
& - \frac{b^2 c^2}{3 d x} + \frac{b^2 c^3 \operatorname{ArcTanh}[c x]}{3 d} - \frac{b c (a + b \operatorname{ArcTanh}[c x])}{3 d x^2} + \frac{b c^2 (a + b \operatorname{ArcTanh}[c x])}{d x} \\
& + \frac{5 c^3 (a + b \operatorname{ArcTanh}[c x])^2}{6 d} - \frac{(a + b \operatorname{ArcTanh}[c x])^2}{3 d x^3} + \frac{c (a + b \operatorname{ArcTanh}[c x])^2}{2 d x^2} - \frac{c^2 (a + b \operatorname{ArcTanh}[c x])^2}{d x} \\
& + \frac{b^2 c^3 \operatorname{Log}[x]}{d} + \frac{b^2 c^3 \operatorname{Log}[1 - c^2 x^2]}{2 d} + \frac{8 b c^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[2 - \frac{2}{1+c x}\right]}{3 d} - \frac{c^3 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[2 - \frac{2}{1+c x}\right]}{d} \\
& + \frac{4 b^2 c^3 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+c x}\right]}{3 d} + \frac{b c^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+c x}\right]}{d} + \frac{b^2 c^3 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1+c x}\right]}{2 d}
\end{aligned}$$

Result (type 4, 388 leaves):

$$\begin{aligned}
& \frac{1}{24 d} \left(-\frac{8 a^2}{x^3} + \frac{12 a^2 c}{x^2} - \frac{24 a^2 c^2}{x} - 24 a^2 c^3 \operatorname{Log}[x] + \right. \\
& 24 a^2 c^3 \operatorname{Log}[1 + c x] - \frac{1}{x^3} 8 a b \left(\operatorname{ArcTanh}[c x] \left(2 - 3 c x + 6 c^2 x^2 + 3 c^3 x^3 + 6 c^3 x^3 \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right]\right) - \right. \\
& \left. c x \left(-1 + 3 c x + c^2 x^2 + 8 c^2 x^2 \operatorname{Log}\left[\frac{c x}{\sqrt{1 - c^2 x^2}}\right]\right) - 3 c^3 x^3 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] \right) + \\
& b^2 c^3 \left(-i \pi^3 - \frac{8}{c x} + 8 \operatorname{ArcTanh}[c x] - \frac{8 \operatorname{ArcTanh}[c x]}{c^2 x^2} + \frac{24 \operatorname{ArcTanh}[c x]}{c x} + 20 \operatorname{ArcTanh}[c x]^2 - \frac{8 \operatorname{ArcTanh}[c x]^2}{c^3 x^3} + \frac{12 \operatorname{ArcTanh}[c x]^2}{c^2 x^2} - \right. \\
& \left. \frac{24 \operatorname{ArcTanh}[c x]^2}{c x} + 16 \operatorname{ArcTanh}[c x]^3 + 64 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] - 24 \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] - \right. \\
& \left. \left. 24 \operatorname{Log}\left[\frac{c x}{\sqrt{1 - c^2 x^2}}\right] - 32 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] - 24 \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] + 12 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right] \right) \right)
\end{aligned}$$

Problem 108: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x (d + c d x)^2} dx$$

Optimal (type 4, 295 leaves, 19 steps):

$$\begin{aligned}
& \frac{b^2}{2 d^2 (1 + c x)} - \frac{b^2 \operatorname{ArcTanh}[c x]}{2 d^2} + \frac{b (a + b \operatorname{ArcTanh}[c x])}{d^2 (1 + c x)} - \frac{(a + b \operatorname{ArcTanh}[c x])^2}{2 d^2} + \\
& \frac{(a + b \operatorname{ArcTanh}[c x])^2}{d^2 (1 + c x)} + \frac{2 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x}\right]}{d^2} + \frac{(a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2}{1 + c x}\right]}{d^2} - \\
& \frac{b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right]}{d^2} + \frac{b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x}\right]}{d^2} - \\
& \frac{b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c x}\right]}{d^2} + \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x}\right]}{2 d^2} - \frac{b^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x}\right]}{2 d^2} - \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 + c x}\right]}{2 d^2}
\end{aligned}$$

Result (type 4, 254 leaves):

$$\frac{1}{24 d^2} \left(\frac{24 a^2}{1+c x} + 24 a^2 \operatorname{Log}[c x] - 24 a^2 \operatorname{Log}[1+c x] + 12 a b \left(\operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] - 2 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] + \right. \right. \\ \left. \left. 2 \operatorname{ArcTanh}[c x] \left(\operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] + 2 \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] - \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] \right) - \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] \right) + \right. \\ \left. b^2 \left(i \pi^3 - 16 \operatorname{ArcTanh}[c x]^3 + 6 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] + 12 \operatorname{ArcTanh}[c x] \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] + 12 \operatorname{ArcTanh}[c x]^2 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] + \right. \right. \\ \left. \left. 24 \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x]}\right] + 24 \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] - 12 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right] - \right. \right. \\ \left. \left. 6 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] - 12 \operatorname{ArcTanh}[c x] \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] - 12 \operatorname{ArcTanh}[c x]^2 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] \right) \right)$$

Problem 109: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x^2 (d + c d x)^2} dx$$

Optimal (type 4, 371 leaves, 23 steps):

$$-\frac{b^2 c}{2 d^2 (1+c x)} + \frac{b^2 c \operatorname{ArcTanh}[c x]}{2 d^2} - \frac{b c (a + b \operatorname{ArcTanh}[c x])}{d^2 (1+c x)} + \frac{3 c (a + b \operatorname{ArcTanh}[c x])^2}{2 d^2} - \frac{(a + b \operatorname{ArcTanh}[c x])^2}{d^2 x} - \\ \frac{c (a + b \operatorname{ArcTanh}[c x])^2}{d^2 (1+c x)} - \frac{4 c (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1-c x}\right]}{d^2} - \frac{2 c (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2}{1+c x}\right]}{d^2} + \\ \frac{2 b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[2 - \frac{2}{1+c x}\right]}{d^2} + \frac{2 b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c x}\right]}{d^2} - \\ \frac{2 b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1-c x}\right]}{d^2} + \frac{2 b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{d^2} - \\ \frac{b^2 c \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+c x}\right]}{d^2} - \frac{b^2 c \operatorname{PolyLog}\left[3, 1 - \frac{2}{1-c x}\right]}{d^2} + \frac{b^2 c \operatorname{PolyLog}\left[3, -1 + \frac{2}{1-c x}\right]}{d^2} + \frac{b^2 c \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+c x}\right]}{d^2}$$

Result (type 4, 347 leaves):

$$\frac{1}{12 d^2} \left(-\frac{12 a^2}{x} - \frac{12 a^2 c}{1+c x} - 24 a^2 c \operatorname{Log}[x] + 24 a^2 c \operatorname{Log}[1+c x] + \right. \\ \left. b^2 c \left(-i \pi^3 + 12 \operatorname{ArcTanh}[c x]^2 - \frac{12 \operatorname{ArcTanh}[c x]^2}{c x} + 16 \operatorname{ArcTanh}[c x]^3 - 3 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] - 6 \operatorname{ArcTanh}[c x] \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] - \right. \right. \\ \left. \left. 6 \operatorname{ArcTanh}[c x]^2 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] + 24 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] - 24 \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x]}\right] - \right. \right. \\ \left. \left. 12 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] - 24 \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] + 12 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right] + \right. \right. \\ \left. \left. 3 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] + 6 \operatorname{ArcTanh}[c x] \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] + 6 \operatorname{ArcTanh}[c x]^2 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] \right) \right) + \\ \left. 6 a b c \left(-\operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] + 4 \operatorname{Log}\left[\frac{c x}{\sqrt{1-c^2 x^2}}\right] + 4 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] + \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] \right) + \right. \\ \left. \operatorname{ArcTanh}[c x] \left(-\frac{4}{c x} - 2 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] - 8 \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] + 2 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] \right) \right) \right)$$

Problem 110: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x^3 (d + c d x)^2} dx$$

Optimal (type 4, 480 leaves, 31 steps):

$$\frac{b^2 c^2}{2 d^2 (1+c x)} - \frac{b^2 c^2 \operatorname{ArcTanh}[c x]}{2 d^2} - \frac{b c (a + b \operatorname{ArcTanh}[c x])}{d^2 x} + \frac{b c^2 (a + b \operatorname{ArcTanh}[c x])}{d^2 (1+c x)} - \\ \frac{2 c^2 (a + b \operatorname{ArcTanh}[c x])^2}{d^2} - \frac{(a + b \operatorname{ArcTanh}[c x])^2}{2 d^2 x^2} + \frac{2 c (a + b \operatorname{ArcTanh}[c x])^2}{d^2 x} + \frac{c^2 (a + b \operatorname{ArcTanh}[c x])^2}{d^2 (1+c x)} + \\ \frac{6 c^2 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1-c x}\right]}{d^2} + \frac{b^2 c^2 \operatorname{Log}[x]}{d^2} + \frac{3 c^2 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2}{1+c x}\right]}{d^2} - \\ \frac{b^2 c^2 \operatorname{Log}\left[1 - c^2 x^2\right]}{2 d^2} - \frac{4 b c^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[2 - \frac{2}{1+c x}\right]}{d^2} - \frac{3 b c^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c x}\right]}{d^2} + \\ \frac{3 b c^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1-c x}\right]}{d^2} - \frac{3 b c^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{d^2} + \\ \frac{2 b^2 c^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+c x}\right]}{d^2} + \frac{3 b^2 c^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1-c x}\right]}{2 d^2} - \frac{3 b^2 c^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1-c x}\right]}{2 d^2} - \frac{3 b^2 c^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+c x}\right]}{2 d^2}$$

Result (type 4, 452 leaves):

$$\begin{aligned}
& \frac{1}{8 d^2} \left(-\frac{4 a^2}{x^2} + \frac{16 a^2 c}{x} + \frac{8 a^2 c^2}{1+c x} + 24 a^2 c^2 \operatorname{Log}[x] - 24 a^2 c^2 \operatorname{Log}[1+c x] + \right. \\
& b^2 c^2 \left(i \pi^3 - \frac{8 \operatorname{ArcTanh}[c x]}{c x} - 12 \operatorname{ArcTanh}[c x]^2 - \frac{4 \operatorname{ArcTanh}[c x]^2}{c^2 x^2} + \frac{16 \operatorname{ArcTanh}[c x]^2}{c x} - 16 \operatorname{ArcTanh}[c x]^3 + 2 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] + \right. \\
& 4 \operatorname{ArcTanh}[c x] \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] + 4 \operatorname{ArcTanh}[c x]^2 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] - 32 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] + \\
& 24 \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x]}\right] + 8 \operatorname{Log}\left[\frac{c x}{\sqrt{1-c^2 x^2}}\right] + 16 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] + 24 \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] - \\
& \left. 12 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right] - 2 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] - 4 \operatorname{ArcTanh}[c x] \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] - 4 \operatorname{ArcTanh}[c x]^2 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]]\right) + \frac{1}{x^2} \\
& 4 a b \left(-6 c^2 x^2 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] + c x \left(-2 + c x \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] - 8 c x \operatorname{Log}\left[\frac{c x}{\sqrt{1-c^2 x^2}}\right] - c x \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]]\right) + \right. \\
& \left. 2 \operatorname{ArcTanh}[c x] \left(-1 + 4 c x + c^2 x^2 + c^2 x^2 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] + 6 c^2 x^2 \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] - c^2 x^2 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]]\right) \right) \left. \right)
\end{aligned}$$

Problem 116: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x (d + c d x)^3} dx$$

Optimal (type 4, 362 leaves, 32 steps):

$$\begin{aligned}
& \frac{b^2}{16 d^3 (1+c x)^2} + \frac{11 b^2}{16 d^3 (1+c x)} - \frac{11 b^2 \operatorname{ArcTanh}[c x]}{16 d^3} + \frac{b (a + b \operatorname{ArcTanh}[c x])}{4 d^3 (1+c x)^2} + \frac{5 b (a + b \operatorname{ArcTanh}[c x])}{4 d^3 (1+c x)} - \\
& \frac{5 (a + b \operatorname{ArcTanh}[c x])^2}{8 d^3} + \frac{(a + b \operatorname{ArcTanh}[c x])^2}{2 d^3 (1+c x)^2} + \frac{(a + b \operatorname{ArcTanh}[c x])^2}{d^3 (1+c x)} + \frac{2 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1-c x}\right]}{d^3} + \\
& \frac{(a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2}{1+c x}\right]}{d^3} - \frac{b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c x}\right]}{d^3} + \frac{b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1-c x}\right]}{d^3} - \\
& \frac{b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{d^3} + \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1-c x}\right]}{2 d^3} - \frac{b^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1-c x}\right]}{2 d^3} - \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+c x}\right]}{2 d^3}
\end{aligned}$$

Result (type 4, 376 leaves):

$$\frac{1}{192 d^3} \left(\frac{96 a^2}{(1 + c x)^2} + \frac{192 a^2}{1 + c x} + 192 a^2 \operatorname{Log}[c x] - 192 a^2 \operatorname{Log}[1 + c x] + \right.$$

$$12 a b \left(12 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] + \operatorname{Cosh}[4 \operatorname{ArcTanh}[c x]] - 16 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] - 12 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] + \right.$$

$$4 \operatorname{ArcTanh}[c x] \left(6 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] + \operatorname{Cosh}[4 \operatorname{ArcTanh}[c x]] + 8 \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] - 6 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] - \operatorname{Sinh}[4 \operatorname{ArcTanh}[c x]] \right) -$$

$$\operatorname{Sinh}[4 \operatorname{ArcTanh}[c x]] \left. \right) + b^2 \left(8 i \pi^3 - 128 \operatorname{ArcTanh}[c x]^3 + 72 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] + 144 \operatorname{ArcTanh}[c x] \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] + \right.$$

$$144 \operatorname{ArcTanh}[c x]^2 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] + 3 \operatorname{Cosh}[4 \operatorname{ArcTanh}[c x]] + 12 \operatorname{ArcTanh}[c x] \operatorname{Cosh}[4 \operatorname{ArcTanh}[c x]] +$$

$$24 \operatorname{ArcTanh}[c x]^2 \operatorname{Cosh}[4 \operatorname{ArcTanh}[c x]] + 192 \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c x]}\right] + 192 \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] -$$

$$96 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right] - 72 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] - 144 \operatorname{ArcTanh}[c x] \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] - 144 \operatorname{ArcTanh}[c x]^2 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] -$$

$$\left. \left. 3 \operatorname{Sinh}[4 \operatorname{ArcTanh}[c x]] - 12 \operatorname{ArcTanh}[c x] \operatorname{Sinh}[4 \operatorname{ArcTanh}[c x]] - 24 \operatorname{ArcTanh}[c x]^2 \operatorname{Sinh}[4 \operatorname{ArcTanh}[c x]] \right) \right)$$

Problem 117: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x^2 (d + c d x)^3} dx$$

Optimal (type 4, 448 leaves, 36 steps):

$$-\frac{b^2 c}{16 d^3 (1 + c x)^2} - \frac{19 b^2 c}{16 d^3 (1 + c x)} + \frac{19 b^2 c \operatorname{ArcTanh}[c x]}{16 d^3} - \frac{b c (a + b \operatorname{ArcTanh}[c x])}{4 d^3 (1 + c x)^2} - \frac{9 b c (a + b \operatorname{ArcTanh}[c x])}{4 d^3 (1 + c x)} + \frac{17 c (a + b \operatorname{ArcTanh}[c x])^2}{8 d^3}$$

$$\frac{(a + b \operatorname{ArcTanh}[c x])^2}{d^3 x} - \frac{c (a + b \operatorname{ArcTanh}[c x])^2}{2 d^3 (1 + c x)^2} - \frac{2 c (a + b \operatorname{ArcTanh}[c x])^2}{d^3 (1 + c x)} - \frac{6 c (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c x}\right]}{d^3}$$

$$\frac{3 c (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{-2}{1 + c x}\right]}{d^3} + \frac{2 b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[2 - \frac{2}{1 + c x}\right]}{d^3} + \frac{3 b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right]}{d^3}$$

$$\frac{3 b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c x}\right]}{d^3} + \frac{3 b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c x}\right]}{d^3}$$

$$\frac{b^2 c \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c x}\right]}{d^3} - \frac{3 b^2 c \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c x}\right]}{2 d^3} + \frac{3 b^2 c \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c x}\right]}{2 d^3} + \frac{3 b^2 c \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 + c x}\right]}{2 d^3}$$

Result (type 4, 479 leaves):

$$\frac{1}{64 d^3} \left(-\frac{64 a^2}{x} - \frac{32 a^2 c}{(1+c x)^2} - \frac{128 a^2 c}{1+c x} - 192 a^2 c \operatorname{Log}[x] + 192 a^2 c \operatorname{Log}[1+c x] + b^2 c \left(-8 i \pi^3 + 64 \operatorname{ArcTanh}[c x]^2 - \frac{64 \operatorname{ArcTanh}[c x]^2}{c x} + 128 \operatorname{ArcTanh}[c x]^3 - \right. \right. \\ \left. \left. 40 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] - 80 \operatorname{ArcTanh}[c x] \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] - 80 \operatorname{ArcTanh}[c x]^2 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] - \operatorname{Cosh}[4 \operatorname{ArcTanh}[c x]] - \right. \right. \\ \left. \left. 4 \operatorname{ArcTanh}[c x] \operatorname{Cosh}[4 \operatorname{ArcTanh}[c x]] - 8 \operatorname{ArcTanh}[c x]^2 \operatorname{Cosh}[4 \operatorname{ArcTanh}[c x]] + 128 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] - \right. \right. \\ \left. \left. 192 \operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] - 64 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] - 192 \operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c x]}\right] + \right. \right. \\ \left. \left. 96 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c x]}\right] + 40 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] + 80 \operatorname{ArcTanh}[c x] \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] + 80 \operatorname{ArcTanh}[c x]^2 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] + \right. \right. \\ \left. \left. \operatorname{Sinh}[4 \operatorname{ArcTanh}[c x]] + 4 \operatorname{ArcTanh}[c x] \operatorname{Sinh}[4 \operatorname{ArcTanh}[c x]] + 8 \operatorname{ArcTanh}[c x]^2 \operatorname{Sinh}[4 \operatorname{ArcTanh}[c x]] \right) + \right. \\ \left. \frac{1}{x} 4 a b \left(48 c x \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] + c x \left(-20 \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]] - \operatorname{Cosh}[4 \operatorname{ArcTanh}[c x]] + 32 \operatorname{Log}\left[\frac{c x}{\sqrt{1-c^2 x^2}}\right] + \right. \right. \right. \\ \left. \left. 20 \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] + \operatorname{Sinh}[4 \operatorname{ArcTanh}[c x]] \right) - 4 \operatorname{ArcTanh}[c x] (8 + 10 c x \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]]) + \right. \\ \left. \left. c x \operatorname{Cosh}[4 \operatorname{ArcTanh}[c x]] + 24 c x \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] - 10 c x \operatorname{Sinh}[2 \operatorname{ArcTanh}[c x]] - c x \operatorname{Sinh}[4 \operatorname{ArcTanh}[c x]] \right) \right) \right)$$

Problem 120: Result more than twice size of optimal antiderivative.

$$\int (1+c x)^3 (a+b \operatorname{ArcTanh}[c x])^3 dx$$

Optimal (type 4, 306 leaves, 26 steps):

$$3 a b^2 x + \frac{b^3 x}{4} - \frac{b^3 \operatorname{ArcTanh}[c x]}{4 c} + 3 b^3 x \operatorname{ArcTanh}[c x] + \frac{1}{4} b^2 c x^2 (a+b \operatorname{ArcTanh}[c x]) + \\ \frac{4 b (a+b \operatorname{ArcTanh}[c x])^2}{c} + \frac{21}{4} b x (a+b \operatorname{ArcTanh}[c x])^2 + \frac{3}{2} b c x^2 (a+b \operatorname{ArcTanh}[c x])^2 + \frac{1}{4} b c^2 x^3 (a+b \operatorname{ArcTanh}[c x])^2 + \\ \frac{(1+c x)^4 (a+b \operatorname{ArcTanh}[c x])^3}{4 c} - \frac{11 b^2 (a+b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1-c x}\right]}{c} - \frac{6 b (a+b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2}{1-c x}\right]}{c} + \\ \frac{3 b^3 \operatorname{Log}\left[1-c^2 x^2\right]}{2 c} - \frac{11 b^3 \operatorname{PolyLog}\left[2, 1-\frac{2}{1-c x}\right]}{2 c} - \frac{6 b^2 (a+b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1-\frac{2}{1-c x}\right]}{c} + \frac{3 b^3 \operatorname{PolyLog}\left[3, 1-\frac{2}{1-c x}\right]}{c}$$

Result (type 4, 644 leaves):

$$\frac{1}{8c} \left(-2ab^2 + 8a^3cx + 42a^2bcx + 24ab^2cx + 2b^3cx + 12a^3c^2x^2 + 12a^2bc^2x^2 + 2ab^2c^2x^2 + 8a^3c^3x^3 + 2a^2bc^3x^3 + 2a^3c^4x^4 - 24ab^2 \operatorname{ArcTanh}[cx] - \right. \\ \left. 2b^3 \operatorname{ArcTanh}[cx] + 24a^2bcx \operatorname{ArcTanh}[cx] + 84ab^2cx \operatorname{ArcTanh}[cx] + 24b^3cx \operatorname{ArcTanh}[cx] + 36a^2bc^2x^2 \operatorname{ArcTanh}[cx] + \right. \\ \left. 24ab^2c^2x^2 \operatorname{ArcTanh}[cx] + 2b^3c^2x^2 \operatorname{ArcTanh}[cx] + 24a^2bc^3x^3 \operatorname{ArcTanh}[cx] + 4ab^2c^3x^3 \operatorname{ArcTanh}[cx] + 6a^2bc^4x^4 \operatorname{ArcTanh}[cx] - \right. \\ \left. 90ab^2 \operatorname{ArcTanh}[cx]^2 - 56b^3 \operatorname{ArcTanh}[cx]^2 + 24a^2bcx \operatorname{ArcTanh}[cx]^2 + 42b^3cx \operatorname{ArcTanh}[cx]^2 + 36a^2bc^2x^2 \operatorname{ArcTanh}[cx]^2 + \right. \\ \left. 12b^3c^2x^2 \operatorname{ArcTanh}[cx]^2 + 24a^2bc^3x^3 \operatorname{ArcTanh}[cx]^2 + 2b^3c^3x^3 \operatorname{ArcTanh}[cx]^2 + 6ab^2c^4x^4 \operatorname{ArcTanh}[cx]^2 - 30b^3 \operatorname{ArcTanh}[cx]^3 + \right. \\ \left. 8b^3cx \operatorname{ArcTanh}[cx]^3 + 12b^3c^2x^2 \operatorname{ArcTanh}[cx]^3 + 8b^3c^3x^3 \operatorname{ArcTanh}[cx]^3 + 2b^3c^4x^4 \operatorname{ArcTanh}[cx]^3 - 96ab^2 \operatorname{ArcTanh}[cx] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[cx]}\right] - \right. \\ \left. 88b^3 \operatorname{ArcTanh}[cx] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[cx]}\right] - 48b^3 \operatorname{ArcTanh}[cx]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[cx]}\right] + 45a^2b \operatorname{Log}[1 - cx] + 3a^2b \operatorname{Log}[1 + cx] + \right. \\ \left. 44ab^2 \operatorname{Log}[1 - c^2x^2] + 12b^3 \operatorname{Log}[1 - c^2x^2] + 4b^2(12a + 11b + 12b \operatorname{ArcTanh}[cx]) \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[cx]}\right] + 24b^3 \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}[cx]}\right] \right)$$

Problem 121: Result more than twice size of optimal antiderivative.

$$\int (1 + cx)^2 (a + b \operatorname{ArcTanh}[cx])^3 dx$$

Optimal (type 4, 240 leaves, 17 steps):

$$ab^2x + b^3x \operatorname{ArcTanh}[cx] + \frac{5b(a + b \operatorname{ArcTanh}[cx])^2}{2c} + 3bx(a + b \operatorname{ArcTanh}[cx])^2 + \frac{1}{2}bcx^2(a + b \operatorname{ArcTanh}[cx])^2 + \\ \frac{(1 + cx)^3(a + b \operatorname{ArcTanh}[cx])^3}{3c} - \frac{6b^2(a + b \operatorname{ArcTanh}[cx]) \operatorname{Log}\left[\frac{2}{1 - cx}\right]}{c} - \frac{4b(a + b \operatorname{ArcTanh}[cx])^2 \operatorname{Log}\left[\frac{2}{1 - cx}\right]}{c} + \\ \frac{b^3 \operatorname{Log}[1 - c^2x^2]}{2c} - \frac{3b^3 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - cx}\right]}{c} - \frac{4b^2(a + b \operatorname{ArcTanh}[cx]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - cx}\right]}{c} + \frac{2b^3 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - cx}\right]}{c}$$

Result (type 4, 488 leaves):

$$\frac{1}{6c} \left(6a^3cx + 18a^2bcx + 6ab^2cx + 6a^3c^2x^2 + 3a^2bc^2x^2 + 2a^3c^3x^3 - 6ab^2 \operatorname{ArcTanh}[cx] + 18a^2bcx \operatorname{ArcTanh}[cx] + \right. \\ \left. 36ab^2cx \operatorname{ArcTanh}[cx] + 6b^3cx \operatorname{ArcTanh}[cx] + 18a^2bc^2x^2 \operatorname{ArcTanh}[cx] + 6ab^2c^2x^2 \operatorname{ArcTanh}[cx] + 6a^2bc^3x^3 \operatorname{ArcTanh}[cx] - \right. \\ \left. 42ab^2 \operatorname{ArcTanh}[cx]^2 - 21b^3 \operatorname{ArcTanh}[cx]^2 + 18a^2bcx \operatorname{ArcTanh}[cx]^2 + 18b^3cx \operatorname{ArcTanh}[cx]^2 + 18ab^2c^2x^2 \operatorname{ArcTanh}[cx]^2 + \right. \\ \left. 3b^3c^2x^2 \operatorname{ArcTanh}[cx]^2 + 6ab^2c^3x^3 \operatorname{ArcTanh}[cx]^2 - 14b^3 \operatorname{ArcTanh}[cx]^3 + 6b^3cx \operatorname{ArcTanh}[cx]^3 + 6b^3c^2x^2 \operatorname{ArcTanh}[cx]^3 + \right. \\ \left. 2b^3c^3x^3 \operatorname{ArcTanh}[cx]^3 - 48ab^2 \operatorname{ArcTanh}[cx] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[cx]}\right] - 36b^3 \operatorname{ArcTanh}[cx] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[cx]}\right] - \right. \\ \left. 24b^3 \operatorname{ArcTanh}[cx]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[cx]}\right] + 21a^2b \operatorname{Log}[1 - cx] + 3a^2b \operatorname{Log}[1 + cx] + 18ab^2 \operatorname{Log}[1 - c^2x^2] + \right. \\ \left. 3b^3 \operatorname{Log}[1 - c^2x^2] + 6b^2(4a + 3b + 4b \operatorname{ArcTanh}[cx]) \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[cx]}\right] + 12b^3 \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}[cx]}\right] \right)$$

Problem 132: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh}[ax]^3}{x^2(c + acx)} dx$$

Optimal (type 4, 191 leaves, 10 steps):

$$\frac{a \operatorname{ArcTanh}[a x]^3}{c} - \frac{\operatorname{ArcTanh}[a x]^3}{c x} + \frac{3 a \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[2 - \frac{2}{1+a x}\right]}{c} - \frac{a \operatorname{ArcTanh}[a x]^3 \operatorname{Log}\left[2 - \frac{2}{1+a x}\right]}{c} - \frac{3 a \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+a x}\right]}{c} +$$

$$\frac{3 a \operatorname{ArcTanh}[a x]^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+a x}\right]}{2 c} - \frac{3 a \operatorname{PolyLog}\left[3, -1 + \frac{2}{1+a x}\right]}{2 c} + \frac{3 a \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[3, -1 + \frac{2}{1+a x}\right]}{2 c} + \frac{3 a \operatorname{PolyLog}\left[4, -1 + \frac{2}{1+a x}\right]}{4 c}$$

Result (type 4, 154 leaves):

$$\frac{1}{c} a \left(\frac{i \pi^3}{8} - \frac{\pi^4}{64} - \operatorname{ArcTanh}[a x]^3 - \frac{\operatorname{ArcTanh}[a x]^3}{a x} + \frac{1}{2} \operatorname{ArcTanh}[a x]^4 + 3 \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[a x]}\right] - \operatorname{ArcTanh}[a x]^3 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[a x]}\right] - \right.$$

$$\left. \frac{3}{2} (-2 + \operatorname{ArcTanh}[a x]) \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[a x]}\right] + \frac{3}{2} (-1 + \operatorname{ArcTanh}[a x]) \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[a x]}\right] - \frac{3}{4} \operatorname{PolyLog}\left[4, e^{2 \operatorname{ArcTanh}[a x]}\right] \right)$$

Problem 133: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh}[a x]^3}{x^3 (c + a c x)} dx$$

Optimal (type 4, 305 leaves, 18 steps):

$$\frac{3 a^2 \operatorname{ArcTanh}[a x]^2}{2 c} - \frac{3 a \operatorname{ArcTanh}[a x]^2}{2 c x} - \frac{a^2 \operatorname{ArcTanh}[a x]^3}{2 c} - \frac{\operatorname{ArcTanh}[a x]^3}{2 c x^2} + \frac{a \operatorname{ArcTanh}[a x]^3}{c x} + \frac{3 a^2 \operatorname{ArcTanh}[a x] \operatorname{Log}\left[2 - \frac{2}{1+a x}\right]}{c} -$$

$$\frac{3 a^2 \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[2 - \frac{2}{1+a x}\right]}{c} + \frac{a^2 \operatorname{ArcTanh}[a x]^3 \operatorname{Log}\left[2 - \frac{2}{1+a x}\right]}{c} - \frac{3 a^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+a x}\right]}{2 c} + \frac{3 a^2 \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+a x}\right]}{c} -$$

$$\frac{3 a^2 \operatorname{ArcTanh}[a x]^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+a x}\right]}{2 c} + \frac{3 a^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1+a x}\right]}{2 c} - \frac{3 a^2 \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[3, -1 + \frac{2}{1+a x}\right]}{2 c} - \frac{3 a^2 \operatorname{PolyLog}\left[4, -1 + \frac{2}{1+a x}\right]}{4 c}$$

Result (type 4, 222 leaves):

$$\frac{1}{64 c} a^2 \left(-8 i \pi^3 + \pi^4 + 96 \operatorname{ArcTanh}[a x]^2 - \frac{96 \operatorname{ArcTanh}[a x]^2}{a x} + 96 \operatorname{ArcTanh}[a x]^3 - \frac{32 \operatorname{ArcTanh}[a x]^3}{a^2 x^2} + \right.$$

$$\left. \frac{64 \operatorname{ArcTanh}[a x]^3}{a x} - 32 \operatorname{ArcTanh}[a x]^4 + 192 \operatorname{ArcTanh}[a x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[a x]}\right] - 192 \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[a x]}\right] + \right.$$

$$\left. 64 \operatorname{ArcTanh}[a x]^3 \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[a x]}\right] - 96 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[a x]}\right] + 96 (-2 + \operatorname{ArcTanh}[a x]) \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[a x]}\right] + \right.$$

$$\left. 96 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[a x]}\right] - 96 \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[a x]}\right] + 48 \operatorname{PolyLog}\left[4, e^{2 \operatorname{ArcTanh}[a x]}\right] \right)$$

Problem 139: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh}[a x]^4}{x^2 (c - a c x)} dx$$

Optimal (type 4, 239 leaves, 12 steps):

$$\frac{a \operatorname{ArcTanh}[a x]^4}{c} - \frac{\operatorname{ArcTanh}[a x]^4}{c x} + \frac{a \operatorname{ArcTanh}[a x]^4 \operatorname{Log}\left[2 - \frac{2}{1-a x}\right]}{c} + \frac{4 a \operatorname{ArcTanh}[a x]^3 \operatorname{Log}\left[2 - \frac{2}{1+a x}\right]}{c} +$$

$$\frac{2 a \operatorname{ArcTanh}[a x]^3 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1-a x}\right]}{c} - \frac{6 a \operatorname{ArcTanh}[a x]^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+a x}\right]}{c} - \frac{3 a \operatorname{ArcTanh}[a x]^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1-a x}\right]}{c} -$$

$$\frac{6 a \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[3, -1 + \frac{2}{1+a x}\right]}{c} + \frac{3 a \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[4, -1 + \frac{2}{1-a x}\right]}{c} - \frac{3 a \operatorname{PolyLog}\left[4, -1 + \frac{2}{1+a x}\right]}{c} - \frac{3 a \operatorname{PolyLog}\left[5, -1 + \frac{2}{1-a x}\right]}{2 c}$$

Result (type 4, 172 leaves):

$$-\frac{1}{c} a \left(-\frac{\pi^4}{16} + \frac{i \pi^5}{160} + \operatorname{ArcTanh}[a x]^4 + \frac{\operatorname{ArcTanh}[a x]^4}{a x} - 4 \operatorname{ArcTanh}[a x]^3 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[a x]}\right] - \operatorname{ArcTanh}[a x]^4 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[a x]}\right] - \right.$$

$$2 \operatorname{ArcTanh}[a x]^2 \left(3 + \operatorname{ArcTanh}[a x]\right) \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[a x]}\right] + 3 \operatorname{ArcTanh}[a x] \left(2 + \operatorname{ArcTanh}[a x]\right) \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[a x]}\right] -$$

$$\left. 3 \operatorname{PolyLog}\left[4, e^{2 \operatorname{ArcTanh}[a x]}\right] - 3 \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[4, e^{2 \operatorname{ArcTanh}[a x]}\right] + \frac{3}{2} \operatorname{PolyLog}\left[5, e^{2 \operatorname{ArcTanh}[a x]}\right] \right)$$

Problem 140: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh}[a x]^4}{x^3 (c - a c x)} dx$$

Optimal (type 4, 380 leaves, 21 steps):

$$\frac{2 a^2 \operatorname{ArcTanh}[a x]^3}{c} - \frac{2 a \operatorname{ArcTanh}[a x]^3}{c x} + \frac{3 a^2 \operatorname{ArcTanh}[a x]^4}{2 c} - \frac{\operatorname{ArcTanh}[a x]^4}{2 c x^2} - \frac{a \operatorname{ArcTanh}[a x]^4}{c x} +$$

$$\frac{a^2 \operatorname{ArcTanh}[a x]^4 \operatorname{Log}\left[2 - \frac{2}{1-a x}\right]}{c} + \frac{6 a^2 \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[2 - \frac{2}{1+a x}\right]}{c} + \frac{4 a^2 \operatorname{ArcTanh}[a x]^3 \operatorname{Log}\left[2 - \frac{2}{1+a x}\right]}{c} +$$

$$\frac{2 a^2 \operatorname{ArcTanh}[a x]^3 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1-a x}\right]}{c} - \frac{6 a^2 \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+a x}\right]}{c} - \frac{6 a^2 \operatorname{ArcTanh}[a x]^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+a x}\right]}{c} -$$

$$\frac{3 a^2 \operatorname{ArcTanh}[a x]^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1-a x}\right]}{c} - \frac{3 a^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1+a x}\right]}{c} - \frac{6 a^2 \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[3, -1 + \frac{2}{1+a x}\right]}{c} +$$

$$\frac{3 a^2 \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[4, -1 + \frac{2}{1-a x}\right]}{c} - \frac{3 a^2 \operatorname{PolyLog}\left[4, -1 + \frac{2}{1+a x}\right]}{c} - \frac{3 a^2 \operatorname{PolyLog}\left[5, -1 + \frac{2}{1-a x}\right]}{2 c}$$

Result (type 4, 250 leaves):

$$\begin{aligned}
& -\frac{1}{c} a^2 \left(-\frac{i \pi^3}{4} - \frac{\pi^4}{16} + \frac{i \pi^5}{160} + 2 \operatorname{ArcTanh}[a x]^3 + \frac{2 \operatorname{ArcTanh}[a x]^3}{a x} + \frac{1}{2} \operatorname{ArcTanh}[a x]^4 + \frac{\operatorname{ArcTanh}[a x]^4}{2 a^2 x^2} + \frac{\operatorname{ArcTanh}[a x]^4}{a x} - \right. \\
& 6 \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[a x]}\right] - 4 \operatorname{ArcTanh}[a x]^3 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[a x]}\right] - \operatorname{ArcTanh}[a x]^4 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[a x]}\right] - \\
& 2 \operatorname{ArcTanh}[a x] \left(3 + 3 \operatorname{ArcTanh}[a x] + \operatorname{ArcTanh}[a x]^2\right) \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[a x]}\right] + 3 \left(1 + \operatorname{ArcTanh}[a x]\right)^2 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[a x]}\right] - \\
& \left. 3 \operatorname{PolyLog}\left[4, e^{2 \operatorname{ArcTanh}[a x]}\right] - 3 \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[4, e^{2 \operatorname{ArcTanh}[a x]}\right] + \frac{3}{2} \operatorname{PolyLog}\left[5, e^{2 \operatorname{ArcTanh}[a x]}\right]\right)
\end{aligned}$$

Problem 147: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 (a + b \operatorname{ArcTanh}[c x])}{d + e x} dx$$

Optimal (type 4, 275 leaves, 16 steps):

$$\begin{aligned}
& \frac{a d^2 x}{e^3} - \frac{b d x}{2 c e^2} + \frac{b x^2}{6 c e} + \frac{b d \operatorname{ArcTanh}[c x]}{2 c^2 e^2} + \frac{b d^2 x \operatorname{ArcTanh}[c x]}{e^3} - \frac{d x^2 (a + b \operatorname{ArcTanh}[c x])}{2 e^2} + \\
& \frac{x^3 (a + b \operatorname{ArcTanh}[c x])}{3 e} + \frac{d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1+c x}\right]}{e^4} - \frac{d^3 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2 c (d+e x)}{(c d+e) (1+c x)}\right]}{e^4} + \\
& \frac{b d^2 \operatorname{Log}\left[1 - c^2 x^2\right]}{2 c e^3} + \frac{b \operatorname{Log}\left[1 - c^2 x^2\right]}{6 c^3 e} - \frac{b d^3 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{2 e^4} + \frac{b d^3 \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d+e x)}{(c d+e) (1+c x)}\right]}{2 e^4}
\end{aligned}$$

Result (type 4, 474 leaves):

$$\frac{1}{12 e^4} \left(-\frac{2 b e^3}{c^3} + 12 a d^2 e x - \frac{6 b d e^2 x}{c} - 6 a d e^2 x^2 + \frac{2 b e^3 x^2}{c} + 4 a e^3 x^3 + \frac{6 b d e^2 \operatorname{ArcTanh}[c x]}{c^2} - 6 i b d^3 \pi \operatorname{ArcTanh}[c x] + 12 b d^2 e x \operatorname{ArcTanh}[c x] - \right.$$

$$6 b d e^2 x^2 \operatorname{ArcTanh}[c x] + 4 b e^3 x^3 \operatorname{ArcTanh}[c x] - 12 b d^3 \operatorname{ArcTanh}\left[\frac{c d}{e}\right] \operatorname{ArcTanh}[c x] + 6 b d^3 \operatorname{ArcTanh}[c x]^2 - \frac{6 b d^2 e \operatorname{ArcTanh}[c x]^2}{c} +$$

$$\frac{6 b d^2 \sqrt{1 - \frac{c^2 d^2}{e^2}} e^{-\operatorname{ArcTanh}\left[\frac{c d}{e}\right]} \operatorname{ArcTanh}[c x]^2}{c} + 12 b d^3 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c x]}\right] + 6 i b d^3 \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[c x]}\right] -$$

$$12 b d^3 \operatorname{ArcTanh}\left[\frac{c d}{e}\right] \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right)}\right] - 12 b d^3 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right)}\right] - 12 a d^3 \operatorname{Log}[d + e x] +$$

$$\frac{6 b d^2 e \operatorname{Log}\left[1 - c^2 x^2\right]}{c} + \frac{2 b e^3 \operatorname{Log}\left[1 - c^2 x^2\right]}{c^3} + 3 i b d^3 \pi \operatorname{Log}\left[1 - c^2 x^2\right] + 12 b d^3 \operatorname{ArcTanh}\left[\frac{c d}{e}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right]\right] -$$

$$\left. 6 b d^3 \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[c x]}\right] + 6 b d^3 \operatorname{PolyLog}\left[2, e^{-2 \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right)}\right] \right)$$

Problem 148: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 (a + b \operatorname{ArcTanh}[c x])}{d + e x} dx$$

Optimal (type 4, 214 leaves, 12 steps):

$$-\frac{a d x}{e^2} + \frac{b x}{2 c e} - \frac{b \operatorname{ArcTanh}[c x]}{2 c^2 e} - \frac{b d x \operatorname{ArcTanh}[c x]}{e^2} + \frac{x^2 (a + b \operatorname{ArcTanh}[c x])}{2 e} - \frac{d^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1+c x}\right]}{e^3} +$$

$$\frac{d^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{e^3} - \frac{b d \operatorname{Log}\left[1 - c^2 x^2\right]}{2 c e^2} + \frac{b d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{2 e^3} - \frac{b d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{2 e^3}$$

Result (type 4, 394 leaves):

$$\frac{1}{2e^3} \left(-2ade^2x + \frac{be^2x}{c} + ae^2x^2 - \frac{be^2 \operatorname{ArcTanh}[cx]}{c^2} + i b d^2 \pi \operatorname{ArcTanh}[cx] - 2 b d e x \operatorname{ArcTanh}[cx] + b e^2 x^2 \operatorname{ArcTanh}[cx] + \right.$$

$$2 b d^2 \operatorname{ArcTanh}\left[\frac{cd}{e}\right] \operatorname{ArcTanh}[cx] - b d^2 \operatorname{ArcTanh}[cx]^2 + \frac{b d e \operatorname{ArcTanh}[cx]^2}{c} - \frac{b d \sqrt{1 - \frac{c^2 d^2}{e^2}} e^{-\operatorname{ArcTanh}\left[\frac{cd}{e}\right]} \operatorname{ArcTanh}[cx]^2}{c} -$$

$$2 b d^2 \operatorname{ArcTanh}[cx] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[cx]}\right] - i b d^2 \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[cx]}\right] + 2 b d^2 \operatorname{ArcTanh}\left[\frac{cd}{e}\right] \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right)}\right] +$$

$$2 b d^2 \operatorname{ArcTanh}[cx] \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right)}\right] + 2 a d^2 \operatorname{Log}[d + e x] - \frac{b d e \operatorname{Log}\left[1 - c^2 x^2\right]}{c} - \frac{1}{2} i b d^2 \pi \operatorname{Log}\left[1 - c^2 x^2\right] -$$

$$\left. 2 b d^2 \operatorname{ArcTanh}\left[\frac{cd}{e}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right]\right] + b d^2 \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[cx]}\right] - b d^2 \operatorname{PolyLog}\left[2, e^{-2 \left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right)}\right] \right)$$

Problem 149: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x (a + b \operatorname{ArcTanh}[cx])}{d + e x} dx$$

Optimal (type 4, 156 leaves, 9 steps):

$$\frac{a x}{e} + \frac{b x \operatorname{ArcTanh}[cx]}{e} + \frac{d (a + b \operatorname{ArcTanh}[cx]) \operatorname{Log}\left[\frac{2}{1+cx}\right]}{e^2} -$$

$$\frac{d (a + b \operatorname{ArcTanh}[cx]) \operatorname{Log}\left[\frac{2c(d+ex)}{(c d+e)(1+cx)}\right]}{e^2} + \frac{b \operatorname{Log}\left[1 - c^2 x^2\right]}{2 c e} - \frac{b d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+cx}\right]}{2 e^2} + \frac{b d \operatorname{PolyLog}\left[2, 1 - \frac{2c(d+ex)}{(c d+e)(1+cx)}\right]}{2 e^2}$$

Result (type 4, 315 leaves):

$$\frac{1}{2e^2} \left(2aex - 2ad \operatorname{Log}[d + ex] + \right.$$

$$\frac{1}{c} b \left(-i cd \pi \operatorname{ArcTanh}[cx] + 2cex \operatorname{ArcTanh}[cx] - 2cd \operatorname{ArcTanh}\left[\frac{cd}{e}\right] \operatorname{ArcTanh}[cx] + cd \operatorname{ArcTanh}[cx]^2 - e \operatorname{ArcTanh}[cx]^2 + \right.$$

$$\sqrt{1 - \frac{c^2 d^2}{e^2}} e^{-\operatorname{ArcTanh}\left[\frac{cd}{e}\right]} \operatorname{ArcTanh}[cx]^2 + 2cd \operatorname{ArcTanh}[cx] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[cx]}\right] + i cd \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[cx]}\right] -$$

$$2cd \operatorname{ArcTanh}\left[\frac{cd}{e}\right] \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right)}\right] - 2cd \operatorname{ArcTanh}[cx] \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right)}\right] +$$

$$e \operatorname{Log}\left[1 - c^2 x^2\right] + \frac{1}{2} i cd \pi \operatorname{Log}\left[1 - c^2 x^2\right] + 2cd \operatorname{ArcTanh}\left[\frac{cd}{e}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right]\right] -$$

$$\left. \left. cd \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[cx]}\right] + cd \operatorname{PolyLog}\left[2, e^{-2 \left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right)}\right] \right) \right)$$

Problem 150: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{ArcTanh}[cx]}{d + ex} dx$$

Optimal (type 4, 114 leaves, 4 steps):

$$-\frac{(a + b \operatorname{ArcTanh}[cx]) \operatorname{Log}\left[\frac{2}{1+cx}\right]}{e} + \frac{(a + b \operatorname{ArcTanh}[cx]) \operatorname{Log}\left[\frac{2c(d+ex)}{(cd+e)(1+cx)}\right]}{e} + \frac{b \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+cx}\right]}{2e} - \frac{b \operatorname{PolyLog}\left[2, 1 - \frac{2c(d+ex)}{(cd+e)(1+cx)}\right]}{2e}$$

Result (type 4, 257 leaves):

$$\frac{1}{e} \left(a \operatorname{Log}[d + ex] + b \operatorname{ArcTanh}[cx] \left(\frac{1}{2} \operatorname{Log}[1 - c^2 x^2] + \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right]\right] \right) - \right.$$

$$\frac{1}{2} i b \left(-\frac{1}{4} i (\pi - 2 i \operatorname{ArcTanh}[cx])^2 + i \left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx] \right)^2 + (\pi - 2 i \operatorname{ArcTanh}[cx]) \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[cx]}\right] + \right.$$

$$2 i \left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx] \right) \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right)}\right] - (\pi - 2 i \operatorname{ArcTanh}[cx]) \operatorname{Log}\left[\frac{2}{\sqrt{1 - c^2 x^2}}\right] -$$

$$2 i \left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx] \right) \operatorname{Log}\left[2 i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right]\right] -$$

$$\left. \left. i \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcTanh}[cx]}\right] - i \operatorname{PolyLog}\left[2, e^{-2 \left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right)}\right] \right) \right)$$

Problem 151: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcTanh}[c x]}{x (d + e x)} dx$$

Optimal (type 4, 148 leaves, 7 steps):

$$\frac{a \operatorname{Log}[x]}{d} + \frac{(a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1+cx}\right]}{d} - \frac{(a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2c(d+ex)}{(cd+e)(1+cx)}\right]}{d} -$$

$$\frac{b \operatorname{PolyLog}[2, -cx]}{2d} + \frac{b \operatorname{PolyLog}[2, cx]}{2d} - \frac{b \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+cx}\right]}{2d} + \frac{b \operatorname{PolyLog}\left[2, 1 - \frac{2c(d+ex)}{(cd+e)(1+cx)}\right]}{2d}$$

Result (type 4, 294 leaves):

$$\frac{1}{2d^2} \left(2ad \operatorname{Log}[x] - 2ad \operatorname{Log}[d+ex] + \frac{1}{c} b \left(-i c d \pi \operatorname{ArcTanh}[cx] - 2cd \operatorname{ArcTanh}\left[\frac{cd}{e}\right] \operatorname{ArcTanh}[cx] + cd \operatorname{ArcTanh}[cx]^2 - e \operatorname{ArcTanh}[cx]^2 + \sqrt{1 - \frac{c^2 d^2}{e^2}} \right. \right.$$

$$e^{-\operatorname{ArcTanh}\left[\frac{cd}{e}\right]} \operatorname{ArcTanh}[cx]^2 + 2cd \operatorname{ArcTanh}[cx] \operatorname{Log}\left[1 - e^{-2\operatorname{ArcTanh}[cx]}\right] + i c d \pi \operatorname{Log}\left[1 + e^{2\operatorname{ArcTanh}[cx]}\right] -$$

$$2cd \operatorname{ArcTanh}\left[\frac{cd}{e}\right] \operatorname{Log}\left[1 - e^{-2\left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right)}\right] - 2cd \operatorname{ArcTanh}[cx] \operatorname{Log}\left[1 - e^{-2\left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right)}\right] + \frac{1}{2} i c d \pi \operatorname{Log}\left[1 - c^2 x^2\right] +$$

$$\left. \left. 2cd \operatorname{ArcTanh}\left[\frac{cd}{e}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right]\right] - cd \operatorname{PolyLog}\left[2, e^{-2\operatorname{ArcTanh}[cx]}\right] + cd \operatorname{PolyLog}\left[2, e^{-2\left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right)}\right] \right) \right)$$

Problem 152: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcTanh}[c x]}{x^2 (d + e x)} dx$$

Optimal (type 4, 200 leaves, 12 steps):

$$-\frac{a + b \operatorname{ArcTanh}[c x]}{dx} + \frac{bc \operatorname{Log}[x]}{d} - \frac{ae \operatorname{Log}[x]}{d^2} - \frac{e(a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1+cx}\right]}{d^2} + \frac{e(a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2c(d+ex)}{(cd+e)(1+cx)}\right]}{d^2} -$$

$$\frac{bc \operatorname{Log}\left[1 - c^2 x^2\right]}{2d} + \frac{be \operatorname{PolyLog}[2, -cx]}{2d^2} - \frac{be \operatorname{PolyLog}[2, cx]}{2d^2} + \frac{be \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+cx}\right]}{2d^2} - \frac{be \operatorname{PolyLog}\left[2, 1 - \frac{2c(d+ex)}{(cd+e)(1+cx)}\right]}{2d^2}$$

Result (type 4, 360 leaves):

$$\begin{aligned}
& -\frac{1}{2d^3} \left(\frac{2ad^2}{x} - i b d e \pi \operatorname{ArcTanh}[cx] + \frac{2bd^2 \operatorname{ArcTanh}[cx]}{x} - 2bd e \operatorname{ArcTanh}\left[\frac{cd}{e}\right] \operatorname{ArcTanh}[cx] + b d e \operatorname{ArcTanh}[cx]^2 - \frac{b e^2 \operatorname{ArcTanh}[cx]^2}{c} + \right. \\
& \frac{b \sqrt{1 - \frac{c^2 d^2}{e^2}} e^2 e^{-\operatorname{ArcTanh}\left[\frac{cd}{e}\right]} \operatorname{ArcTanh}[cx]^2}{c} + 2bd e \operatorname{ArcTanh}[cx] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[cx]}\right] + i b d e \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[cx]}\right] - \\
& 2bd e \operatorname{ArcTanh}\left[\frac{cd}{e}\right] \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right)}\right] - 2bd e \operatorname{ArcTanh}[cx] \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right)}\right] + \\
& 2ade \operatorname{Log}[x] - 2ade \operatorname{Log}[d + ex] - 2bcd^2 \operatorname{Log}\left[\frac{cx}{\sqrt{1 - c^2 x^2}}\right] + \frac{1}{2} i b d e \pi \operatorname{Log}\left[1 - c^2 x^2\right] + \\
& \left. 2bd e \operatorname{ArcTanh}\left[\frac{cd}{e}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right]\right] - b d e \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[cx]}\right] + b d e \operatorname{PolyLog}\left[2, e^{-2 \left(\operatorname{ArcTanh}\left[\frac{cd}{e}\right] + \operatorname{ArcTanh}[cx]\right)}\right] \right)
\end{aligned}$$

Problem 153: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcTanh}[cx]}{x^3 (d + ex)} dx$$

Optimal (type 4, 261 leaves, 15 steps):

$$\begin{aligned}
& -\frac{bc}{2dx} + \frac{bc^2 \operatorname{ArcTanh}[cx]}{2d} - \frac{a + b \operatorname{ArcTanh}[cx]}{2dx^2} + \frac{e(a + b \operatorname{ArcTanh}[cx])}{d^2 x} - \frac{bce \operatorname{Log}[x]}{d^2} + \\
& \frac{ae^2 \operatorname{Log}[x]}{d^3} + \frac{e^2(a + b \operatorname{ArcTanh}[cx]) \operatorname{Log}\left[\frac{2}{1+cx}\right]}{d^3} - \frac{e^2(a + b \operatorname{ArcTanh}[cx]) \operatorname{Log}\left[\frac{2c(d+ex)}{(cd+e)(1+cx)}\right]}{d^3} + \frac{bce \operatorname{Log}\left[1 - c^2 x^2\right]}{2d^2} - \\
& \frac{be^2 \operatorname{PolyLog}\left[2, -cx\right]}{2d^3} + \frac{be^2 \operatorname{PolyLog}\left[2, cx\right]}{2d^3} - \frac{be^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+cx}\right]}{2d^3} + \frac{be^2 \operatorname{PolyLog}\left[2, 1 - \frac{2c(d+ex)}{(cd+e)(1+cx)}\right]}{2d^3}
\end{aligned}$$

Result (type 4, 435 leaves):

$$\frac{1}{4 d^4} \left(-\frac{2 a d^3}{x^2} - \frac{2 b c d^3}{x} + \frac{4 a d^2 e}{x} + 2 b c^2 d^3 \operatorname{ArcTanh}[c x] - 2 i b d e^2 \pi \operatorname{ArcTanh}[c x] - \frac{2 b d^3 \operatorname{ArcTanh}[c x]}{x^2} + \right. \\ \left. \frac{4 b d^2 e \operatorname{ArcTanh}[c x]}{x} - 4 b d e^2 \operatorname{ArcTanh}\left[\frac{c d}{e}\right] \operatorname{ArcTanh}[c x] + 2 b d e^2 \operatorname{ArcTanh}[c x]^2 - \frac{2 b e^3 \operatorname{ArcTanh}[c x]^2}{c} + \right. \\ \left. \frac{2 b \sqrt{1 - \frac{c^2 d^2}{e^2}} e^3 e^{-\operatorname{ArcTanh}\left[\frac{c d}{e}\right]} \operatorname{ArcTanh}[c x]^2}{c} + 4 b d e^2 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] + 2 i b d e^2 \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[c x]}\right] - \right. \\ \left. 4 b d e^2 \operatorname{ArcTanh}\left[\frac{c d}{e}\right] \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right)}\right] - 4 b d e^2 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right)}\right] + 4 a d e^2 \operatorname{Log}[x] - \right. \\ \left. 4 a d e^2 \operatorname{Log}[d + e x] - 4 b c d^2 e \operatorname{Log}\left[\frac{c x}{\sqrt{1 - c^2 x^2}}\right] + i b d e^2 \pi \operatorname{Log}\left[1 - c^2 x^2\right] + 4 b d e^2 \operatorname{ArcTanh}\left[\frac{c d}{e}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right]\right] - \right. \\ \left. 2 b d e^2 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c x]}\right] + 2 b d e^2 \operatorname{PolyLog}\left[2, e^{-2 \left(\operatorname{ArcTanh}\left[\frac{c d}{e}\right] + \operatorname{ArcTanh}[c x]\right)}\right] \right)$$

Problem 154: Unable to integrate problem.

$$\int \frac{x^2 (a + b \operatorname{ArcTanh}[c x])^2}{d + e x} dx$$

Optimal (type 4, 385 leaves, 14 steps):

$$\frac{a b x}{c e} + \frac{b^2 x \operatorname{ArcTanh}[c x]}{c e} - \frac{d (a + b \operatorname{ArcTanh}[c x])^2}{c e^2} - \frac{(a + b \operatorname{ArcTanh}[c x])^2}{2 c^2 e} - \frac{d x (a + b \operatorname{ArcTanh}[c x])^2}{e^2} + \\ \frac{x^2 (a + b \operatorname{ArcTanh}[c x])^2}{2 e} + \frac{2 b d (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1 - c x}\right]}{c e^2} - \frac{d^2 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2}{1 + c x}\right]}{e^3} + \\ \frac{d^2 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2 c (d + e x)}{(c d + e) (1 + c x)}\right]}{e^3} + \frac{b^2 \operatorname{Log}\left[1 - c^2 x^2\right]}{2 c^2 e} + \frac{b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c x}\right]}{c e^2} + \frac{b d^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c x}\right]}{e^3} - \\ \frac{b d^2 (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d + e x)}{(c d + e) (1 + c x)}\right]}{e^3} + \frac{b^2 d^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 + c x}\right]}{2 e^3} - \frac{b^2 d^2 \operatorname{PolyLog}\left[3, 1 - \frac{2 c (d + e x)}{(c d + e) (1 + c x)}\right]}{2 e^3}$$

Result (type 8, 23 leaves):

$$\int \frac{x^2 (a + b \operatorname{ArcTanh}[c x])^2}{d + e x} dx$$

Problem 155: Unable to integrate problem.

$$\int \frac{x (a + b \operatorname{ArcTanh}[c x])^2}{d + e x} dx$$

Optimal (type 4, 279 leaves, 8 steps):

$$\begin{aligned} & \frac{(a + b \operatorname{ArcTanh}[c x])^2}{c e} + \frac{x (a + b \operatorname{ArcTanh}[c x])^2}{e} - \frac{2 b (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[\frac{2}{1-c x}\right]}{c e} + \frac{d (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2}{1+c x}\right]}{e^2} - \\ & \frac{d (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{e^2} - \frac{b^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c x}\right]}{c e} - \frac{b d (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{e^2} + \\ & \frac{b d (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{e^2} - \frac{b^2 d \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+c x}\right]}{2 e^2} + \frac{b^2 d \operatorname{PolyLog}\left[3, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{2 e^2} \end{aligned}$$

Result (type 8, 21 leaves):

$$\int \frac{x (a + b \operatorname{ArcTanh}[c x])^2}{d + e x} dx$$

Problem 156: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{d + e x} dx$$

Optimal (type 4, 188 leaves, 1 step):

$$\begin{aligned} & - \frac{(a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2}{1+c x}\right]}{e} + \frac{(a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{e} + \frac{b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{e} - \\ & \frac{b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{e} + \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+c x}\right]}{2 e} - \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{2 e} \end{aligned}$$

Result (type 8, 20 leaves):

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{d + e x} dx$$

Problem 157: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x (d + e x)} dx$$

Optimal (type 4, 319 leaves, 9 steps):

$$\begin{aligned} & \frac{2 (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1-cx}\right]}{d} + \frac{(a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2}{1+cx}\right]}{d} - \frac{(a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2c(d+ex)}{(cd+e)(1+cx)}\right]}{d} \\ & - \frac{b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-cx}\right]}{d} + \frac{b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1-cx}\right]}{d} \\ & - \frac{b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+cx}\right]}{d} + \frac{b (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2c(d+ex)}{(cd+e)(1+cx)}\right]}{d} \\ & - \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1-cx}\right]}{2d} - \frac{b^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1-cx}\right]}{2d} - \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+cx}\right]}{2d} + \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2c(d+ex)}{(cd+e)(1+cx)}\right]}{2d} \end{aligned}$$

Result (type 8, 23 leaves):

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x (d + e x)} dx$$

Problem 158: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x^2 (d + e x)} dx$$

Optimal (type 4, 412 leaves, 13 steps):

$$\begin{aligned}
& \frac{c (a + b \operatorname{ArcTanh}[c x])^2}{d} - \frac{(a + b \operatorname{ArcTanh}[c x])^2}{d x} - \frac{2 e (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1-c x}\right]}{d^2} - \\
& \frac{e (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2}{1+c x}\right]}{d^2} + \frac{e (a + b \operatorname{ArcTanh}[c x])^2 \operatorname{Log}\left[\frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{d^2} + \frac{2 b c (a + b \operatorname{ArcTanh}[c x]) \operatorname{Log}\left[2 - \frac{2}{1+c x}\right]}{d} + \\
& \frac{b e (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c x}\right]}{d^2} - \frac{b e (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1-c x}\right]}{d^2} + \\
& \frac{b e (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{d^2} - \frac{b^2 c \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+c x}\right]}{d} - \frac{b e (a + b \operatorname{ArcTanh}[c x]) \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{d^2} - \\
& \frac{b^2 e \operatorname{PolyLog}\left[3, 1 - \frac{2}{1-c x}\right]}{2 d^2} + \frac{b^2 e \operatorname{PolyLog}\left[3, -1 + \frac{2}{1-c x}\right]}{2 d^2} + \frac{b^2 e \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+c x}\right]}{2 d^2} - \frac{b^2 e \operatorname{PolyLog}\left[3, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{2 d^2}
\end{aligned}$$

Result (type 8, 23 leaves):

$$\int \frac{(a + b \operatorname{ArcTanh}[c x])^2}{x^2 (d + e x)} dx$$

Problem 159: Unable to integrate problem.

$$\int \frac{\operatorname{ArcTanh}[c x]^2}{x (d + e x)} dx$$

Optimal (type 4, 275 leaves, 9 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTanh}[c x]^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1-c x}\right]}{d} + \frac{\operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[\frac{2}{1+c x}\right]}{d} - \frac{\operatorname{ArcTanh}[c x]^2 \operatorname{Log}\left[\frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{d} - \frac{\operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c x}\right]}{d} + \\
& \frac{\operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, -1 + \frac{2}{1-c x}\right]}{d} - \frac{\operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{d} + \frac{\operatorname{ArcTanh}[c x] \operatorname{PolyLog}\left[2, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{d} + \\
& \frac{\operatorname{PolyLog}\left[3, 1 - \frac{2}{1-c x}\right]}{2 d} - \frac{\operatorname{PolyLog}\left[3, -1 + \frac{2}{1-c x}\right]}{2 d} - \frac{\operatorname{PolyLog}\left[3, 1 - \frac{2}{1+c x}\right]}{2 d} + \frac{\operatorname{PolyLog}\left[3, 1 - \frac{2 c (d+e x)}{(c d+e)(1+c x)}\right]}{2 d}
\end{aligned}$$

Result (type 8, 19 leaves):

$$\int \frac{\operatorname{ArcTanh}[c x]^2}{x (d + e x)} dx$$

Problem 212: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(1 - a^2 x^2)^2 \operatorname{ArcTanh}[a x]^2}{x^5} dx$$

Optimal (type 4, 214 leaves, 29 steps):

$$\begin{aligned} & -\frac{a^2}{12 x^2} - \frac{a \operatorname{ArcTanh}[a x]}{6 x^3} + \frac{3 a^3 \operatorname{ArcTanh}[a x]}{2 x} - \frac{3}{4} a^4 \operatorname{ArcTanh}[a x]^2 - \frac{\operatorname{ArcTanh}[a x]^2}{4 x^4} + \frac{a^2 \operatorname{ArcTanh}[a x]^2}{x^2} + \\ & 2 a^4 \operatorname{ArcTanh}[a x]^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - a x}\right] - \frac{4}{3} a^4 \operatorname{Log}[x] + \frac{2}{3} a^4 \operatorname{Log}[1 - a^2 x^2] - a^4 \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - a x}\right] + \\ & a^4 \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - a x}\right] + \frac{1}{2} a^4 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - a x}\right] - \frac{1}{2} a^4 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - a x}\right] \end{aligned}$$

Result (type 4, 238 leaves):

$$\begin{aligned} & \frac{1}{24} \left(2 a^4 + i a^4 \pi^3 - \frac{2 a^2}{x^2} - \frac{4 a \operatorname{ArcTanh}[a x]}{x^3} + \frac{36 a^3 \operatorname{ArcTanh}[a x]}{x} - 18 a^4 \operatorname{ArcTanh}[a x]^2 - \right. \\ & \left. \frac{6 \operatorname{ArcTanh}[a x]^2}{x^4} + \frac{24 a^2 \operatorname{ArcTanh}[a x]^2}{x^2} - 16 a^4 \operatorname{ArcTanh}[a x]^3 - 24 a^4 \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[a x]}\right] + \right. \\ & \left. 24 a^4 \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[a x]}\right] - 32 a^4 \operatorname{Log}\left[\frac{a x}{\sqrt{1 - a^2 x^2}}\right] + 24 a^4 \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[a x]}\right] + \right. \\ & \left. 24 a^4 \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[a x]}\right] + 12 a^4 \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}[a x]}\right] - 12 a^4 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[a x]}\right] \right) \end{aligned}$$

Problem 240: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh}[a x]^2}{x^3 (1 - a^2 x^2)} dx$$

Optimal (type 4, 138 leaves, 13 steps):

$$\begin{aligned} & -\frac{a \operatorname{ArcTanh}[a x]}{x} + \frac{1}{2} a^2 \operatorname{ArcTanh}[a x]^2 - \frac{\operatorname{ArcTanh}[a x]^2}{2 x^2} + \frac{1}{3} a^2 \operatorname{ArcTanh}[a x]^3 + a^2 \operatorname{Log}[x] - \frac{1}{2} a^2 \operatorname{Log}[1 - a^2 x^2] + \\ & a^2 \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[2 - \frac{2}{1 + a x}\right] - a^2 \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + a x}\right] - \frac{1}{2} a^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 + a x}\right] \end{aligned}$$

Result (type 4, 133 leaves):

$$-a^2 \left(-\frac{i\pi^3}{24} + \frac{\text{ArcTanh}[ax]}{ax} + \frac{(1-a^2x^2)\text{ArcTanh}[ax]^2}{2a^2x^2} + \frac{1}{3}\text{ArcTanh}[ax]^3 - \right. \\ \left. \text{ArcTanh}[ax]^2 \text{Log}[1 - e^{2\text{ArcTanh}[ax]}] - \text{Log}\left[\frac{ax}{\sqrt{1-a^2x^2}}\right] - \text{ArcTanh}[ax] \text{PolyLog}[2, e^{2\text{ArcTanh}[ax]}] + \frac{1}{2}\text{PolyLog}[3, e^{2\text{ArcTanh}[ax]}] \right)$$

Problem 246: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTanh}[ax]^3}{x^2(1-a^2x^2)} dx$$

Optimal (type 4, 90 leaves, 7 steps):

$$a \text{ArcTanh}[ax]^3 - \frac{\text{ArcTanh}[ax]^3}{x} + \frac{1}{4} a \text{ArcTanh}[ax]^4 + \\ 3 a \text{ArcTanh}[ax]^2 \text{Log}\left[2 - \frac{2}{1+ax}\right] - 3 a \text{ArcTanh}[ax] \text{PolyLog}\left[2, -1 + \frac{2}{1+ax}\right] - \frac{3}{2} a \text{PolyLog}\left[3, -1 + \frac{2}{1+ax}\right]$$

Result (type 4, 93 leaves):

$$-a \left(-\frac{i\pi^3}{8} + \text{ArcTanh}[ax]^3 + \frac{\text{ArcTanh}[ax]^3}{ax} - \frac{1}{4} \text{ArcTanh}[ax]^4 - \right. \\ \left. 3 \text{ArcTanh}[ax]^2 \text{Log}[1 - e^{2\text{ArcTanh}[ax]}] - 3 \text{ArcTanh}[ax] \text{PolyLog}[2, e^{2\text{ArcTanh}[ax]}] + \frac{3}{2} \text{PolyLog}[3, e^{2\text{ArcTanh}[ax]}] \right)$$

Problem 270: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTanh}[ax]^2}{x(1-a^2x^2)^2} dx$$

Optimal (type 4, 136 leaves, 8 steps):

$$\frac{1}{4(1-a^2x^2)} - \frac{ax \text{ArcTanh}[ax]}{2(1-a^2x^2)} - \frac{1}{4} \text{ArcTanh}[ax]^2 + \frac{\text{ArcTanh}[ax]^2}{2(1-a^2x^2)} + \frac{1}{3} \text{ArcTanh}[ax]^3 + \\ \text{ArcTanh}[ax]^2 \text{Log}\left[2 - \frac{2}{1+ax}\right] - \text{ArcTanh}[ax] \text{PolyLog}\left[2, -1 + \frac{2}{1+ax}\right] - \frac{1}{2} \text{PolyLog}\left[3, -1 + \frac{2}{1+ax}\right]$$

Result (type 4, 106 leaves):

$$\frac{1}{24} \left(i\pi^3 - 8 \text{ArcTanh}[ax]^3 + 3 \text{Cosh}[2 \text{ArcTanh}[ax]] + 6 \text{ArcTanh}[ax]^2 \text{Cosh}[2 \text{ArcTanh}[ax]] + 24 \text{ArcTanh}[ax]^2 \text{Log}[1 - e^{2\text{ArcTanh}[ax]}] + \right. \\ \left. 24 \text{ArcTanh}[ax] \text{PolyLog}[2, e^{2\text{ArcTanh}[ax]}] - 12 \text{PolyLog}[3, e^{2\text{ArcTanh}[ax]}] - 6 \text{ArcTanh}[ax] \text{Sinh}[2 \text{ArcTanh}[ax]] \right)$$

Problem 272: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTanh}[a x]^2}{x^3 (1 - a^2 x^2)^2} dx$$

Optimal (type 4, 205 leaves, 22 steps):

$$\frac{a^2}{4(1 - a^2 x^2)} - \frac{a \text{ArcTanh}[a x]}{x} - \frac{a^3 x \text{ArcTanh}[a x]}{2(1 - a^2 x^2)} + \frac{1}{4} a^2 \text{ArcTanh}[a x]^2 - \frac{\text{ArcTanh}[a x]^2}{2 x^2} + \frac{a^2 \text{ArcTanh}[a x]^2}{2(1 - a^2 x^2)} + \frac{2}{3} a^2 \text{ArcTanh}[a x]^3 + a^2 \text{Log}[x] - \frac{1}{2} a^2 \text{Log}[1 - a^2 x^2] + 2 a^2 \text{ArcTanh}[a x]^2 \text{Log}\left[2 - \frac{2}{1 + a x}\right] - 2 a^2 \text{ArcTanh}[a x] \text{PolyLog}\left[2, -1 + \frac{2}{1 + a x}\right] - a^2 \text{PolyLog}\left[3, -1 + \frac{2}{1 + a x}\right]$$

Result (type 4, 146 leaves):

$$a^2 \left(2 \text{ArcTanh}[a x] \text{PolyLog}\left[2, e^{2 \text{ArcTanh}[a x]}\right] + \frac{1}{24} \left(2 i \pi^3 - 16 \text{ArcTanh}[a x]^3 + 3 \text{Cosh}\left[2 \text{ArcTanh}[a x]\right] + 6 \text{ArcTanh}[a x]^2 \left(2 - \frac{2}{a^2 x^2} + \text{Cosh}\left[2 \text{ArcTanh}[a x]\right] + 8 \text{Log}\left[1 - e^{2 \text{ArcTanh}[a x]}\right] \right) + 24 \text{Log}\left[\frac{a x}{\sqrt{1 - a^2 x^2}}\right] - 24 \text{PolyLog}\left[3, e^{2 \text{ArcTanh}[a x]}\right] - \frac{6 \text{ArcTanh}[a x] \left(4 + a x \text{Sinh}\left[2 \text{ArcTanh}[a x]\right] \right)}{a x} \right) \right)$$

Problem 278: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTanh}[a x]^3}{x^2 (1 - a^2 x^2)^2} dx$$

Optimal (type 4, 191 leaves, 12 steps):

$$-\frac{3 a}{8(1 - a^2 x^2)} + \frac{3 a^2 x \text{ArcTanh}[a x]}{4(1 - a^2 x^2)} + \frac{3}{8} a \text{ArcTanh}[a x]^2 - \frac{3 a \text{ArcTanh}[a x]^2}{4(1 - a^2 x^2)} + a \text{ArcTanh}[a x]^3 - \frac{\text{ArcTanh}[a x]^3}{x} + \frac{a^2 x \text{ArcTanh}[a x]^3}{2(1 - a^2 x^2)} + \frac{3}{8} a \text{ArcTanh}[a x]^4 + 3 a \text{ArcTanh}[a x]^2 \text{Log}\left[2 - \frac{2}{1 + a x}\right] - 3 a \text{ArcTanh}[a x] \text{PolyLog}\left[2, -1 + \frac{2}{1 + a x}\right] - \frac{3}{2} a \text{PolyLog}\left[3, -1 + \frac{2}{1 + a x}\right]$$

Result (type 4, 144 leaves):

$$\frac{1}{16} a \left(2 i \pi^3 - 16 \text{ArcTanh}[a x]^3 - \frac{16 \text{ArcTanh}[a x]^3}{a x} + 6 \text{ArcTanh}[a x]^4 - 3 \text{Cosh}\left[2 \text{ArcTanh}[a x]\right] - 6 \text{ArcTanh}[a x]^2 \text{Cosh}\left[2 \text{ArcTanh}[a x]\right] + 48 \text{ArcTanh}[a x]^2 \text{Log}\left[1 - e^{2 \text{ArcTanh}[a x]}\right] + 48 \text{ArcTanh}[a x] \text{PolyLog}\left[2, e^{2 \text{ArcTanh}[a x]}\right] - 24 \text{PolyLog}\left[3, e^{2 \text{ArcTanh}[a x]}\right] + 6 \text{ArcTanh}[a x] \text{Sinh}\left[2 \text{ArcTanh}[a x]\right] + 4 \text{ArcTanh}[a x]^3 \text{Sinh}\left[2 \text{ArcTanh}[a x]\right] \right)$$

Problem 282: Attempted integration timed out after 120 seconds.

$$\int \frac{x^3}{(1 - a^2 x^2)^2 \operatorname{ArcTanh}[a x]} dx$$

Optimal (type 9, 42 leaves, 0 steps):

$$\frac{\operatorname{SinhIntegral}[2 \operatorname{ArcTanh}[a x]]}{2 a^4} - \frac{\operatorname{Unintegrable}\left[\frac{x}{(1 - a^2 x^2) \operatorname{ArcTanh}[a x]}, x\right]}{a^2}$$

Result (type 1, 1 leaves):

???

Problem 312: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh}[a x]^2}{x (1 - a^2 x^2)^3} dx$$

Optimal (type 4, 196 leaves, 13 steps):

$$\frac{1}{32 (1 - a^2 x^2)^2} + \frac{11}{32 (1 - a^2 x^2)} - \frac{a x \operatorname{ArcTanh}[a x]}{8 (1 - a^2 x^2)^2} - \frac{11 a x \operatorname{ArcTanh}[a x]}{16 (1 - a^2 x^2)} - \frac{11}{32} \operatorname{ArcTanh}[a x]^2 + \frac{\operatorname{ArcTanh}[a x]^2}{4 (1 - a^2 x^2)^2} + \frac{\operatorname{ArcTanh}[a x]^2}{2 (1 - a^2 x^2)} + \frac{1}{3} \operatorname{ArcTanh}[a x]^3 + \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[2 - \frac{2}{1 + a x}\right] - \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + a x}\right] - \frac{1}{2} \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 + a x}\right]$$

Result (type 4, 129 leaves):

$$\operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[a x]}\right] + \frac{1}{768} \left(32 i \pi^3 - 256 \operatorname{ArcTanh}[a x]^3 + 144 \operatorname{Cosh}[2 \operatorname{ArcTanh}[a x]] + 3 \operatorname{Cosh}[4 \operatorname{ArcTanh}[a x]] + 24 \operatorname{ArcTanh}[a x]^2 (12 \operatorname{Cosh}[2 \operatorname{ArcTanh}[a x]] + \operatorname{Cosh}[4 \operatorname{ArcTanh}[a x]]) + 32 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[a x]}]\right) - 384 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[a x]}\right] - 12 \operatorname{ArcTanh}[a x] (24 \operatorname{Sinh}[2 \operatorname{ArcTanh}[a x]] + \operatorname{Sinh}[4 \operatorname{ArcTanh}[a x]])$$

Problem 319: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh}[a x]^3}{x^2 (1 - a^2 x^2)^3} dx$$

Optimal (type 4, 281 leaves, 21 steps):

$$\begin{aligned}
& -\frac{3a}{128(1-a^2x^2)^2} - \frac{93a}{128(1-a^2x^2)} + \frac{3a^2x \operatorname{ArcTanh}[ax]}{32(1-a^2x^2)^2} + \frac{93a^2x \operatorname{ArcTanh}[ax]}{64(1-a^2x^2)} + \frac{93}{128}a \operatorname{ArcTanh}[ax]^2 - \\
& \frac{3a \operatorname{ArcTanh}[ax]^2}{16(1-a^2x^2)^2} - \frac{21a \operatorname{ArcTanh}[ax]^2}{16(1-a^2x^2)} + a \operatorname{ArcTanh}[ax]^3 - \frac{\operatorname{ArcTanh}[ax]^3}{x} + \frac{a^2x \operatorname{ArcTanh}[ax]^3}{4(1-a^2x^2)^2} + \frac{7a^2x \operatorname{ArcTanh}[ax]^3}{8(1-a^2x^2)} + \\
& \frac{15}{32}a \operatorname{ArcTanh}[ax]^4 + 3a \operatorname{ArcTanh}[ax]^2 \operatorname{Log}\left[2 - \frac{2}{1+ax}\right] - 3a \operatorname{ArcTanh}[ax] \operatorname{PolyLog}\left[2, -1 + \frac{2}{1+ax}\right] - \frac{3}{2}a \operatorname{PolyLog}\left[3, -1 + \frac{2}{1+ax}\right]
\end{aligned}$$

Result (type 4, 218 leaves):

$$\begin{aligned}
& -a \left(-\frac{i\pi^3}{8} + \operatorname{ArcTanh}[ax]^3 + \frac{\operatorname{ArcTanh}[ax]^3}{ax} - \frac{ax \operatorname{ArcTanh}[ax]^3}{1-a^2x^2} - \frac{15}{32} \operatorname{ArcTanh}[ax]^4 + \frac{3}{8} \operatorname{Cosh}[2 \operatorname{ArcTanh}[ax]] + \right. \\
& \left. \frac{3}{4} \operatorname{ArcTanh}[ax]^2 \operatorname{Cosh}[2 \operatorname{ArcTanh}[ax]] + \frac{3 \operatorname{Cosh}[4 \operatorname{ArcTanh}[ax]]}{1024} + \frac{3}{128} \operatorname{ArcTanh}[ax]^2 \operatorname{Cosh}[4 \operatorname{ArcTanh}[ax]] - \right. \\
& \left. 3 \operatorname{ArcTanh}[ax]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[ax]}] - 3 \operatorname{ArcTanh}[ax] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[ax]}] + \frac{3}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[ax]}] - \right. \\
& \left. \frac{3}{4} \operatorname{ArcTanh}[ax] \operatorname{Sinh}[2 \operatorname{ArcTanh}[ax]] - \frac{3}{256} \operatorname{ArcTanh}[ax] \operatorname{Sinh}[4 \operatorname{ArcTanh}[ax]] - \frac{1}{32} \operatorname{ArcTanh}[ax]^3 \operatorname{Sinh}[4 \operatorname{ArcTanh}[ax]] \right)
\end{aligned}$$

Problem 383: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcTanh}[ax]^3}{\sqrt{1-a^2x^2}} dx$$

Optimal (type 4, 153 leaves, 10 steps):

$$\begin{aligned}
& \frac{2 \operatorname{ArcTan}[e^{\operatorname{ArcTanh}[ax]}] \operatorname{ArcTanh}[ax]^3}{a} - \frac{3i \operatorname{ArcTanh}[ax]^2 \operatorname{PolyLog}[2, -ie^{\operatorname{ArcTanh}[ax]}]}{a} + \\
& \frac{3i \operatorname{ArcTanh}[ax]^2 \operatorname{PolyLog}[2, ie^{\operatorname{ArcTanh}[ax]}]}{a} + \frac{6i \operatorname{ArcTanh}[ax] \operatorname{PolyLog}[3, -ie^{\operatorname{ArcTanh}[ax]}]}{a} - \\
& \frac{6i \operatorname{ArcTanh}[ax] \operatorname{PolyLog}[3, ie^{\operatorname{ArcTanh}[ax]}]}{a} - \frac{6i \operatorname{PolyLog}[4, -ie^{\operatorname{ArcTanh}[ax]}]}{a} + \frac{6i \operatorname{PolyLog}[4, ie^{\operatorname{ArcTanh}[ax]}]}{a}
\end{aligned}$$

Result (type 4, 451 leaves):

$$\begin{aligned}
& -\frac{1}{64 a} i \left(7 \pi^4 + 8 i \pi^3 \operatorname{ArcTanh}[a x] + 24 \pi^2 \operatorname{ArcTanh}[a x]^2 - 32 i \pi \operatorname{ArcTanh}[a x]^3 - 16 \operatorname{ArcTanh}[a x]^4 + 8 i \pi^3 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcTanh}[a x]}\right] + \right. \\
& 48 \pi^2 \operatorname{ArcTanh}[a x] \operatorname{Log}\left[1 + i e^{-\operatorname{ArcTanh}[a x]}\right] - 96 i \pi \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcTanh}[a x]}\right] - 64 \operatorname{ArcTanh}[a x]^3 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcTanh}[a x]}\right] - \\
& 48 \pi^2 \operatorname{ArcTanh}[a x] \operatorname{Log}\left[1 - i e^{\operatorname{ArcTanh}[a x]}\right] + 96 i \pi \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[1 - i e^{\operatorname{ArcTanh}[a x]}\right] - 8 i \pi^3 \operatorname{Log}\left[1 + i e^{\operatorname{ArcTanh}[a x]}\right] + \\
& 64 \operatorname{ArcTanh}[a x]^3 \operatorname{Log}\left[1 + i e^{\operatorname{ArcTanh}[a x]}\right] + 8 i \pi^3 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{4}(\pi + 2 i \operatorname{ArcTanh}[a x])\right]\right] - 48 (\pi - 2 i \operatorname{ArcTanh}[a x])^2 \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcTanh}[a x]}\right] + \\
& 192 \operatorname{ArcTanh}[a x]^2 \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcTanh}[a x]}\right] - 48 \pi^2 \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcTanh}[a x]}\right] + 192 i \pi \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcTanh}[a x]}\right] + \\
& 192 i \pi \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcTanh}[a x]}\right] + 384 \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcTanh}[a x]}\right] - 384 \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcTanh}[a x]}\right] - \\
& \left. 192 i \pi \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcTanh}[a x]}\right] + 384 \operatorname{PolyLog}\left[4, -i e^{-\operatorname{ArcTanh}[a x]}\right] + 384 \operatorname{PolyLog}\left[4, -i e^{\operatorname{ArcTanh}[a x]}\right] \right)
\end{aligned}$$

Problem 405: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 \operatorname{ArcTanh}[a x]^3}{(1 - a^2 x^2)^{3/2}} dx$$

Optimal (type 4, 246 leaves, 13 steps):

$$\begin{aligned}
& -\frac{6}{a^3 \sqrt{1 - a^2 x^2}} + \frac{6 x \operatorname{ArcTanh}[a x]}{a^2 \sqrt{1 - a^2 x^2}} - \frac{3 \operatorname{ArcTanh}[a x]^2}{a^3 \sqrt{1 - a^2 x^2}} + \frac{x \operatorname{ArcTanh}[a x]^3}{a^2 \sqrt{1 - a^2 x^2}} - \frac{2 \operatorname{ArcTan}\left[e^{\operatorname{ArcTanh}[a x]}\right] \operatorname{ArcTanh}[a x]^3}{a^3} + \\
& \frac{3 i \operatorname{ArcTanh}[a x]^2 \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcTanh}[a x]}\right]}{a^3} - \frac{3 i \operatorname{ArcTanh}[a x]^2 \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcTanh}[a x]}\right]}{a^3} - \frac{6 i \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcTanh}[a x]}\right]}{a^3} + \\
& \frac{6 i \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcTanh}[a x]}\right]}{a^3} + \frac{6 i \operatorname{PolyLog}\left[4, -i e^{\operatorname{ArcTanh}[a x]}\right]}{a^3} - \frac{6 i \operatorname{PolyLog}\left[4, i e^{\operatorname{ArcTanh}[a x]}\right]}{a^3}
\end{aligned}$$

Result (type 4, 541 leaves):

$$\begin{aligned}
& \frac{1}{64 a^3} \left(7 i \pi^4 - \frac{384}{\sqrt{1 - a^2 x^2}} - 8 \pi^3 \operatorname{ArcTanh}[a x] + \frac{384 a x \operatorname{ArcTanh}[a x]}{\sqrt{1 - a^2 x^2}} + 24 i \pi^2 \operatorname{ArcTanh}[a x]^2 - \frac{192 \operatorname{ArcTanh}[a x]^2}{\sqrt{1 - a^2 x^2}} + 32 \pi \operatorname{ArcTanh}[a x]^3 + \right. \\
& \frac{64 a x \operatorname{ArcTanh}[a x]^3}{\sqrt{1 - a^2 x^2}} - 16 i \operatorname{ArcTanh}[a x]^4 - 8 \pi^3 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcTanh}[a x]}\right] + 48 i \pi^2 \operatorname{ArcTanh}[a x] \operatorname{Log}\left[1 + i e^{-\operatorname{ArcTanh}[a x]}\right] + \\
& 96 \pi \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcTanh}[a x]}\right] - 64 i \operatorname{ArcTanh}[a x]^3 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcTanh}[a x]}\right] - 48 i \pi^2 \operatorname{ArcTanh}[a x] \operatorname{Log}\left[1 - i e^{\operatorname{ArcTanh}[a x]}\right] - \\
& 96 \pi \operatorname{ArcTanh}[a x]^2 \operatorname{Log}\left[1 - i e^{\operatorname{ArcTanh}[a x]}\right] + 8 \pi^3 \operatorname{Log}\left[1 + i e^{\operatorname{ArcTanh}[a x]}\right] + 64 i \operatorname{ArcTanh}[a x]^3 \operatorname{Log}\left[1 + i e^{\operatorname{ArcTanh}[a x]}\right] - \\
& 8 \pi^3 \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{4}(\pi + 2 i \operatorname{ArcTanh}[a x])\right]\right] - 48 i (\pi - 2 i \operatorname{ArcTanh}[a x])^2 \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcTanh}[a x]}\right] + \\
& 192 i \operatorname{ArcTanh}[a x]^2 \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcTanh}[a x]}\right] - 48 i \pi^2 \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcTanh}[a x]}\right] - 192 \pi \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcTanh}[a x]}\right] - \\
& 192 \pi \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcTanh}[a x]}\right] + 384 i \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcTanh}[a x]}\right] - 384 i \operatorname{ArcTanh}[a x] \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcTanh}[a x]}\right] + \\
& \left. 192 \pi \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcTanh}[a x]}\right] + 384 i \operatorname{PolyLog}\left[4, -i e^{-\operatorname{ArcTanh}[a x]}\right] + 384 i \operatorname{PolyLog}\left[4, -i e^{\operatorname{ArcTanh}[a x]}\right] \right)
\end{aligned}$$

Problem 412: Attempted integration timed out after 120 seconds.

$$\int \frac{x^2}{(1 - a^2 x^2)^{3/2} \text{ArcTanh}[a x]} dx$$

Optimal (type 9, 26 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{x^2}{(1 - a^2 x^2)^{3/2} \text{ArcTanh}[a x]}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 458: Result more than twice size of optimal antiderivative.

$$\int \frac{(1 - a^2 x^2)^{3/2} \text{ArcTanh}[a x]}{x^7} dx$$

Optimal (type 4, 243 leaves, 24 steps):

$$\begin{aligned} & -\frac{a \sqrt{1 - a^2 x^2}}{30 x^5} + \frac{19 a^3 \sqrt{1 - a^2 x^2}}{360 x^3} + \frac{31 a^5 \sqrt{1 - a^2 x^2}}{720 x} - \frac{\sqrt{1 - a^2 x^2} \text{ArcTanh}[a x]}{6 x^6} + \frac{7 a^2 \sqrt{1 - a^2 x^2} \text{ArcTanh}[a x]}{24 x^4} \\ & - \frac{a^4 \sqrt{1 - a^2 x^2} \text{ArcTanh}[a x]}{16 x^2} - \frac{1}{8} a^6 \text{ArcTanh}[a x] \text{ArcTanh}\left[\frac{\sqrt{1 - a x}}{\sqrt{1 + a x}}\right] + \frac{1}{16} a^6 \text{PolyLog}\left[2, -\frac{\sqrt{1 - a x}}{\sqrt{1 + a x}}\right] - \frac{1}{16} a^6 \text{PolyLog}\left[2, \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}}\right] \end{aligned}$$

Result (type 4, 530 leaves):

$$\begin{aligned}
& -\frac{1}{192} a^6 \left(-8 \operatorname{Coth} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right] - 6 \operatorname{ArcTanh} [a x] \operatorname{Csch} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^2 - \frac{a x \operatorname{Csch} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^4}{\sqrt{1-a^2 x^2}} - \right. \\
& 3 \operatorname{ArcTanh} [a x] \operatorname{Csch} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^4 - 24 \operatorname{ArcTanh} [a x] \operatorname{Log} \left[1 - e^{-\operatorname{ArcTanh} [a x]} \right] + 24 \operatorname{ArcTanh} [a x] \operatorname{Log} \left[1 + e^{-\operatorname{ArcTanh} [a x]} \right] - \\
& 24 \operatorname{PolyLog} \left[2, -e^{-\operatorname{ArcTanh} [a x]} \right] + 24 \operatorname{PolyLog} \left[2, e^{-\operatorname{ArcTanh} [a x]} \right] - 6 \operatorname{ArcTanh} [a x] \operatorname{Sech} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^2 + \\
& \left. 3 \operatorname{ArcTanh} [a x] \operatorname{Sech} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^4 - \frac{16 (1-a^2 x^2)^{3/2} \operatorname{Sinh} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^4}{a^3 x^3} + 8 \operatorname{Tanh} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right] \right) + \\
& \frac{1}{5760} a^6 \left(-76 \operatorname{Coth} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right] - 90 \operatorname{ArcTanh} [a x] \operatorname{Csch} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^2 - \frac{26 a x \operatorname{Csch} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^4}{\sqrt{1-a^2 x^2}} - \right. \\
& 90 \operatorname{ArcTanh} [a x] \operatorname{Csch} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^4 - \frac{3 a x \operatorname{Csch} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^6}{\sqrt{1-a^2 x^2}} - 15 \operatorname{ArcTanh} [a x] \operatorname{Csch} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^6 - 360 \operatorname{ArcTanh} [a x] \\
& \operatorname{Log} \left[1 - e^{-\operatorname{ArcTanh} [a x]} \right] + 360 \operatorname{ArcTanh} [a x] \operatorname{Log} \left[1 + e^{-\operatorname{ArcTanh} [a x]} \right] - 360 \operatorname{PolyLog} \left[2, -e^{-\operatorname{ArcTanh} [a x]} \right] + 360 \operatorname{PolyLog} \left[2, e^{-\operatorname{ArcTanh} [a x]} \right] - \\
& 90 \operatorname{ArcTanh} [a x] \operatorname{Sech} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^2 + 90 \operatorname{ArcTanh} [a x] \operatorname{Sech} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^4 - 15 \operatorname{ArcTanh} [a x] \operatorname{Sech} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^6 - \\
& \left. \frac{416 (1-a^2 x^2)^{3/2} \operatorname{Sinh} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^4}{a^3 x^3} + 76 \operatorname{Tanh} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right] + 6 \operatorname{Sech} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right]^4 \operatorname{Tanh} \left[\frac{1}{2} \operatorname{ArcTanh} [a x] \right] \right)
\end{aligned}$$

Problem 502: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh} [a x]}{c+d x^2} dx$$

Optimal (type 4, 429 leaves, 17 steps):

$$\begin{aligned}
& -\frac{\operatorname{Log} [1-a x] \operatorname{Log} \left[\frac{a(\sqrt{-c}-\sqrt{d} x)}{a\sqrt{-c}-\sqrt{d}} \right]}{4\sqrt{-c}\sqrt{d}} + \frac{\operatorname{Log} [1+a x] \operatorname{Log} \left[\frac{a(\sqrt{-c}+\sqrt{d} x)}{a\sqrt{-c}+\sqrt{d}} \right]}{4\sqrt{-c}\sqrt{d}} - \frac{\operatorname{Log} [1+a x] \operatorname{Log} \left[\frac{a(\sqrt{-c}-\sqrt{d} x)}{a\sqrt{-c}-\sqrt{d}} \right]}{4\sqrt{-c}\sqrt{d}} + \\
& \frac{\operatorname{Log} [1-a x] \operatorname{Log} \left[\frac{a(\sqrt{-c}+\sqrt{d} x)}{a\sqrt{-c}+\sqrt{d}} \right]}{4\sqrt{-c}\sqrt{d}} - \frac{\operatorname{PolyLog} \left[2, -\frac{\sqrt{d}(1-ax)}{a\sqrt{-c}-\sqrt{d}} \right]}{4\sqrt{-c}\sqrt{d}} + \frac{\operatorname{PolyLog} \left[2, \frac{\sqrt{d}(1+ax)}{a\sqrt{-c}+\sqrt{d}} \right]}{4\sqrt{-c}\sqrt{d}} - \frac{\operatorname{PolyLog} \left[2, -\frac{\sqrt{d}(1-ax)}{a\sqrt{-c}-\sqrt{d}} \right]}{4\sqrt{-c}\sqrt{d}} + \frac{\operatorname{PolyLog} \left[2, \frac{\sqrt{d}(1+ax)}{a\sqrt{-c}+\sqrt{d}} \right]}{4\sqrt{-c}\sqrt{d}}
\end{aligned}$$

Result (type 4, 662 leaves):

$$\begin{aligned}
& -\frac{1}{4\sqrt{a^2 c d}} a \left(-2 i \operatorname{ArcCos}\left[\frac{-a^2 c + d}{a^2 c + d}\right] \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] + 4 \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d} x}\right] \operatorname{ArcTanh}[a x] - \left(\operatorname{ArcCos}\left[\frac{-a^2 c + d}{a^2 c + d}\right] + 2 \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] \right) \right. \\
& \quad \operatorname{Log}\left[\frac{2 i a c (i d + \sqrt{a^2 c d}) (-1 + a x)}{(a^2 c + d) (a c + i \sqrt{a^2 c d} x)}\right] - \left(\operatorname{ArcCos}\left[\frac{-a^2 c + d}{a^2 c + d}\right] - 2 \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] \right) \operatorname{Log}\left[\frac{2 a c (d + i \sqrt{a^2 c d}) (1 + a x)}{(a^2 c + d) (a c + i \sqrt{a^2 c d} x)}\right] + \\
& \quad \left(\operatorname{ArcCos}\left[\frac{-a^2 c + d}{a^2 c + d}\right] + 2 \left(\operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d} x}\right] + \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{a^2 c d} e^{-\operatorname{ArcTanh}[a x]}}{\sqrt{a^2 c + d} \sqrt{a^2 c - d + (a^2 c + d) \operatorname{Cosh}[2 \operatorname{ArcTanh}[a x]]}}\right] + \\
& \quad \left(\operatorname{ArcCos}\left[\frac{-a^2 c + d}{a^2 c + d}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d} x}\right] + \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{a^2 c d} e^{\operatorname{ArcTanh}[a x]}}{\sqrt{a^2 c + d} \sqrt{a^2 c - d + (a^2 c + d) \operatorname{Cosh}[2 \operatorname{ArcTanh}[a x]]}}\right] + \\
& \quad \left. i \left(-\operatorname{PolyLog}\left[2, \frac{(-a^2 c + d - 2 i \sqrt{a^2 c d}) (i a c + \sqrt{a^2 c d} x)}{(a^2 c + d) (-i a c + \sqrt{a^2 c d} x)}\right] + \operatorname{PolyLog}\left[2, \frac{(-a^2 c + d + 2 i \sqrt{a^2 c d}) (i a c + \sqrt{a^2 c d} x)}{(a^2 c + d) (-i a c + \sqrt{a^2 c d} x)}\right] \right) \right)
\end{aligned}$$

Problem 504: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcTanh}[a x]}{(c + d x^2)^3} dx$$

Optimal (type 4, 657 leaves, 23 steps):

$$\begin{aligned}
& \frac{a}{8 c (a^2 c + d) (c + d x^2)} + \frac{x \operatorname{ArcTanh}[a x]}{4 c (c + d x^2)^2} + \frac{3 x \operatorname{ArcTanh}[a x]}{8 c^2 (c + d x^2)} + \frac{3 \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right] \operatorname{ArcTanh}[a x]}{8 c^{5/2} \sqrt{d}} + \\
& \frac{3 i \operatorname{Log}\left[\frac{\sqrt{d} (1 - a x)}{i a \sqrt{c} + \sqrt{d}}\right] \operatorname{Log}\left[1 - \frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}} - \frac{3 i \operatorname{Log}\left[-\frac{\sqrt{d} (1 + a x)}{i a \sqrt{c} - \sqrt{d}}\right] \operatorname{Log}\left[1 - \frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}} - \frac{3 i \operatorname{Log}\left[-\frac{\sqrt{d} (1 - a x)}{i a \sqrt{c} - \sqrt{d}}\right] \operatorname{Log}\left[1 + \frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}} + \\
& \frac{3 i \operatorname{Log}\left[\frac{\sqrt{d} (1 + a x)}{i a \sqrt{c} + \sqrt{d}}\right] \operatorname{Log}\left[1 + \frac{i \sqrt{d} x}{\sqrt{c}}\right]}{32 c^{5/2} \sqrt{d}} + \frac{a (5 a^2 c + 3 d) \operatorname{Log}[1 - a^2 x^2]}{16 c^2 (a^2 c + d)^2} - \frac{a (5 a^2 c + 3 d) \operatorname{Log}[c + d x^2]}{16 c^2 (a^2 c + d)^2} + \\
& \frac{3 i \operatorname{PolyLog}\left[2, \frac{a (\sqrt{c} - i \sqrt{d} x)}{a \sqrt{c} - i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}} - \frac{3 i \operatorname{PolyLog}\left[2, \frac{a (\sqrt{c} - i \sqrt{d} x)}{a \sqrt{c} + i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}} + \frac{3 i \operatorname{PolyLog}\left[2, \frac{a (\sqrt{c} + i \sqrt{d} x)}{a \sqrt{c} - i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}} - \frac{3 i \operatorname{PolyLog}\left[2, \frac{a (\sqrt{c} + i \sqrt{d} x)}{a \sqrt{c} + i \sqrt{d}}\right]}{32 c^{5/2} \sqrt{d}}
\end{aligned}$$

Result (type 4, 1840 leaves):

$$\begin{aligned}
& a^5 \left(-\frac{5 \operatorname{Log}\left[1 + \frac{(a^2 c + d) \operatorname{Cosh}[2 \operatorname{ArcTanh}[a x]]}{a^2 c - d}\right]}{16 a^2 c (a^2 c + d)^2} - \frac{3 d \operatorname{Log}\left[1 + \frac{(a^2 c + d) \operatorname{Cosh}[2 \operatorname{ArcTanh}[a x]]}{a^2 c - d}\right]}{16 a^4 c^2 (a^2 c + d)^2} - \right. \\
& \frac{1}{32 a^2 c \sqrt{a^2 c d} (a^2 c + d)} 3 \left(-2 i \operatorname{ArcCos}\left[-\frac{a^2 c - d}{a^2 c + d}\right] \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] + 4 \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d} x}\right] \operatorname{ArcTanh}[a x] - \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{a^2 c - d}{a^2 c + d}\right] - 2 \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] \right) \operatorname{Log}\left[1 - \frac{(a^2 c - d - 2 i \sqrt{a^2 c d}) (2 a^2 c - 2 i a \sqrt{a^2 c d} x)}{(a^2 c + d) (2 a^2 c + 2 i a \sqrt{a^2 c d} x)}\right] + \right. \\
& \left. \left(-\operatorname{ArcCos}\left[-\frac{a^2 c - d}{a^2 c + d}\right] - 2 \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] \right) \operatorname{Log}\left[1 - \frac{(a^2 c - d + 2 i \sqrt{a^2 c d}) (2 a^2 c - 2 i a \sqrt{a^2 c d} x)}{(a^2 c + d) (2 a^2 c + 2 i a \sqrt{a^2 c d} x)}\right] + \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{a^2 c - d}{a^2 c + d}\right] + 2 i \left(-i \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d} x}\right] - i \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{a^2 c d} e^{-\operatorname{ArcTanh}[a x]}}{\sqrt{a^2 c + d} \sqrt{a^2 c - d + (a^2 c + d) \operatorname{Cosh}[2 \operatorname{ArcTanh}[a x]]}}\right] + \right. \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{a^2 c - d}{a^2 c + d}\right] - 2 i \left(-i \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d} x}\right] - i \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{a^2 c d} e^{\operatorname{ArcTanh}[a x]}}{\sqrt{a^2 c + d} \sqrt{a^2 c - d + (a^2 c + d) \operatorname{Cosh}[2 \operatorname{ArcTanh}[a x]]}}\right] + \right. \\
& \left. i \left(\operatorname{PolyLog}\left[2, \frac{(a^2 c - d - 2 i \sqrt{a^2 c d}) (2 a^2 c - 2 i a \sqrt{a^2 c d} x)}{(a^2 c + d) (2 a^2 c + 2 i a \sqrt{a^2 c d} x)}\right] - \operatorname{PolyLog}\left[2, \frac{(a^2 c - d + 2 i \sqrt{a^2 c d}) (2 a^2 c - 2 i a \sqrt{a^2 c d} x)}{(a^2 c + d) (2 a^2 c + 2 i a \sqrt{a^2 c d} x)}\right] \right) \right) - \\
& \frac{1}{32 a^4 c^2 \sqrt{a^2 c d} (a^2 c + d)} 3 d \left(-2 i \operatorname{ArcCos}\left[-\frac{a^2 c - d}{a^2 c + d}\right] \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] + 4 \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d} x}\right] \operatorname{ArcTanh}[a x] - \right. \\
& \left(\operatorname{ArcCos}\left[-\frac{a^2 c - d}{a^2 c + d}\right] - 2 \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] \right) \operatorname{Log}\left[1 - \frac{(a^2 c - d - 2 i \sqrt{a^2 c d}) (2 a^2 c - 2 i a \sqrt{a^2 c d} x)}{(a^2 c + d) (2 a^2 c + 2 i a \sqrt{a^2 c d} x)}\right] + \\
& \left(-\operatorname{ArcCos}\left[-\frac{a^2 c - d}{a^2 c + d}\right] - 2 \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] \right) \operatorname{Log}\left[1 - \frac{(a^2 c - d + 2 i \sqrt{a^2 c d}) (2 a^2 c - 2 i a \sqrt{a^2 c d} x)}{(a^2 c + d) (2 a^2 c + 2 i a \sqrt{a^2 c d} x)}\right] + \\
& \left(\operatorname{ArcCos}\left[-\frac{a^2 c - d}{a^2 c + d}\right] + 2 i \left(-i \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d} x}\right] - i \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{a^2 c d} e^{-\operatorname{ArcTanh}[a x]}}{\sqrt{a^2 c + d} \sqrt{a^2 c - d + (a^2 c + d) \operatorname{Cosh}[2 \operatorname{ArcTanh}[a x]]}}\right] + \\
& \left. \left(\operatorname{ArcCos}\left[-\frac{a^2 c - d}{a^2 c + d}\right] - 2 i \left(-i \operatorname{ArcTan}\left[\frac{a c}{\sqrt{a^2 c d} x}\right] - i \operatorname{ArcTan}\left[\frac{a d x}{\sqrt{a^2 c d}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} \sqrt{a^2 c d} e^{\operatorname{ArcTanh}[a x]}}{\sqrt{a^2 c + d} \sqrt{a^2 c - d + (a^2 c + d) \operatorname{Cosh}[2 \operatorname{ArcTanh}[a x]]}}\right] \right) +
\end{aligned}$$

$$\begin{aligned}
& i \left(\text{PolyLog}\left[2, \frac{(a^2 c - d - 2 i \sqrt{a^2 c d}) (2 a^2 c - 2 i a \sqrt{a^2 c d} x)}{(a^2 c + d) (2 a^2 c + 2 i a \sqrt{a^2 c d} x)}\right] - \text{PolyLog}\left[2, \frac{(a^2 c - d + 2 i \sqrt{a^2 c d}) (2 a^2 c - 2 i a \sqrt{a^2 c d} x)}{(a^2 c + d) (2 a^2 c + 2 i a \sqrt{a^2 c d} x)}\right] \right) + \\
& \frac{d \text{ArcTanh}[a x] \text{Sinh}[2 \text{ArcTanh}[a x]]}{2 a^2 c (a^2 c + d) (a^2 c - d + a^2 c \text{Cosh}[2 \text{ArcTanh}[a x]] + d \text{Cosh}[2 \text{ArcTanh}[a x]])^2 +} \\
& (2 a^2 c d + 5 a^4 c^2 \text{ArcTanh}[a x] \text{Sinh}[2 \text{ArcTanh}[a x]] + 8 a^2 c d \text{ArcTanh}[a x] \text{Sinh}[2 \text{ArcTanh}[a x]] + 3 d^2 \text{ArcTanh}[a x] \text{Sinh}[2 \text{ArcTanh}[a x]]) / \\
& \left(8 a^4 c^2 (a^2 c + d)^2 (a^2 c - d + a^2 c \text{Cosh}[2 \text{ArcTanh}[a x]] + d \text{Cosh}[2 \text{ArcTanh}[a x]]) \right)
\end{aligned}$$

Problem 506: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTanh}[b x]}{1 - x^2} dx$$

Optimal (type 4, 171 leaves, 17 steps):

$$\begin{aligned}
& \frac{1}{4} \text{Log}\left[-\frac{b(1-x)}{1-b}\right] \text{Log}[1-bx] - \frac{1}{4} \text{Log}\left[\frac{b(1+x)}{1+b}\right] \text{Log}[1-bx] - \frac{1}{4} \text{Log}\left[\frac{b(1-x)}{1+b}\right] \text{Log}[1+bx] + \\
& \frac{1}{4} \text{Log}\left[-\frac{b(1+x)}{1-b}\right] \text{Log}[1+bx] + \frac{1}{4} \text{PolyLog}\left[2, \frac{1-bx}{1-b}\right] - \frac{1}{4} \text{PolyLog}\left[2, \frac{1-bx}{1+b}\right] + \frac{1}{4} \text{PolyLog}\left[2, \frac{1+bx}{1-b}\right] - \frac{1}{4} \text{PolyLog}\left[2, \frac{1+bx}{1+b}\right]
\end{aligned}$$

Result (type 4, 576 leaves):

$$\begin{aligned}
& -\frac{1}{4\sqrt{-b^2}} \\
& b \left(2i \operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] \operatorname{ArcTan}\left[\frac{bx}{\sqrt{-b^2}}\right] - 4 \operatorname{ArcTan}\left[\frac{\sqrt{-b^2}}{bx}\right] \operatorname{ArcTanh}[bx] - \left(\operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] - 2 \operatorname{ArcTan}\left[\frac{bx}{\sqrt{-b^2}}\right] \right) \operatorname{Log}\left[\frac{2b(-i+\sqrt{-b^2})(-1+bx)}{(-1+b^2)(-ib+\sqrt{-b^2}x)}\right] - \right. \\
& \left. \left(\operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 \operatorname{ArcTan}\left[\frac{bx}{\sqrt{-b^2}}\right] \right) \operatorname{Log}\left[\frac{2b(i+\sqrt{-b^2})(1+bx)}{(-1+b^2)(-ib+\sqrt{-b^2}x)}\right] + \right. \\
& \left. \left(\operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{\sqrt{-b^2}}{bx}\right] + \operatorname{ArcTan}\left[\frac{bx}{\sqrt{-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2}\sqrt{-b^2}e^{-\operatorname{ArcTanh}[bx]}}{\sqrt{-1+b^2}\sqrt{1+b^2+(-1+b^2)\operatorname{Cosh}[2\operatorname{ArcTanh}[bx]]}}\right] + \right. \\
& \left. \left(\operatorname{ArcCos}\left[\frac{1+b^2}{1-b^2}\right] + 2 \left(\operatorname{ArcTan}\left[\frac{\sqrt{-b^2}}{bx}\right] + \operatorname{ArcTan}\left[\frac{bx}{\sqrt{-b^2}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2}\sqrt{-b^2}e^{\operatorname{ArcTanh}[bx]}}{\sqrt{-1+b^2}\sqrt{1+b^2+(-1+b^2)\operatorname{Cosh}[2\operatorname{ArcTanh}[bx]]}}\right] + \right. \\
& \left. i \left(\operatorname{PolyLog}\left[2, \frac{(1+b^2-2i\sqrt{-b^2})(b-i\sqrt{-b^2}x)}{(-1+b^2)(b+i\sqrt{-b^2}x)}\right] - \operatorname{PolyLog}\left[2, \frac{(1+b^2+2i\sqrt{-b^2})(b-i\sqrt{-b^2}x)}{(-1+b^2)(b+i\sqrt{-b^2}x)}\right] \right) \right)
\end{aligned}$$

Problem 507: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcTanh}[a+bx]}{1-x^2} dx$$

Optimal (type 4, 203 leaves, 17 steps):

$$\begin{aligned}
& \frac{1}{4} \operatorname{Log}\left[-\frac{b(1-x)}{1-a-b}\right] \operatorname{Log}[1-a-bx] - \frac{1}{4} \operatorname{Log}\left[\frac{b(1+x)}{1-a+b}\right] \operatorname{Log}[1-a-bx] - \frac{1}{4} \operatorname{Log}\left[\frac{b(1-x)}{1+a+b}\right] \operatorname{Log}[1+a+bx] + \frac{1}{4} \operatorname{Log}\left[-\frac{b(1+x)}{1+a-b}\right] \operatorname{Log}[1+a+bx] + \\
& \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{1-a-bx}{1-a-b}\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{1-a-bx}{1-a+b}\right] + \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{1+a+bx}{1+a-b}\right] - \frac{1}{4} \operatorname{PolyLog}\left[2, \frac{1+a+bx}{1+a+b}\right]
\end{aligned}$$

Result (type 4, 646 leaves):

$$\begin{aligned}
& -\frac{1}{4(-1+a^2)} \left(-2 \operatorname{ArcTanh}\left[\frac{-1+a}{b}\right] \operatorname{ArcTanh}[x] + 2a^2 \operatorname{ArcTanh}\left[\frac{-1+a}{b}\right] \operatorname{ArcTanh}[x] + 2 \operatorname{ArcTanh}\left[\frac{1+a}{b}\right] \operatorname{ArcTanh}[x] - 2a^2 \operatorname{ArcTanh}\left[\frac{1+a}{b}\right] \operatorname{ArcTanh}[x] - \right. \\
& 2b \operatorname{ArcTanh}[x]^2 + b \sqrt{\frac{-1+2a-a^2+b^2}{b^2}} e^{\operatorname{ArcTanh}\left[\frac{-1+a}{b}\right]} \operatorname{ArcTanh}[x]^2 + a b \sqrt{\frac{-1+2a-a^2+b^2}{b^2}} e^{\operatorname{ArcTanh}\left[\frac{-1+a}{b}\right]} \operatorname{ArcTanh}[x]^2 + \\
& b \sqrt{\frac{1+2a+a^2-b^2}{b^2}} e^{\operatorname{ArcTanh}\left[\frac{1+a}{b}\right]} \operatorname{ArcTanh}[x]^2 - a b \sqrt{\frac{1+2a+a^2-b^2}{b^2}} e^{\operatorname{ArcTanh}\left[\frac{1+a}{b}\right]} \operatorname{ArcTanh}[x]^2 + 4 \operatorname{ArcTanh}[x] \operatorname{ArcTanh}[a+bx] - \\
& 4a^2 \operatorname{ArcTanh}[x] \operatorname{ArcTanh}[a+bx] + 2 \operatorname{ArcTanh}\left[\frac{-1+a}{b}\right] \operatorname{Log}\left[1 - e^{2\left(\operatorname{ArcTanh}\left[\frac{-1+a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] - 2a^2 \operatorname{ArcTanh}\left[\frac{-1+a}{b}\right] \operatorname{Log}\left[1 - e^{2\left(\operatorname{ArcTanh}\left[\frac{-1+a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] + \\
& 2 \operatorname{ArcTanh}[x] \operatorname{Log}\left[1 - e^{2\left(\operatorname{ArcTanh}\left[\frac{-1+a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] - 2a^2 \operatorname{ArcTanh}[x] \operatorname{Log}\left[1 - e^{2\left(\operatorname{ArcTanh}\left[\frac{-1+a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] - \\
& 2 \operatorname{ArcTanh}\left[\frac{1+a}{b}\right] \operatorname{Log}\left[1 - e^{2\left(\operatorname{ArcTanh}\left[\frac{1+a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] + 2a^2 \operatorname{ArcTanh}\left[\frac{1+a}{b}\right] \operatorname{Log}\left[1 - e^{2\left(\operatorname{ArcTanh}\left[\frac{1+a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] - \\
& 2 \operatorname{ArcTanh}[x] \operatorname{Log}\left[1 - e^{2\left(\operatorname{ArcTanh}\left[\frac{1+a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] + 2a^2 \operatorname{ArcTanh}[x] \operatorname{Log}\left[1 - e^{2\left(\operatorname{ArcTanh}\left[\frac{1+a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] - \\
& 2 \operatorname{ArcTanh}\left[\frac{-1+a}{b}\right] \operatorname{Log}\left[\operatorname{i} \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{1-a}{b}\right] - \operatorname{ArcTanh}[x]\right]\right] + 2a^2 \operatorname{ArcTanh}\left[\frac{-1+a}{b}\right] \operatorname{Log}\left[\operatorname{i} \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{1-a}{b}\right] - \operatorname{ArcTanh}[x]\right]\right] + \\
& 2 \operatorname{ArcTanh}\left[\frac{1+a}{b}\right] \operatorname{Log}\left[-\operatorname{i} \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{1+a}{b}\right] + \operatorname{ArcTanh}[x]\right]\right] - 2a^2 \operatorname{ArcTanh}\left[\frac{1+a}{b}\right] \operatorname{Log}\left[-\operatorname{i} \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{1+a}{b}\right] + \operatorname{ArcTanh}[x]\right]\right] - \\
& \left. (-1+a^2) \operatorname{PolyLog}\left[2, e^{2\left(\operatorname{ArcTanh}\left[\frac{-1+a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] + (-1+a^2) \operatorname{PolyLog}\left[2, e^{2\left(\operatorname{ArcTanh}\left[\frac{1+a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] \right)
\end{aligned}$$

Problem 508: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcTanh}[x]}{a+bx} dx$$

Optimal (type 4, 86 leaves, 4 steps):

$$-\frac{\operatorname{ArcTanh}[x] \operatorname{Log}\left[\frac{2}{1+x}\right]}{b} + \frac{\operatorname{ArcTanh}[x] \operatorname{Log}\left[\frac{2(a+bx)}{(a+b)(1+x)}\right]}{b} + \frac{\operatorname{PolyLog}\left[2, 1 - \frac{2}{1+x}\right]}{2b} - \frac{\operatorname{PolyLog}\left[2, 1 - \frac{2(a+bx)}{(a+b)(1+x)}\right]}{2b}$$

Result (type 4, 260 leaves):

$$\begin{aligned} & \frac{1}{8b} \left(-\pi^2 + 4 \operatorname{ArcTanh}\left[\frac{a}{b}\right]^2 + 4i\pi \operatorname{ArcTanh}[x] + 8 \operatorname{ArcTanh}\left[\frac{a}{b}\right] \operatorname{ArcTanh}[x] + 8 \operatorname{ArcTanh}[x]^2 - \right. \\ & 4i\pi \operatorname{Log}\left[1 + e^{2\operatorname{ArcTanh}[x]}\right] - 8 \operatorname{ArcTanh}[x] \operatorname{Log}\left[1 + e^{2\operatorname{ArcTanh}[x]}\right] + 8 \operatorname{ArcTanh}\left[\frac{a}{b}\right] \operatorname{Log}\left[1 - e^{-2\left(\operatorname{ArcTanh}\left[\frac{a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] + \\ & 8 \operatorname{ArcTanh}[x] \operatorname{Log}\left[1 - e^{-2\left(\operatorname{ArcTanh}\left[\frac{a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] + 4i\pi \operatorname{Log}\left[\frac{2}{\sqrt{1-x^2}}\right] + 8 \operatorname{ArcTanh}[x] \operatorname{Log}\left[\frac{2}{\sqrt{1-x^2}}\right] + 4 \operatorname{ArcTanh}[x] \operatorname{Log}\left[1-x^2\right] + \\ & 8 \operatorname{ArcTanh}[x] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{a}{b}\right] + \operatorname{ArcTanh}[x]\right]\right] - 8 \operatorname{ArcTanh}\left[\frac{a}{b}\right] \operatorname{Log}\left[2i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{a}{b}\right] + \operatorname{ArcTanh}[x]\right]\right] - \\ & \left. 8 \operatorname{ArcTanh}[x] \operatorname{Log}\left[2i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{a}{b}\right] + \operatorname{ArcTanh}[x]\right]\right] - 4 \operatorname{PolyLog}\left[2, -e^{2\operatorname{ArcTanh}[x]}\right] - 4 \operatorname{PolyLog}\left[2, e^{-2\left(\operatorname{ArcTanh}\left[\frac{a}{b}\right] + \operatorname{ArcTanh}[x]\right)}\right] \right) \end{aligned}$$

Problem 509: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh}[x]}{a + b x^2} dx$$

Optimal (type 4, 397 leaves, 17 steps):

$$\begin{aligned} & -\frac{\operatorname{Log}[1-x] \operatorname{Log}\left[\frac{\sqrt{-a}-\sqrt{b}x}{\sqrt{-a}-\sqrt{b}}\right]}{4\sqrt{-a}\sqrt{b}} + \frac{\operatorname{Log}[1+x] \operatorname{Log}\left[\frac{\sqrt{-a}-\sqrt{b}x}{\sqrt{-a}+\sqrt{b}}\right]}{4\sqrt{-a}\sqrt{b}} - \frac{\operatorname{Log}[1+x] \operatorname{Log}\left[\frac{\sqrt{-a}+\sqrt{b}x}{\sqrt{-a}-\sqrt{b}}\right]}{4\sqrt{-a}\sqrt{b}} + \\ & \frac{\operatorname{Log}[1-x] \operatorname{Log}\left[\frac{\sqrt{-a}+\sqrt{b}x}{\sqrt{-a}+\sqrt{b}}\right]}{4\sqrt{-a}\sqrt{b}} - \frac{\operatorname{PolyLog}\left[2, -\frac{\sqrt{b}(1-x)}{\sqrt{-a}-\sqrt{b}}\right]}{4\sqrt{-a}\sqrt{b}} + \frac{\operatorname{PolyLog}\left[2, \frac{\sqrt{b}(1-x)}{\sqrt{-a}+\sqrt{b}}\right]}{4\sqrt{-a}\sqrt{b}} - \frac{\operatorname{PolyLog}\left[2, -\frac{\sqrt{b}(1+x)}{\sqrt{-a}-\sqrt{b}}\right]}{4\sqrt{-a}\sqrt{b}} + \frac{\operatorname{PolyLog}\left[2, \frac{\sqrt{b}(1+x)}{\sqrt{-a}+\sqrt{b}}\right]}{4\sqrt{-a}\sqrt{b}} \end{aligned}$$

Result (type 4, 485 leaves):

$$\begin{aligned}
& -\frac{1}{4\sqrt{ab}} \left(-2i \operatorname{ArcCos}\left[\frac{-a+b}{a+b}\right] \operatorname{ArcTan}\left[\frac{bx}{\sqrt{ab}}\right] + 4 \operatorname{ArcTan}\left[\frac{a}{\sqrt{ab}x}\right] \operatorname{ArcTanh}[x] - \right. \\
& \left. \left(\operatorname{ArcCos}\left[\frac{-a+b}{a+b}\right] + 2 \operatorname{ArcTan}\left[\frac{bx}{\sqrt{ab}}\right] \right) \operatorname{Log}\left[\frac{2ia(i b + \sqrt{ab})(-1+x)}{(a+b)(a+i\sqrt{ab}x)}\right] - \left(\operatorname{ArcCos}\left[\frac{-a+b}{a+b}\right] - 2 \operatorname{ArcTan}\left[\frac{bx}{\sqrt{ab}}\right] \right) \operatorname{Log}\left[\frac{2a(b+i\sqrt{ab})(1+x)}{(a+b)(a+i\sqrt{ab}x)}\right] \right) + \\
& \left(\operatorname{ArcCos}\left[\frac{-a+b}{a+b}\right] + 2 \left(\operatorname{ArcTan}\left[\frac{a}{\sqrt{ab}x}\right] + \operatorname{ArcTan}\left[\frac{bx}{\sqrt{ab}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2}\sqrt{ab}e^{-\operatorname{ArcTanh}[x]}}{\sqrt{a+b}\sqrt{a-b+(a+b)\operatorname{Cosh}[2\operatorname{ArcTanh}[x]]}}\right] + \\
& \left(\operatorname{ArcCos}\left[\frac{-a+b}{a+b}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{a}{\sqrt{ab}x}\right] + \operatorname{ArcTan}\left[\frac{bx}{\sqrt{ab}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2}\sqrt{ab}e^{\operatorname{ArcTanh}[x]}}{\sqrt{a+b}\sqrt{a-b+(a+b)\operatorname{Cosh}[2\operatorname{ArcTanh}[x]]}}\right] + \\
& i \left(-\operatorname{PolyLog}\left[2, \frac{(-a+b-2i\sqrt{ab})(ia+\sqrt{ab}x)}{(a+b)(-ia+\sqrt{ab}x)}\right] + \operatorname{PolyLog}\left[2, \frac{(-a+b+2i\sqrt{ab})(ia+\sqrt{ab}x)}{(a+b)(-ia+\sqrt{ab}x)}\right] \right)
\end{aligned}$$

Problem 510: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcTanh}[x]}{a+bx+cx^2} dx$$

Optimal (type 4, 258 leaves, 10 steps):

$$\begin{aligned}
& \frac{\operatorname{ArcTanh}[x] \operatorname{Log}\left[\frac{2(b-\sqrt{b^2-4ac}+2cx)}{(b+2c-\sqrt{b^2-4ac})(1+x)}\right]}{\sqrt{b^2-4ac}} - \frac{\operatorname{ArcTanh}[x] \operatorname{Log}\left[\frac{2(b+\sqrt{b^2-4ac}+2cx)}{(b+2c+\sqrt{b^2-4ac})(1+x)}\right]}{\sqrt{b^2-4ac}} - \\
& \frac{\operatorname{PolyLog}\left[2, 1 - \frac{2(b-\sqrt{b^2-4ac}+2cx)}{(b+2c-\sqrt{b^2-4ac})(1+x)}\right]}{2\sqrt{b^2-4ac}} + \frac{\operatorname{PolyLog}\left[2, 1 - \frac{2(b+\sqrt{b^2-4ac}+2cx)}{(b+2c+\sqrt{b^2-4ac})(1+x)}\right]}{2\sqrt{b^2-4ac}}
\end{aligned}$$

Result (type 4, 910 leaves):

$$\begin{aligned}
& \frac{1}{2\sqrt{-b^2+4ac}} \frac{1}{(b^2-4c^2)} \left(2 \left(\sqrt{-b^2+4ac} \left(b \left(\sqrt{\frac{c(a+b+c)}{-b^2+4ac}} e^{i \operatorname{ArcTan}\left[\frac{-b-2c}{\sqrt{-b^2+4ac}}\right]} - \sqrt{\frac{c(a-b+c)}{-b^2+4ac}} e^{i \operatorname{ArcTan}\left[\frac{-b+2c}{\sqrt{-b^2+4ac}}\right]} \right) \right. \right. \\
& \left. \left. 2c \left(-1 + \sqrt{\frac{c(a+b+c)}{-b^2+4ac}} e^{i \operatorname{ArcTan}\left[\frac{-b-2c}{\sqrt{-b^2+4ac}}\right]} + \sqrt{\frac{c(a-b+c)}{-b^2+4ac}} e^{i \operatorname{ArcTan}\left[\frac{-b+2c}{\sqrt{-b^2+4ac}}\right]} \right) \right) \operatorname{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right]^2 + \right. \\
& (b^2-4c^2) \operatorname{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right] \left(-i \operatorname{ArcTan}\left[\frac{-b-2c}{\sqrt{-b^2+4ac}}\right] + i \operatorname{ArcTan}\left[\frac{-b+2c}{\sqrt{-b^2+4ac}}\right] + 2 \operatorname{ArcTanh}[x] + \right. \\
& \left. \left. \operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{-b-2c}{\sqrt{-b^2+4ac}}\right] + \operatorname{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right] \right)}\right] - \operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{-b+2c}{\sqrt{-b^2+4ac}}\right] + \operatorname{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right] \right)}\right] \right) + (b^2-4c^2) \right. \\
& \left(\operatorname{ArcTan}\left[\frac{-b-2c}{\sqrt{-b^2+4ac}}\right] \left(\operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{-b-2c}{\sqrt{-b^2+4ac}}\right] + \operatorname{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right] \right)}\right] - \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{-b-2c}{\sqrt{-b^2+4ac}}\right] + \operatorname{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right]\right]\right] \right) + \right. \\
& \left. \operatorname{ArcTan}\left[\frac{-b+2c}{\sqrt{-b^2+4ac}}\right] \left(-\operatorname{Log}\left[1 - e^{2i \left(\operatorname{ArcTan}\left[\frac{-b+2c}{\sqrt{-b^2+4ac}}\right] + \operatorname{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right] \right)}\right] + \operatorname{Log}\left[\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{-b+2c}{\sqrt{-b^2+4ac}}\right] + \operatorname{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right]\right]\right] \right) \right) \right) - \\
& i (b^2-4c^2) \operatorname{PolyLog}\left[2, e^{2i \left(\operatorname{ArcTan}\left[\frac{-b-2c}{\sqrt{-b^2+4ac}}\right] + \operatorname{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right] \right)}\right] + i (b^2-4c^2) \operatorname{PolyLog}\left[2, e^{2i \left(\operatorname{ArcTan}\left[\frac{-b+2c}{\sqrt{-b^2+4ac}}\right] + \operatorname{ArcTan}\left[\frac{b+2cx}{\sqrt{-b^2+4ac}}\right] \right)}\right] \right)
\end{aligned}$$

Problem 527: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[cx]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x} dx$$

Optimal (type 4, 216 leaves, 14 steps):

$$\begin{aligned}
& a d \operatorname{Log}[x] - \frac{1}{2} b e \operatorname{Log}[cx] \operatorname{Log}[1 - cx]^2 + \frac{1}{2} b e \operatorname{Log}[-cx] \operatorname{Log}[1 + cx]^2 - \frac{1}{2} b d \operatorname{PolyLog}[2, -cx] + \\
& \frac{1}{2} b e (\operatorname{Log}[1 - cx] + \operatorname{Log}[1 + cx] - \operatorname{Log}[1 - c^2 x^2]) \operatorname{PolyLog}[2, -cx] + \frac{1}{2} b d \operatorname{PolyLog}[2, cx] - \\
& \frac{1}{2} b e (\operatorname{Log}[1 - cx] + \operatorname{Log}[1 + cx] - \operatorname{Log}[1 - c^2 x^2]) \operatorname{PolyLog}[2, cx] - \frac{1}{2} a e \operatorname{PolyLog}[2, c^2 x^2] - \\
& b e \operatorname{Log}[1 - cx] \operatorname{PolyLog}[2, 1 - cx] + b e \operatorname{Log}[1 + cx] \operatorname{PolyLog}[2, 1 + cx] + b e \operatorname{PolyLog}[3, 1 - cx] - b e \operatorname{PolyLog}[3, 1 + cx]
\end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x} dx$$

Problem 528: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x^2} dx$$

Optimal (type 4, 105 leaves, 6 steps):

$$-\frac{c e (a + b \operatorname{ArcTanh}[c x])^2}{b} - \frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x} + \frac{1}{2} b c (d + e \operatorname{Log}[1 - c^2 x^2]) \operatorname{Log}\left[1 - \frac{1}{1 - c^2 x^2}\right] - \frac{1}{2} b c e \operatorname{PolyLog}\left[2, \frac{1}{1 - c^2 x^2}\right]$$

Result (type 4, 332 leaves):

$$\begin{aligned} & -\frac{1}{4 x} \left(4 a d + 4 b d \operatorname{ArcTanh}[c x] + 8 a c e x \operatorname{ArcTanh}[c x] + 4 b c e x \operatorname{ArcTanh}[c x]^2 - 4 b c d x \operatorname{Log}[x] - b c e x \operatorname{Log}\left[-\frac{1}{c} + x\right]^2 - \right. \\ & b c e x \operatorname{Log}\left[\frac{1}{c} + x\right]^2 - 2 b c e x \operatorname{Log}\left[\frac{1}{c} + x\right] \operatorname{Log}\left[\frac{1}{2}(1 - c x)\right] + 4 b c e x \operatorname{Log}[x] \operatorname{Log}[1 - c x] - 2 b c e x \operatorname{Log}\left[-\frac{1}{c} + x\right] \operatorname{Log}\left[\frac{1}{2}(1 + c x)\right] + \\ & 4 b c e x \operatorname{Log}[x] \operatorname{Log}[1 + c x] + 4 a e \operatorname{Log}[1 - c^2 x^2] + 2 b c d x \operatorname{Log}[1 - c^2 x^2] + 4 b e \operatorname{ArcTanh}[c x] \operatorname{Log}[1 - c^2 x^2] - \\ & 4 b c e x \operatorname{Log}[x] \operatorname{Log}[1 - c^2 x^2] + 2 b c e x \operatorname{Log}\left[-\frac{1}{c} + x\right] \operatorname{Log}[1 - c^2 x^2] + 2 b c e x \operatorname{Log}\left[\frac{1}{c} + x\right] \operatorname{Log}[1 - c^2 x^2] + \\ & \left. 4 b c e x \operatorname{PolyLog}\left[2, -c x\right] + 4 b c e x \operatorname{PolyLog}\left[2, c x\right] - 2 b c e x \operatorname{PolyLog}\left[2, \frac{1}{2} - \frac{c x}{2}\right] - 2 b c e x \operatorname{PolyLog}\left[2, \frac{1}{2}(1 + c x)\right] \right) \end{aligned}$$

Problem 530: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{x^4} dx$$

Optimal (type 4, 197 leaves, 15 steps):

$$\begin{aligned} & \frac{2 c^2 e (a + b \operatorname{ArcTanh}[c x])}{3 x} - \frac{c^3 e (a + b \operatorname{ArcTanh}[c x])^2}{3 b} - b c^3 e \operatorname{Log}[x] + \frac{1}{3} b c^3 e \operatorname{Log}[1 - c^2 x^2] - \frac{b c (1 - c^2 x^2) (d + e \operatorname{Log}[1 - c^2 x^2])}{6 x^2} - \\ & \frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[1 - c^2 x^2])}{3 x^3} + \frac{1}{6} b c^3 (d + e \operatorname{Log}[1 - c^2 x^2]) \operatorname{Log}\left[1 - \frac{1}{1 - c^2 x^2}\right] - \frac{1}{6} b c^3 e \operatorname{PolyLog}\left[2, \frac{1}{1 - c^2 x^2}\right] \end{aligned}$$

Result (type 4, 460 leaves):

$$\frac{1}{6} \left(-\frac{2ad}{x^3} - \frac{bcd}{x^2} + \frac{4ac^2e}{x} - 4ac^3e \operatorname{ArcTanh}[cx] - \frac{2bd \operatorname{ArcTanh}[cx]}{x^3} + \frac{4bc^2e \operatorname{ArcTanh}[cx]}{x} - 2bc^3e \operatorname{ArcTanh}[cx]^2 + 2bc^3d \operatorname{Log}[x] - \right.$$

$$2bc^3e \operatorname{Log}[x] + \frac{1}{2}bc^3e \operatorname{Log}\left[-\frac{1}{c}+x\right]^2 + \frac{1}{2}bc^3e \operatorname{Log}\left[\frac{1}{c}+x\right]^2 + bc^3e \operatorname{Log}\left[\frac{1}{c}+x\right] \operatorname{Log}\left[\frac{1}{2}(1-cx)\right] - 2bc^3e \operatorname{Log}[x] \operatorname{Log}[1-cx] +$$

$$bc^3e \operatorname{Log}\left[-\frac{1}{c}+x\right] \operatorname{Log}\left[\frac{1}{2}(1+cx)\right] - 2bc^3e \operatorname{Log}[x] \operatorname{Log}[1+cx] - 4bc^3e \operatorname{Log}\left[\frac{cx}{\sqrt{1-c^2x^2}}\right] - bc^3d \operatorname{Log}[1-c^2x^2] + bc^3e \operatorname{Log}[1-c^2x^2] -$$

$$\frac{2ae \operatorname{Log}[1-c^2x^2]}{x^3} - \frac{bce \operatorname{Log}[1-c^2x^2]}{x^2} - \frac{2be \operatorname{ArcTanh}[cx] \operatorname{Log}[1-c^2x^2]}{x^3} + 2bc^3e \operatorname{Log}[x] \operatorname{Log}[1-c^2x^2] - bc^3e \operatorname{Log}\left[-\frac{1}{c}+x\right] \operatorname{Log}[1-c^2x^2] -$$

$$\left. bc^3e \operatorname{Log}\left[\frac{1}{c}+x\right] \operatorname{Log}[1-c^2x^2] - 2bc^3e \operatorname{PolyLog}[2, -cx] - 2bc^3e \operatorname{PolyLog}[2, cx] + bc^3e \operatorname{PolyLog}\left[2, \frac{1}{2} - \frac{cx}{2}\right] + bc^3e \operatorname{PolyLog}\left[2, \frac{1}{2}(1+cx)\right] \right)$$

Problem 532: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[cx]) (d + e \operatorname{Log}[1 - c^2x^2])}{x^6} dx$$

Optimal (type 4, 256 leaves, 24 steps):

$$\frac{7bc^3e}{60x^2} + \frac{2c^2e(a + b \operatorname{ArcTanh}[cx])}{15x^3} + \frac{2c^4e(a + b \operatorname{ArcTanh}[cx])}{5x} - \frac{c^5e(a + b \operatorname{ArcTanh}[cx])^2}{5b} -$$

$$\frac{5}{6}bc^5e \operatorname{Log}[x] + \frac{19}{60}bc^5e \operatorname{Log}[1 - c^2x^2] - \frac{bc(d + e \operatorname{Log}[1 - c^2x^2])}{20x^4} - \frac{bc^3(1 - c^2x^2)(d + e \operatorname{Log}[1 - c^2x^2])}{10x^2} -$$

$$\frac{(a + b \operatorname{ArcTanh}[cx]) (d + e \operatorname{Log}[1 - c^2x^2])}{5x^5} + \frac{1}{10}bc^5(d + e \operatorname{Log}[1 - c^2x^2]) \operatorname{Log}\left[1 - \frac{1}{1 - c^2x^2}\right] - \frac{1}{10}bc^5e \operatorname{PolyLog}\left[2, \frac{1}{1 - c^2x^2}\right]$$

Result (type 8, 29 leaves):

$$\int \frac{(a + b \operatorname{ArcTanh}[cx]) (d + e \operatorname{Log}[1 - c^2x^2])}{x^6} dx$$

Problem 533: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int x (a + b \operatorname{ArcTanh}[cx]) (d + e \operatorname{Log}[f + gx^2]) dx$$

Optimal (type 4, 512 leaves, 22 steps):

$$\begin{aligned}
& \frac{b(d-e)x}{2c} - \frac{bex}{c} + \frac{be\sqrt{f}\operatorname{ArcTan}\left[\frac{\sqrt{g}x}{\sqrt{f}}\right]}{c\sqrt{g}} - \frac{b(d-e)\operatorname{ArcTanh}[cx]}{2c^2} + \frac{1}{2}dx^2(a+b\operatorname{ArcTanh}[cx]) - \frac{1}{2}ex^2(a+b\operatorname{ArcTanh}[cx]) - \\
& \frac{be(c^2f+g)\operatorname{ArcTanh}[cx]\operatorname{Log}\left[\frac{2}{1+cx}\right]}{c^2g} + \frac{be(c^2f+g)\operatorname{ArcTanh}[cx]\operatorname{Log}\left[\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(1+cx)}\right]}{2c^2g} + \frac{be(c^2f+g)\operatorname{ArcTanh}[cx]\operatorname{Log}\left[\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})(1+cx)}\right]}{2c^2g} + \\
& \frac{bex\operatorname{Log}[f+gx^2]}{2c} - \frac{be(c^2f+g)\operatorname{ArcTanh}[cx]\operatorname{Log}[f+gx^2]}{2c^2g} + \frac{e(f+gx^2)(a+b\operatorname{ArcTanh}[cx])\operatorname{Log}[f+gx^2]}{2g} + \\
& \frac{be(c^2f+g)\operatorname{PolyLog}\left[2, 1-\frac{2}{1+cx}\right]}{2c^2g} - \frac{be(c^2f+g)\operatorname{PolyLog}\left[2, 1-\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(1+cx)}\right]}{4c^2g} - \frac{be(c^2f+g)\operatorname{PolyLog}\left[2, 1-\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+\sqrt{g})(1+cx)}\right]}{4c^2g}
\end{aligned}$$

Result (type 4, 1145 leaves):

$$\begin{aligned}
& \frac{1}{4c^2g} \left(2bcdgx - 6bcegx + 2ac^2dgx^2 - 2ac^2egx^2 + 4bce\sqrt{f}\sqrt{g}\operatorname{ArcTan}\left[\frac{\sqrt{g}x}{\sqrt{f}}\right] - 2bdg\operatorname{ArcTanh}[cx] + \right. \\
& 2beg\operatorname{ArcTanh}[cx] + 2bc^2dgx^2\operatorname{ArcTanh}[cx] - 2bc^2egx^2\operatorname{ArcTanh}[cx] - 4ibc^2ef\operatorname{ArcSin}\left[\sqrt{\frac{c^2f}{c^2f+g}}\right]\operatorname{ArcTanh}\left[\frac{c gx}{\sqrt{-c^2fg}}\right] - \\
& 4ibeg\operatorname{ArcSin}\left[\sqrt{\frac{c^2f}{c^2f+g}}\right]\operatorname{ArcTanh}\left[\frac{c gx}{\sqrt{-c^2fg}}\right] - 4bc^2ef\operatorname{ArcTanh}[cx]\operatorname{Log}\left[1+e^{-2\operatorname{ArcTanh}[cx]}\right] - 4beg\operatorname{ArcTanh}[cx]\operatorname{Log}\left[1+e^{-2\operatorname{ArcTanh}[cx]}\right] - \\
& 2ibc^2ef\operatorname{ArcSin}\left[\sqrt{\frac{c^2f}{c^2f+g}}\right]\operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[cx]}\left(c^2(1+e^{2\operatorname{ArcTanh}[cx]})f+(-1+e^{2\operatorname{ArcTanh}[cx]})g-2\sqrt{-c^2fg}\right)}{c^2f+g}\right] - \\
& 2ibeg\operatorname{ArcSin}\left[\sqrt{\frac{c^2f}{c^2f+g}}\right]\operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[cx]}\left(c^2(1+e^{2\operatorname{ArcTanh}[cx]})f+(-1+e^{2\operatorname{ArcTanh}[cx]})g-2\sqrt{-c^2fg}\right)}{c^2f+g}\right] + \\
& 2bc^2ef\operatorname{ArcTanh}[cx]\operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[cx]}\left(c^2(1+e^{2\operatorname{ArcTanh}[cx]})f+(-1+e^{2\operatorname{ArcTanh}[cx]})g-2\sqrt{-c^2fg}\right)}{c^2f+g}\right] + \\
& 2beg\operatorname{ArcTanh}[cx]\operatorname{Log}\left[\frac{e^{-2\operatorname{ArcTanh}[cx]}\left(c^2(1+e^{2\operatorname{ArcTanh}[cx]})f+(-1+e^{2\operatorname{ArcTanh}[cx]})g-2\sqrt{-c^2fg}\right)}{c^2f+g}\right] +
\end{aligned}$$

$$\begin{aligned}
& 2 i b c^2 e f \operatorname{ArcSin}\left[\sqrt{\frac{c^2 f}{c^2 f + g}}\right] \operatorname{Log}\left[\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(c^2\left(1+e^{2 \operatorname{ArcTanh}[c x]}\right) f+\left(-1+e^{2 \operatorname{ArcTanh}[c x]}\right) g+2 \sqrt{-c^2 f g}\right)}{c^2 f + g}\right] + \\
& 2 i b e g \operatorname{ArcSin}\left[\sqrt{\frac{c^2 f}{c^2 f + g}}\right] \operatorname{Log}\left[\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(c^2\left(1+e^{2 \operatorname{ArcTanh}[c x]}\right) f+\left(-1+e^{2 \operatorname{ArcTanh}[c x]}\right) g+2 \sqrt{-c^2 f g}\right)}{c^2 f + g}\right] + \\
& 2 b c^2 e f \operatorname{ArcTanh}[c x] \operatorname{Log}\left[\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(c^2\left(1+e^{2 \operatorname{ArcTanh}[c x]}\right) f+\left(-1+e^{2 \operatorname{ArcTanh}[c x]}\right) g+2 \sqrt{-c^2 f g}\right)}{c^2 f + g}\right] + \\
& 2 b e g \operatorname{ArcTanh}[c x] \operatorname{Log}\left[\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(c^2\left(1+e^{2 \operatorname{ArcTanh}[c x]}\right) f+\left(-1+e^{2 \operatorname{ArcTanh}[c x]}\right) g+2 \sqrt{-c^2 f g}\right)}{c^2 f + g}\right] + 2 a c^2 e f \operatorname{Log}\left[f+g x^2\right] + \\
& 2 b c e g x \operatorname{Log}\left[f+g x^2\right] + 2 a c^2 e g x^2 \operatorname{Log}\left[f+g x^2\right] - 2 b e g \operatorname{ArcTanh}[c x] \operatorname{Log}\left[f+g x^2\right] + 2 b c^2 e g x^2 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[f+g x^2\right] + \\
& 2 b e\left(c^2 f+g\right) \operatorname{PolyLog}\left[2,-e^{-2 \operatorname{ArcTanh}[c x]}\right]-b e\left(c^2 f+g\right) \operatorname{PolyLog}\left[2,\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(-c^2 f+g-2 \sqrt{-c^2 f g}\right)}{c^2 f+g}\right]- \\
& \left. b c^2 e f \operatorname{PolyLog}\left[2,\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(-c^2 f+g+2 \sqrt{-c^2 f g}\right)}{c^2 f+g}\right]-b e g \operatorname{PolyLog}\left[2,\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(-c^2 f+g+2 \sqrt{-c^2 f g}\right)}{c^2 f+g}\right]\right)
\end{aligned}$$

Problem 534: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[f + g x^2]) dx$$

Optimal (type 4, 599 leaves, 28 steps):

$$\begin{aligned}
& -2 a e x + \frac{2 a e \sqrt{f} \operatorname{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right]}{\sqrt{g}} - 2 b e x \operatorname{ArcTanh}[c x] + \frac{b e \sqrt{-f} \operatorname{Log}[1-c x] \operatorname{Log}\left[\frac{c(\sqrt{-f}-\sqrt{g} x)}{c \sqrt{-f}-\sqrt{g}}\right]}{2 \sqrt{g}} - \\
& \frac{b e \sqrt{-f} \operatorname{Log}[1+c x] \operatorname{Log}\left[\frac{c(\sqrt{-f}-\sqrt{g} x)}{c \sqrt{-f}+\sqrt{g}}\right]}{2 \sqrt{g}} + \frac{b e \sqrt{-f} \operatorname{Log}[1+c x] \operatorname{Log}\left[\frac{c(\sqrt{-f}+\sqrt{g} x)}{c \sqrt{-f}-\sqrt{g}}\right]}{2 \sqrt{g}} - \frac{b e \sqrt{-f} \operatorname{Log}[1-c x] \operatorname{Log}\left[\frac{c(\sqrt{-f}+\sqrt{g} x)}{c \sqrt{-f}+\sqrt{g}}\right]}{2 \sqrt{g}} - \\
& \frac{b e \operatorname{Log}\left[1-c^2 x^2\right]}{c} + x(a+b \operatorname{ArcTanh}[c x]) (d+e \operatorname{Log}[f+g x^2]) + \frac{b \operatorname{Log}\left[\frac{g(1-c^2 x^2)}{c^2 f+g}\right] (d+e \operatorname{Log}[f+g x^2])}{2 c} + \frac{b e \sqrt{-f} \operatorname{PolyLog}\left[2, -\frac{\sqrt{g}(1-c x)}{c \sqrt{-f}-\sqrt{g}}\right]}{2 \sqrt{g}} - \\
& \frac{b e \sqrt{-f} \operatorname{PolyLog}\left[2, \frac{\sqrt{g}(1-c x)}{c \sqrt{-f}+\sqrt{g}}\right]}{2 \sqrt{g}} + \frac{b e \sqrt{-f} \operatorname{PolyLog}\left[2, -\frac{\sqrt{g}(1+c x)}{c \sqrt{-f}-\sqrt{g}}\right]}{2 \sqrt{g}} - \frac{b e \sqrt{-f} \operatorname{PolyLog}\left[2, \frac{\sqrt{g}(1+c x)}{c \sqrt{-f}+\sqrt{g}}\right]}{2 \sqrt{g}} + \frac{b e \operatorname{PolyLog}\left[2, \frac{c^2(f+g x^2)}{c^2 f+g}\right]}{2 c}
\end{aligned}$$

Result (type 4, 1251 leaves):

$$\begin{aligned}
& a d x - 2 a e x + \frac{2 a e \sqrt{f} \operatorname{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right]}{\sqrt{g}} + b d x \operatorname{ArcTanh}[c x] + \\
& \frac{b d \operatorname{Log}\left[1-c^2 x^2\right]}{2 c} + a e x \operatorname{Log}[f+g x^2] + b e \left(x \operatorname{ArcTanh}[c x] + \frac{\operatorname{Log}\left[1-c^2 x^2\right]}{2 c}\right) \operatorname{Log}[f+g x^2] - \frac{1}{c} \\
& b e g \left(\frac{\left(-\operatorname{Log}\left[-\frac{1}{c}+x\right] - \operatorname{Log}\left[\frac{1}{c}+x\right] + \operatorname{Log}\left[1-c^2 x^2\right]\right) \operatorname{Log}[f+g x^2]}{2 g} + \frac{\operatorname{Log}\left[-\frac{1}{c}+x\right] \operatorname{Log}\left[1-\frac{\sqrt{g}\left(-\frac{1}{c}+x\right)}{-i \sqrt{f}-\frac{\sqrt{g}}{c}}\right] + \operatorname{PolyLog}\left[2, \frac{\sqrt{g}\left(-\frac{1}{c}+x\right)}{-i \sqrt{f}-\frac{\sqrt{g}}{c}}\right]}{2 g} + \right. \\
& \frac{\operatorname{Log}\left[-\frac{1}{c}+x\right] \operatorname{Log}\left[1-\frac{\sqrt{g}\left(-\frac{1}{c}+x\right)}{i \sqrt{f}-\frac{\sqrt{g}}{c}}\right] + \operatorname{PolyLog}\left[2, \frac{\sqrt{g}\left(-\frac{1}{c}+x\right)}{i \sqrt{f}-\frac{\sqrt{g}}{c}}\right]}{2 g} + \frac{\operatorname{Log}\left[\frac{1}{c}+x\right] \operatorname{Log}\left[1-\frac{\sqrt{g}\left(\frac{1}{c}+x\right)}{-i \sqrt{f}+\frac{\sqrt{g}}{c}}\right] + \operatorname{PolyLog}\left[2, \frac{\sqrt{g}\left(\frac{1}{c}+x\right)}{-i \sqrt{f}+\frac{\sqrt{g}}{c}}\right]}{2 g} + \\
& \left. \frac{\operatorname{Log}\left[\frac{1}{c}+x\right] \operatorname{Log}\left[1-\frac{\sqrt{g}\left(\frac{1}{c}+x\right)}{i \sqrt{f}+\frac{\sqrt{g}}{c}}\right] + \operatorname{PolyLog}\left[2, \frac{\sqrt{g}\left(\frac{1}{c}+x\right)}{i \sqrt{f}+\frac{\sqrt{g}}{c}}\right]}{2 g} \right) - \frac{1}{2 c} b e \left(4 c x \operatorname{ArcTanh}[c x] - 4 \operatorname{Log}\left[\frac{1}{\sqrt{1-c^2 x^2}}\right] + \right. \\
& \left. \frac{1}{g} \sqrt{c^2 f g} \left(-2 i \operatorname{ArcCos}\left[\frac{-c^2 f+g}{c^2 f+g}\right] \operatorname{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right] + 4 \operatorname{ArcTan}\left[\frac{\sqrt{c^2 f g}}{c g x}\right] \operatorname{ArcTanh}[c x] - \left(\operatorname{ArcCos}\left[\frac{-c^2 f+g}{c^2 f+g}\right] - 2 \operatorname{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right]\right)\right)
\end{aligned}$$

$$\begin{aligned} & \text{Log}\left[\frac{2c^2f(g+i\sqrt{c^2fg})(1+cx)}{(c^2f+g)(c^2f+i c\sqrt{c^2fg}x)}\right] - \left(\text{ArcCos}\left[\frac{-c^2f+g}{c^2f+g}\right] + 2\text{ArcTan}\left[\frac{c gx}{\sqrt{c^2fg}}\right]\right) \text{Log}\left[\frac{2c^2f(i g+\sqrt{c^2fg})(-1+cx)}{(c^2f+g)(-i c^2f+c\sqrt{c^2fg}x)}\right] + \\ & \left(\text{ArcCos}\left[\frac{-c^2f+g}{c^2f+g}\right] + 2\left(\text{ArcTan}\left[\frac{\sqrt{c^2fg}}{c gx}\right] + \text{ArcTan}\left[\frac{c gx}{\sqrt{c^2fg}}\right]\right)\right) \text{Log}\left[\frac{\sqrt{2}e^{-\text{ArcTanh}[cx]}\sqrt{c^2fg}}{\sqrt{c^2f+g}\sqrt{c^2f-g+(c^2f+g)\text{Cosh}[2\text{ArcTanh}[cx]]}}\right] + \\ & \left(\text{ArcCos}\left[\frac{-c^2f+g}{c^2f+g}\right] - 2\left(\text{ArcTan}\left[\frac{\sqrt{c^2fg}}{c gx}\right] + \text{ArcTan}\left[\frac{c gx}{\sqrt{c^2fg}}\right]\right)\right) \text{Log}\left[\frac{\sqrt{2}e^{\text{ArcTanh}[cx]}\sqrt{c^2fg}}{\sqrt{c^2f+g}\sqrt{c^2f-g+(c^2f+g)\text{Cosh}[2\text{ArcTanh}[cx]]}}\right] + \\ & i\left(-\text{PolyLog}\left[2,\frac{(-c^2f+g-2i\sqrt{c^2fg})(i c^2f+c\sqrt{c^2fg}x)}{(c^2f+g)(-i c^2f+c\sqrt{c^2fg}x)}\right] + \text{PolyLog}\left[2,\frac{(-c^2f+g+2i\sqrt{c^2fg})(i c^2f+c\sqrt{c^2fg}x)}{(c^2f+g)(-i c^2f+c\sqrt{c^2fg}x)}\right]\right) \end{aligned}$$

Problem 536: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+b\text{ArcTanh}[cx])(d+e\text{Log}[f+gx^2])}{x^2} dx$$

Optimal (type 4, 613 leaves, 28 steps):

$$\begin{aligned} & \frac{2ae\sqrt{g}\text{ArcTan}\left[\frac{\sqrt{g}x}{\sqrt{f}}\right]}{\sqrt{f}} - \frac{be\sqrt{g}\text{Log}[1-cx]\text{Log}\left[\frac{c(\sqrt{-f}-\sqrt{g}x)}{c\sqrt{-f}-\sqrt{g}}\right]}{2\sqrt{-f}} + \frac{be\sqrt{g}\text{Log}[1+cx]\text{Log}\left[\frac{c(\sqrt{-f}+\sqrt{g}x)}{c\sqrt{-f}+\sqrt{g}}\right]}{2\sqrt{-f}} - \frac{be\sqrt{g}\text{Log}[1+cx]\text{Log}\left[\frac{c(\sqrt{-f}+\sqrt{g}x)}{c\sqrt{-f}-\sqrt{g}}\right]}{2\sqrt{-f}} + \\ & \frac{be\sqrt{g}\text{Log}[1-cx]\text{Log}\left[\frac{c(\sqrt{-f}+\sqrt{g}x)}{c\sqrt{-f}+\sqrt{g}}\right]}{2\sqrt{-f}} - \frac{(a+b\text{ArcTanh}[cx])(d+e\text{Log}[f+gx^2])}{x} + \frac{1}{2}bc\text{Log}\left[-\frac{gx^2}{f}\right](d+e\text{Log}[f+gx^2]) - \\ & \frac{1}{2}bc\text{Log}\left[\frac{g(1-c^2x^2)}{c^2f+g}\right](d+e\text{Log}[f+gx^2]) - \frac{be\sqrt{g}\text{PolyLog}\left[2,-\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}-\sqrt{g}}\right]}{2\sqrt{-f}} + \frac{be\sqrt{g}\text{PolyLog}\left[2,\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}+\sqrt{g}}\right]}{2\sqrt{-f}} - \\ & \frac{be\sqrt{g}\text{PolyLog}\left[2,-\frac{\sqrt{g}(1+cx)}{c\sqrt{-f}-\sqrt{g}}\right]}{2\sqrt{-f}} + \frac{be\sqrt{g}\text{PolyLog}\left[2,\frac{\sqrt{g}(1+cx)}{c\sqrt{-f}+\sqrt{g}}\right]}{2\sqrt{-f}} - \frac{1}{2}bce\text{PolyLog}\left[2,\frac{c^2(f+gx^2)}{c^2f+g}\right] + \frac{1}{2}bce\text{PolyLog}\left[2,1+\frac{gx^2}{f}\right] \end{aligned}$$

Result (type 4, 1226 leaves):

$$\begin{aligned}
& -\frac{a d}{x} - \frac{b d \operatorname{ArcTanh}[c x]}{x} + b c d \operatorname{Log}[x] - \frac{1}{2} b c d \operatorname{Log}[1 - c^2 x^2] + \\
& a e \left(\frac{2 \sqrt{g} \operatorname{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right] - \frac{\operatorname{Log}[f + g x^2]}{x}}{\sqrt{f}} \right) + \frac{1}{2} b e \left(-\frac{(2 \operatorname{ArcTanh}[c x] + c x (-2 \operatorname{Log}[x] + \operatorname{Log}[1 - c^2 x^2])) \operatorname{Log}[f + g x^2]}{x} - \right. \\
& 2 c \left(\operatorname{Log}[x] \left(\operatorname{Log}\left[1 - \frac{i \sqrt{g} x}{\sqrt{f}}\right] + \operatorname{Log}\left[1 + \frac{i \sqrt{g} x}{\sqrt{f}}\right] \right) + \operatorname{PolyLog}\left[2, -\frac{i \sqrt{g} x}{\sqrt{f}}\right] + \operatorname{PolyLog}\left[2, \frac{i \sqrt{g} x}{\sqrt{f}}\right] \right) + \\
& c \left(\operatorname{Log}\left[-\frac{1}{c} + x\right] \operatorname{Log}\left[\frac{c(\sqrt{f} - i \sqrt{g} x)}{c \sqrt{f} - i \sqrt{g}}\right] + \operatorname{Log}\left[\frac{1}{c} + x\right] \operatorname{Log}\left[\frac{c(\sqrt{f} + i \sqrt{g} x)}{c \sqrt{f} + i \sqrt{g}}\right] + \operatorname{Log}\left[-\frac{1}{c} + x\right] \operatorname{Log}\left[\frac{c(\sqrt{f} + i \sqrt{g} x)}{c \sqrt{f} + i \sqrt{g}}\right] - \right. \\
& \left. \left(\operatorname{Log}\left[-\frac{1}{c} + x\right] + \operatorname{Log}\left[\frac{1}{c} + x\right] - \operatorname{Log}[1 - c^2 x^2] \right) \operatorname{Log}[f + g x^2] + \operatorname{Log}\left[\frac{1}{c} + x\right] \operatorname{Log}\left[1 - \frac{\sqrt{g}(1 + c x)}{i c \sqrt{f} + \sqrt{g}}\right] + \operatorname{PolyLog}\left[2, \frac{c \sqrt{g} \left(\frac{1}{c} + x\right)}{i c \sqrt{f} + \sqrt{g}}\right] + \right. \\
& \left. \operatorname{PolyLog}\left[2, \frac{i \sqrt{g}(-1 + c x)}{c \sqrt{f} - i \sqrt{g}}\right] + \operatorname{PolyLog}\left[2, -\frac{i \sqrt{g}(-1 + c x)}{c \sqrt{f} + i \sqrt{g}}\right] + \operatorname{PolyLog}\left[2, \frac{i \sqrt{g}(1 + c x)}{c \sqrt{f} + i \sqrt{g}}\right] \right) + \\
& \frac{1}{\sqrt{c^2 f g}} c g \left(2 i \operatorname{ArcCos}\left[\frac{-c^2 f + g}{c^2 f + g}\right] \operatorname{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right] - 4 \operatorname{ArcTan}\left[\frac{c f}{\sqrt{c^2 f g} x}\right] \operatorname{ArcTanh}[c x] + \left(\operatorname{ArcCos}\left[\frac{-c^2 f + g}{c^2 f + g}\right] + 2 \operatorname{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right] \right) \right. \\
& \left. \operatorname{Log}\left[\frac{2 i c f (i g + \sqrt{c^2 f g}) (-1 + c x)}{(c^2 f + g) (c f + i \sqrt{c^2 f g} x)}\right] + \left(\operatorname{ArcCos}\left[\frac{-c^2 f + g}{c^2 f + g}\right] - 2 \operatorname{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right] \right) \operatorname{Log}\left[\frac{2 c f (g + i \sqrt{c^2 f g}) (1 + c x)}{(c^2 f + g) (c f + i \sqrt{c^2 f g} x)}\right] - \right. \\
& \left. \left(\operatorname{ArcCos}\left[\frac{-c^2 f + g}{c^2 f + g}\right] + 2 \left(\operatorname{ArcTan}\left[\frac{c f}{\sqrt{c^2 f g} x}\right] + \operatorname{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} e^{-\operatorname{ArcTanh}[c x]} \sqrt{c^2 f g}}{\sqrt{c^2 f + g} \sqrt{c^2 f - g + (c^2 f + g) \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]]}}\right] - \right. \\
& \left. \left(\operatorname{ArcCos}\left[\frac{-c^2 f + g}{c^2 f + g}\right] - 2 \left(\operatorname{ArcTan}\left[\frac{c f}{\sqrt{c^2 f g} x}\right] + \operatorname{ArcTan}\left[\frac{c g x}{\sqrt{c^2 f g}}\right] \right) \right) \operatorname{Log}\left[\frac{\sqrt{2} e^{\operatorname{ArcTanh}[c x]} \sqrt{c^2 f g}}{\sqrt{c^2 f + g} \sqrt{c^2 f - g + (c^2 f + g) \operatorname{Cosh}[2 \operatorname{ArcTanh}[c x]]}}\right] + \right. \\
& \left. i \left(\operatorname{PolyLog}\left[2, \frac{(-c^2 f + g - 2 i \sqrt{c^2 f g}) (i c f + \sqrt{c^2 f g} x)}{(c^2 f + g) (-i c f + \sqrt{c^2 f g} x)}\right] - \operatorname{PolyLog}\left[2, \frac{(-c^2 f + g + 2 i \sqrt{c^2 f g}) (i c f + \sqrt{c^2 f g} x)}{(c^2 f + g) (-i c f + \sqrt{c^2 f g} x)}\right] \right) \right)
\end{aligned}$$

Problem 537: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[f + g x^2])}{x^3} dx$$

Optimal (type 4, 470 leaves, 20 steps):

$$\begin{aligned} & \frac{b c e \sqrt{g} \operatorname{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right]}{\sqrt{f}} + \frac{a e g \operatorname{Log}[x]}{f} + \frac{b e (c^2 f + g) \operatorname{ArcTanh}[c x] \operatorname{Log}\left[\frac{2}{1+c x}\right]}{f} - \frac{b e (c^2 f + g) \operatorname{ArcTanh}[c x] \operatorname{Log}\left[\frac{2 c (\sqrt{-f} - \sqrt{g} x)}{(c \sqrt{-f} - \sqrt{g}) (1+c x)}\right]}{2 f} \\ & \frac{b e (c^2 f + g) \operatorname{ArcTanh}[c x] \operatorname{Log}\left[\frac{2 c (\sqrt{-f} + \sqrt{g} x)}{(c \sqrt{-f} + \sqrt{g}) (1+c x)}\right]}{2 f} - \frac{a e g \operatorname{Log}[f + g x^2]}{2 f} - \frac{b c (d + e \operatorname{Log}[f + g x^2])}{2 x} + \\ & \frac{1}{2} b c^2 \operatorname{ArcTanh}[c x] (d + e \operatorname{Log}[f + g x^2]) - \frac{(a + b \operatorname{ArcTanh}[c x]) (d + e \operatorname{Log}[f + g x^2])}{2 x^2} - \frac{b e g \operatorname{PolyLog}[2, -c x]}{2 f} + \frac{b e g \operatorname{PolyLog}[2, c x]}{2 f} \\ & \frac{b e (c^2 f + g) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c x}\right]}{2 f} + \frac{b e (c^2 f + g) \operatorname{PolyLog}\left[2, 1 - \frac{2 c (\sqrt{-f} - \sqrt{g} x)}{(c \sqrt{-f} - \sqrt{g}) (1+c x)}\right]}{4 f} + \frac{b e (c^2 f + g) \operatorname{PolyLog}\left[2, 1 - \frac{2 c (\sqrt{-f} + \sqrt{g} x)}{(c \sqrt{-f} + \sqrt{g}) (1+c x)}\right]}{4 f} \end{aligned}$$

Result (type 4, 1211 leaves):

$$\begin{aligned} & \frac{1}{4 f x^2} \left(-2 a d f - 2 b c d f x + 4 b c e \sqrt{f} \sqrt{g} x^2 \operatorname{ArcTan}\left[\frac{\sqrt{g} x}{\sqrt{f}}\right] - 2 b d f \operatorname{ArcTanh}[c x] + 2 b c^2 d f x^2 \operatorname{ArcTanh}[c x] + \right. \\ & 4 i b c^2 e f x^2 \operatorname{ArcSin}\left[\sqrt{\frac{c^2 f}{c^2 f + g}}\right] \operatorname{ArcTanh}\left[\frac{c g x}{\sqrt{-c^2 f g}}\right] + 4 i b e g x^2 \operatorname{ArcSin}\left[\sqrt{\frac{c^2 f}{c^2 f + g}}\right] \operatorname{ArcTanh}\left[\frac{c g x}{\sqrt{-c^2 f g}}\right] + \\ & 4 b e g x^2 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c x]}\right] + 4 b c^2 e f x^2 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c x]}\right] + \\ & 2 i b c^2 e f x^2 \operatorname{ArcSin}\left[\sqrt{\frac{c^2 f}{c^2 f + g}}\right] \operatorname{Log}\left[\frac{e^{-2 \operatorname{ArcTanh}[c x]} \left(c^2 (1 + e^{2 \operatorname{ArcTanh}[c x]}) f + (-1 + e^{2 \operatorname{ArcTanh}[c x]}) g - 2 \sqrt{-c^2 f g}\right)}{c^2 f + g}\right] + \\ & 2 i b e g x^2 \operatorname{ArcSin}\left[\sqrt{\frac{c^2 f}{c^2 f + g}}\right] \operatorname{Log}\left[\frac{e^{-2 \operatorname{ArcTanh}[c x]} \left(c^2 (1 + e^{2 \operatorname{ArcTanh}[c x]}) f + (-1 + e^{2 \operatorname{ArcTanh}[c x]}) g - 2 \sqrt{-c^2 f g}\right)}{c^2 f + g}\right] - \\ & \left. 2 b c^2 e f x^2 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[\frac{e^{-2 \operatorname{ArcTanh}[c x]} \left(c^2 (1 + e^{2 \operatorname{ArcTanh}[c x]}) f + (-1 + e^{2 \operatorname{ArcTanh}[c x]}) g - 2 \sqrt{-c^2 f g}\right)}{c^2 f + g}\right] - \right) \end{aligned}$$

$$\begin{aligned}
& 2 b e g x^2 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(c^2\left(1+e^{2 \operatorname{ArcTanh}[c x]}\right) f+\left(-1+e^{2 \operatorname{ArcTanh}[c x]}\right) g-2 \sqrt{-c^2 f g}\right)}{c^2 f+g}\right]- \\
& 2 i b c^2 e f x^2 \operatorname{ArcSin}\left[\sqrt{\frac{c^2 f}{c^2 f+g}}\right] \operatorname{Log}\left[\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(c^2\left(1+e^{2 \operatorname{ArcTanh}[c x]}\right) f+\left(-1+e^{2 \operatorname{ArcTanh}[c x]}\right) g+2 \sqrt{-c^2 f g}\right)}{c^2 f+g}\right]- \\
& 2 i b e g x^2 \operatorname{ArcSin}\left[\sqrt{\frac{c^2 f}{c^2 f+g}}\right] \operatorname{Log}\left[\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(c^2\left(1+e^{2 \operatorname{ArcTanh}[c x]}\right) f+\left(-1+e^{2 \operatorname{ArcTanh}[c x]}\right) g+2 \sqrt{-c^2 f g}\right)}{c^2 f+g}\right]- \\
& 2 b c^2 e f x^2 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(c^2\left(1+e^{2 \operatorname{ArcTanh}[c x]}\right) f+\left(-1+e^{2 \operatorname{ArcTanh}[c x]}\right) g+2 \sqrt{-c^2 f g}\right)}{c^2 f+g}\right]- \\
& 2 b e g x^2 \operatorname{ArcTanh}[c x] \operatorname{Log}\left[\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(c^2\left(1+e^{2 \operatorname{ArcTanh}[c x]}\right) f+\left(-1+e^{2 \operatorname{ArcTanh}[c x]}\right) g+2 \sqrt{-c^2 f g}\right)}{c^2 f+g}\right]+4 a e g x^2 \operatorname{Log}[x]- \\
& 2 a e f \operatorname{Log}[f+g x^2]-2 b c e f x \operatorname{Log}[f+g x^2]-2 a e g x^2 \operatorname{Log}[f+g x^2]-2 b e f \operatorname{ArcTanh}[c x] \operatorname{Log}[f+g x^2]+ \\
& 2 b c^2 e f x^2 \operatorname{ArcTanh}[c x] \operatorname{Log}[f+g x^2]-2 b c^2 e f x^2 \operatorname{PolyLog}\left[2,-e^{-2 \operatorname{ArcTanh}[c x]}\right]-2 b e g x^2 \operatorname{PolyLog}\left[2,e^{-2 \operatorname{ArcTanh}[c x]}\right]+ \\
& b c^2 e f x^2 \operatorname{PolyLog}\left[2,\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(-c^2 f+g-2 \sqrt{-c^2 f g}\right)}{c^2 f+g}\right]+b e g x^2 \operatorname{PolyLog}\left[2,\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(-c^2 f+g-2 \sqrt{-c^2 f g}\right)}{c^2 f+g}\right]+ \\
& b c^2 e f x^2 \operatorname{PolyLog}\left[2,\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(-c^2 f+g+2 \sqrt{-c^2 f g}\right)}{c^2 f+g}\right]+b e g x^2 \operatorname{PolyLog}\left[2,\frac{e^{-2 \operatorname{ArcTanh}[c x]}\left(-c^2 f+g+2 \sqrt{-c^2 f g}\right)}{c^2 f+g}\right]
\end{aligned}$$

Test results for the 62 problems in "7.3.5 u (a+b arctanh(c+d x))^p.m"

Problem 2: Result more than twice size of optimal antiderivative.

$$\int x^2 \operatorname{ArcTanh}[a+b x]^2 dx$$

Optimal (type 4, 204 leaves, 15 steps):

$$\frac{x}{3b^2} - \frac{\text{ArcTanh}[a+bx]}{3b^3} - \frac{2a(a+bx)\text{ArcTanh}[a+bx]}{b^3} + \frac{(a+bx)^2\text{ArcTanh}[a+bx]}{3b^3} + \frac{a(3+a^2)\text{ArcTanh}[a+bx]^2}{3b^3} + \frac{(1+3a^2)\text{ArcTanh}[a+bx]^2}{3b^3} +$$

$$\frac{1}{3}x^3\text{ArcTanh}[a+bx]^2 - \frac{2(1+3a^2)\text{ArcTanh}[a+bx]\text{Log}\left[\frac{2}{1-a-bx}\right]}{3b^3} - \frac{a\text{Log}[1-(a+bx)^2]}{b^3} - \frac{(1+3a^2)\text{PolyLog}\left[2, -\frac{1+a+bx}{1-a-bx}\right]}{3b^3}$$

Result (type 4, 463 leaves):

$$-\frac{1}{12b^3} \left(1 - (a+bx)^2\right)^{3/2} \left(-\frac{a+bx}{\sqrt{1-(a+bx)^2}} + \frac{6a(a+bx)\text{ArcTanh}[a+bx]}{\sqrt{1-(a+bx)^2}} + \right.$$

$$\frac{3(a+bx)\text{ArcTanh}[a+bx]^2}{\sqrt{1-(a+bx)^2}} - \frac{3a^2(a+bx)\text{ArcTanh}[a+bx]^2}{\sqrt{1-(a+bx)^2}} + \text{ArcTanh}[a+bx]^2 \text{Cosh}[3\text{ArcTanh}[a+bx]] +$$

$$3a^2\text{ArcTanh}[a+bx]^2 \text{Cosh}[3\text{ArcTanh}[a+bx]] + 2\text{ArcTanh}[a+bx] \text{Cosh}[3\text{ArcTanh}[a+bx]] \text{Log}\left[1 + e^{-2\text{ArcTanh}[a+bx]}\right] +$$

$$6a^2\text{ArcTanh}[a+bx] \text{Cosh}[3\text{ArcTanh}[a+bx]] \text{Log}\left[1 + e^{-2\text{ArcTanh}[a+bx]}\right] - 6a \text{Cosh}[3\text{ArcTanh}[a+bx]] \text{Log}\left[\frac{1}{\sqrt{1-(a+bx)^2}}\right] + \frac{1}{\sqrt{1-(a+bx)^2}}$$

$$\left. \left(3(1-4a+3a^2)\text{ArcTanh}[a+bx]^2 + 2\text{ArcTanh}[a+bx] \left(2 + (3+9a^2)\text{Log}\left[1 + e^{-2\text{ArcTanh}[a+bx]}\right]\right) - 18a \text{Log}\left[\frac{1}{\sqrt{1-(a+bx)^2}}\right] \right) - \right.$$

$$\frac{4(1+3a^2)\text{PolyLog}\left[2, -e^{-2\text{ArcTanh}[a+bx]}\right]}{\left(1 - (a+bx)^2\right)^{3/2}} - \text{Sinh}[3\text{ArcTanh}[a+bx]] + 6a\text{ArcTanh}[a+bx] \text{Sinh}[3\text{ArcTanh}[a+bx]] -$$

$$\left. \left. \text{ArcTanh}[a+bx]^2 \text{Sinh}[3\text{ArcTanh}[a+bx]] - 3a^2\text{ArcTanh}[a+bx]^2 \text{Sinh}[3\text{ArcTanh}[a+bx]] \right) \right)$$

Problem 5: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTanh}[a+bx]^2}{x} dx$$

Optimal (type 4, 148 leaves, 2 steps):

$$-\text{ArcTanh}[a+bx]^2 \text{Log}\left[\frac{2}{1+a+bx}\right] + \text{ArcTanh}[a+bx]^2 \text{Log}\left[\frac{2bx}{(1-a)(1+a+bx)}\right] + \text{ArcTanh}[a+bx] \text{PolyLog}\left[2, 1 - \frac{2}{1+a+bx}\right] -$$

$$\text{ArcTanh}[a+bx] \text{PolyLog}\left[2, 1 - \frac{2bx}{(1-a)(1+a+bx)}\right] + \frac{1}{2} \text{PolyLog}\left[3, 1 - \frac{2}{1+a+bx}\right] - \frac{1}{2} \text{PolyLog}\left[3, 1 - \frac{2bx}{(1-a)(1+a+bx)}\right]$$

Result (type 4, 634 leaves):

$$\begin{aligned}
& -\frac{4}{3} \operatorname{ArcTanh}[a+bx]^3 - \frac{2 \operatorname{ArcTanh}[a+bx]^3}{3a} + \frac{2\sqrt{1-a^2} e^{\operatorname{ArcTanh}[a]} \operatorname{ArcTanh}[a+bx]^3}{3a} - \operatorname{ArcTanh}[a+bx]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[a+bx]}\right] - \\
& i \pi \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[\frac{1}{2} \left(e^{-\operatorname{ArcTanh}[a+bx]} + e^{\operatorname{ArcTanh}[a+bx]}\right)\right] + \operatorname{ArcTanh}[a+bx]^2 \operatorname{Log}\left[\frac{1}{2} e^{-\operatorname{ArcTanh}[a+bx]} \left(1+a - e^{2 \operatorname{ArcTanh}[a+bx]} + a e^{2 \operatorname{ArcTanh}[a+bx]}\right)\right] - \\
& \operatorname{ArcTanh}[a+bx]^2 \operatorname{Log}\left[1 + \frac{(-1+a) e^{2 \operatorname{ArcTanh}[a+bx]}}{1+a}\right] + \operatorname{ArcTanh}[a+bx]^2 \operatorname{Log}\left[1 - e^{-\operatorname{ArcTanh}[a] + \operatorname{ArcTanh}[a+bx]}\right] + \\
& \operatorname{ArcTanh}[a+bx]^2 \operatorname{Log}\left[1 + e^{-\operatorname{ArcTanh}[a] + \operatorname{ArcTanh}[a+bx]}\right] - 2 \operatorname{ArcTanh}[a] \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[\frac{1}{2} i \left(-e^{\operatorname{ArcTanh}[a] - \operatorname{ArcTanh}[a+bx]} + e^{-\operatorname{ArcTanh}[a] + \operatorname{ArcTanh}[a+bx]}\right)\right] + \\
& \operatorname{ArcTanh}[a+bx]^2 \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[a] + 2 \operatorname{ArcTanh}[a+bx]}\right] + i \pi \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[\frac{1}{\sqrt{1-(a+bx)^2}}\right] - \\
& \operatorname{ArcTanh}[a+bx]^2 \operatorname{Log}\left[-\frac{bx}{\sqrt{1-(a+bx)^2}}\right] + 2 \operatorname{ArcTanh}[a] \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[-i \operatorname{Sinh}[\operatorname{ArcTanh}[a] - \operatorname{ArcTanh}[a+bx]]\right] + \\
& \operatorname{ArcTanh}[a+bx] \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[a+bx]}\right] - \operatorname{ArcTanh}[a+bx] \operatorname{PolyLog}\left[2, -\frac{(-1+a) e^{2 \operatorname{ArcTanh}[a+bx]}}{1+a}\right] + \\
& 2 \operatorname{ArcTanh}[a+bx] \operatorname{PolyLog}\left[2, -e^{-\operatorname{ArcTanh}[a] + \operatorname{ArcTanh}[a+bx]}\right] + 2 \operatorname{ArcTanh}[a+bx] \operatorname{PolyLog}\left[2, e^{-\operatorname{ArcTanh}[a] + \operatorname{ArcTanh}[a+bx]}\right] + \\
& \operatorname{ArcTanh}[a+bx] \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[a] + 2 \operatorname{ArcTanh}[a+bx]}\right] + \frac{1}{2} \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}[a+bx]}\right] + \frac{1}{2} \operatorname{PolyLog}\left[3, -\frac{(-1+a) e^{2 \operatorname{ArcTanh}[a+bx]}}{1+a}\right] - \\
& 2 \operatorname{PolyLog}\left[3, -e^{-\operatorname{ArcTanh}[a] + \operatorname{ArcTanh}[a+bx]}\right] - 2 \operatorname{PolyLog}\left[3, e^{-\operatorname{ArcTanh}[a] + \operatorname{ArcTanh}[a+bx]}\right] - \frac{1}{2} \operatorname{PolyLog}\left[3, e^{-2 \operatorname{ArcTanh}[a] + 2 \operatorname{ArcTanh}[a+bx]}\right]
\end{aligned}$$

Problem 6: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh}[a+bx]^2}{x^2} dx$$

Optimal (type 4, 251 leaves, 17 steps):

$$\begin{aligned}
& -\frac{\operatorname{ArcTanh}[a+bx]^2}{x} + \frac{b \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[\frac{2}{1-a-bx}\right]}{1-a} + \frac{b \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[\frac{2}{1+a+bx}\right]}{1+a} - \\
& \frac{2b \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[\frac{2}{1+a+bx}\right]}{1-a^2} + \frac{2b \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[\frac{2bx}{(1-a)(1+a+bx)}\right]}{1-a^2} + \frac{b \operatorname{PolyLog}\left[2, -\frac{1+a+bx}{1-a-bx}\right]}{2(1-a)} - \\
& \frac{b \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+a+bx}\right]}{2(1+a)} + \frac{b \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+a+bx}\right]}{1-a^2} - \frac{b \operatorname{PolyLog}\left[2, 1 - \frac{2bx}{(1-a)(1+a+bx)}\right]}{1-a^2}
\end{aligned}$$

Result (type 4, 208 leaves):

$$\frac{1}{a(-1+a^2)x} \left(- \left(-a + a^3 + a^2 b x + b \left(-1 + \sqrt{1-a^2} e^{\text{ArcTanh}[a]} \right) x \right) \text{ArcTanh}[a + b x]^2 + a b x \text{ArcTanh}[a + b x] \right. \\ \left. \left(-i \pi + 2 \text{ArcTanh}[a] - 2 \text{Log}\left[1 - e^{2 \text{ArcTanh}[a] - 2 \text{ArcTanh}[a + b x]}\right] \right) + a b x \left(i \pi \left(\text{Log}\left[1 + e^{2 \text{ArcTanh}[a + b x]}\right] - \text{Log}\left[\frac{1}{\sqrt{1 - (a + b x)^2}}\right] \right) + 2 \text{ArcTanh}[a] \right. \right. \\ \left. \left. \left(\text{Log}\left[1 - e^{2 \text{ArcTanh}[a] - 2 \text{ArcTanh}[a + b x]}\right] - \text{Log}\left[-i \text{Sinh}\left[\text{ArcTanh}[a] - \text{ArcTanh}[a + b x]\right]\right] \right) \right) + a b x \text{PolyLog}\left[2, e^{2 \text{ArcTanh}[a] - 2 \text{ArcTanh}[a + b x]}\right] \right)$$

Problem 7: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTanh}[a + b x]^2}{x^3} dx$$

Optimal (type 4, 370 leaves, 21 steps):

$$-\frac{b \text{ArcTanh}[a + b x]}{(1-a^2)x} - \frac{\text{ArcTanh}[a + b x]^2}{2x^2} + \frac{b^2 \text{Log}[x]}{(1-a^2)^2} + \frac{b^2 \text{ArcTanh}[a + b x] \text{Log}\left[\frac{2}{1-a-bx}\right]}{2(1-a)^2} - \frac{b^2 \text{Log}[1-a-bx]}{2(1-a)^2(1+a)} \\ - \frac{b^2 \text{ArcTanh}[a + b x] \text{Log}\left[\frac{2}{1+a+bx}\right]}{2(1+a)^2} - \frac{2ab^2 \text{ArcTanh}[a + b x] \text{Log}\left[\frac{2}{1+a+bx}\right]}{(1-a^2)^2} + \frac{2ab^2 \text{ArcTanh}[a + b x] \text{Log}\left[\frac{2bx}{(1-a)(1+a+bx)}\right]}{(1-a^2)^2} - \frac{b^2 \text{Log}[1+a+bx]}{2(1-a)(1+a)^2} + \\ \frac{b^2 \text{PolyLog}\left[2, -\frac{1+a+bx}{1-a-bx}\right]}{4(1-a)^2} + \frac{b^2 \text{PolyLog}\left[2, 1 - \frac{2}{1+a+bx}\right]}{4(1+a)^2} + \frac{ab^2 \text{PolyLog}\left[2, 1 - \frac{2}{1+a+bx}\right]}{(1-a^2)^2} - \frac{ab^2 \text{PolyLog}\left[2, 1 - \frac{2bx}{(1-a)(1+a+bx)}\right]}{(1-a^2)^2}$$

Result (type 4, 271 leaves):

$$\frac{1}{2(-1+a^2)^2 x^2} \left(- \left(1 + a^4 - b^2 \left(-1 + 2 \sqrt{1-a^2} e^{\text{ArcTanh}[a]} \right) x^2 - a^2 (2 + b^2 x^2) \right) \text{ArcTanh}[a + b x]^2 + \right. \\ \left. 2 b x \text{ArcTanh}[a + b x] \left(-1 + a^2 + a b x + i a b \pi x - 2 a b x \text{ArcTanh}[a] + 2 a b x \text{Log}\left[1 - e^{2 \text{ArcTanh}[a] - 2 \text{ArcTanh}[a + b x]}\right] \right) + \right. \\ \left. 2 b^2 x^2 \left(-i a \pi \text{Log}\left[1 + e^{2 \text{ArcTanh}[a + b x]}\right] + i a \pi \text{Log}\left[\frac{1}{\sqrt{1 - (a + b x)^2}}\right] + \text{Log}\left[-\frac{b x}{\sqrt{1 - (a + b x)^2}}\right] - 2 a \text{ArcTanh}[a] \right. \right. \\ \left. \left. \left(\text{Log}\left[1 - e^{2 \text{ArcTanh}[a] - 2 \text{ArcTanh}[a + b x]}\right] - \text{Log}\left[-i \text{Sinh}\left[\text{ArcTanh}[a] - \text{ArcTanh}[a + b x]\right]\right] \right) \right) - 2 a b^2 x^2 \text{PolyLog}\left[2, e^{2 \text{ArcTanh}[a] - 2 \text{ArcTanh}[a + b x]}\right] \right)$$

Problem 12: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{ArcTanh}[c + d x]}{c e + d e x} dx$$

Optimal (type 4, 54 leaves, 3 steps):

$$\frac{a \operatorname{Log}[c + d x]}{d e} - \frac{b \operatorname{PolyLog}[2, -c - d x]}{2 d e} + \frac{b \operatorname{PolyLog}[2, c + d x]}{2 d e}$$

Result (type 4, 288 leaves):

$$\begin{aligned} & \frac{a \operatorname{Log}[c + d x]}{d e} - \frac{1}{d e} i b \left(i \operatorname{ArcTanh}[c + d x] \left(-\operatorname{Log}\left[\frac{1}{\sqrt{1 - (c + d x)^2}}\right] + \operatorname{Log}\left[\frac{i(c + d x)}{\sqrt{1 - (c + d x)^2}}\right] \right) + \right. \\ & \left. \frac{1}{2} \left(-\frac{1}{4} i (\pi - 2 i \operatorname{ArcTanh}[c + d x])^2 + i \operatorname{ArcTanh}[c + d x]^2 + (\pi - 2 i \operatorname{ArcTanh}[c + d x]) \operatorname{Log}[1 - e^{i(\pi - 2 i \operatorname{ArcTanh}[c + d x])}] \right) + \right. \\ & \left. 2 i \operatorname{ArcTanh}[c + d x] \operatorname{Log}[1 - e^{-2 \operatorname{ArcTanh}[c + d x]}] - 2 i \operatorname{ArcTanh}[c + d x] \operatorname{Log}\left[\frac{2 i(c + d x)}{\sqrt{1 - (c + d x)^2}}\right] - \right. \\ & \left. (\pi - 2 i \operatorname{ArcTanh}[c + d x]) \operatorname{Log}\left[2 \operatorname{Sin}\left[\frac{1}{2} (\pi - 2 i \operatorname{ArcTanh}[c + d x])\right]\right] - i \operatorname{PolyLog}[2, e^{i(\pi - 2 i \operatorname{ArcTanh}[c + d x])}] - i \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcTanh}[c + d x]}] \right) \end{aligned}$$

Problem 18: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c + d x])^2}{c e + d e x} dx$$

Optimal (type 4, 168 leaves, 8 steps):

$$\begin{aligned} & \frac{2(a + b \operatorname{ArcTanh}[c + d x])^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - c - d x}\right]}{d e} - \frac{b(a + b \operatorname{ArcTanh}[c + d x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c - d x}\right]}{d e} + \\ & \frac{b(a + b \operatorname{ArcTanh}[c + d x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - c - d x}\right]}{d e} + \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c - d x}\right]}{2 d e} - \frac{b^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - c - d x}\right]}{2 d e} \end{aligned}$$

Result (type 4, 424 leaves):

$$\begin{aligned}
& \frac{1}{d e} \left(a^2 \operatorname{Log}[c+d x] + 2 a b \operatorname{ArcTanh}[c+d x] \left(-\operatorname{Log}\left[\frac{1}{\sqrt{1-(c+d x)^2}}\right] + \operatorname{Log}\left[\frac{i(c+d x)}{\sqrt{1-(c+d x)^2}}\right] \right) - \right. \\
& \frac{1}{4} a b \left(\pi^2 - 4 i \pi \operatorname{ArcTanh}[c+d x] - 8 \operatorname{ArcTanh}[c+d x]^2 - 8 \operatorname{ArcTanh}[c+d x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[c+d x]}\right] + 4 i \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[c+d x]}\right] + \right. \\
& 8 \operatorname{ArcTanh}[c+d x] \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[c+d x]}\right] - 4 i \pi \operatorname{Log}\left[\frac{2}{\sqrt{1-(c+d x)^2}}\right] - 8 \operatorname{ArcTanh}[c+d x] \operatorname{Log}\left[\frac{2}{\sqrt{1-(c+d x)^2}}\right] + \\
& \left. 8 \operatorname{ArcTanh}[c+d x] \operatorname{Log}\left[\frac{2 i(c+d x)}{\sqrt{1-(c+d x)^2}}\right] + 4 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcTanh}[c+d x]}\right] + 4 \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcTanh}[c+d x]}\right] \right) + \\
& b^2 \left(\frac{i \pi^3}{24} - \frac{2}{3} \operatorname{ArcTanh}[c+d x]^3 - \operatorname{ArcTanh}[c+d x]^2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c+d x]}\right] + \operatorname{ArcTanh}[c+d x]^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[c+d x]}\right] + \right. \\
& \operatorname{ArcTanh}[c+d x] \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[c+d x]}\right] + \operatorname{ArcTanh}[c+d x] \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcTanh}[c+d x]}\right] + \\
& \left. \left. \frac{1}{2} \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}[c+d x]}\right] - \frac{1}{2} \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcTanh}[c+d x]}\right] \right) \right)
\end{aligned}$$

Problem 25: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a+b \operatorname{ArcTanh}[c+d x])^3}{c e+d e x} d x$$

Optimal (type 4, 257 leaves, 10 steps):

$$\begin{aligned}
& \frac{2(a+b \operatorname{ArcTanh}[c+d x])^3 \operatorname{ArcTanh}\left[1 - \frac{2}{1-c-d x}\right]}{d e} - \frac{3 b(a+b \operatorname{ArcTanh}[c+d x])^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c-d x}\right]}{2 d e} + \\
& \frac{3 b(a+b \operatorname{ArcTanh}[c+d x])^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1-c-d x}\right]}{2 d e} + \frac{3 b^2(a+b \operatorname{ArcTanh}[c+d x]) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1-c-d x}\right]}{2 d e} - \\
& \frac{3 b^2(a+b \operatorname{ArcTanh}[c+d x]) \operatorname{PolyLog}\left[3, -1 + \frac{2}{1-c-d x}\right]}{2 d e} - \frac{3 b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2}{1-c-d x}\right]}{4 d e} + \frac{3 b^3 \operatorname{PolyLog}\left[4, -1 + \frac{2}{1-c-d x}\right]}{4 d e}
\end{aligned}$$

Result (type 4, 599 leaves):

$$\begin{aligned}
& \frac{1}{64 d e} \left(64 a^3 \operatorname{Log}[c + d x] + 192 a^2 b \operatorname{ArcTanh}[c + d x] \left(-\operatorname{Log}\left[\frac{1}{\sqrt{1 - (c + d x)^2}}\right] + \operatorname{Log}\left[\frac{i(c + d x)}{\sqrt{1 - (c + d x)^2}}\right] \right) - \right. \\
& 96 i a^2 b \left(-\frac{1}{4} i (\pi - 2 i \operatorname{ArcTanh}[c + d x])^2 + i \operatorname{ArcTanh}[c + d x]^2 + 2 i \operatorname{ArcTanh}[c + d x] \operatorname{Log}[1 - e^{-2 \operatorname{ArcTanh}[c + d x]}] + \right. \\
& (\pi - 2 i \operatorname{ArcTanh}[c + d x]) \operatorname{Log}[1 + e^{2 \operatorname{ArcTanh}[c + d x]}] - (\pi - 2 i \operatorname{ArcTanh}[c + d x]) \operatorname{Log}\left[\frac{2}{\sqrt{1 - (c + d x)^2}}\right] - \\
& \left. \left. 2 i \operatorname{ArcTanh}[c + d x] \operatorname{Log}\left[\frac{2 i(c + d x)}{\sqrt{1 - (c + d x)^2}}\right] - i \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcTanh}[c + d x]}] - i \operatorname{PolyLog}[2, -e^{2 \operatorname{ArcTanh}[c + d x]}] \right) + \right. \\
& 8 a b^2 (i \pi^3 - 16 \operatorname{ArcTanh}[c + d x]^3 - 24 \operatorname{ArcTanh}[c + d x]^2 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}] + 24 \operatorname{ArcTanh}[c + d x]^2 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c + d x]}] + \\
& 24 \operatorname{ArcTanh}[c + d x] \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c + d x]}] + 24 \operatorname{ArcTanh}[c + d x] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c + d x]}] + \\
& 12 \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c + d x]}] - 12 \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c + d x]}]) + \\
& b^3 (\pi^4 - 32 \operatorname{ArcTanh}[c + d x]^4 - 64 \operatorname{ArcTanh}[c + d x]^3 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}] + 64 \operatorname{ArcTanh}[c + d x]^3 \operatorname{Log}[1 - e^{2 \operatorname{ArcTanh}[c + d x]}] + 96 \operatorname{ArcTanh}[c + d x]^2 \\
& \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c + d x]}] + 96 \operatorname{ArcTanh}[c + d x]^2 \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c + d x]}] + 96 \operatorname{ArcTanh}[c + d x] \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c + d x]}] - \\
& \left. \left. 96 \operatorname{ArcTanh}[c + d x] \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c + d x]}] + 48 \operatorname{PolyLog}[4, -e^{-2 \operatorname{ArcTanh}[c + d x]}] + 48 \operatorname{PolyLog}[4, e^{2 \operatorname{ArcTanh}[c + d x]}] \right) \right)
\end{aligned}$$

Problem 26: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b \operatorname{ArcTanh}[c + d x])^3}{(c e + d e x)^2} dx$$

Optimal (type 4, 143 leaves, 7 steps):

$$\begin{aligned}
& \frac{(a + b \operatorname{ArcTanh}[c + d x])^3}{d e^2} - \frac{(a + b \operatorname{ArcTanh}[c + d x])^3}{d e^2 (c + d x)} + \frac{3 b (a + b \operatorname{ArcTanh}[c + d x])^2 \operatorname{Log}\left[2 - \frac{2}{1 + c + d x}\right]}{d e^2} - \\
& \frac{3 b^2 (a + b \operatorname{ArcTanh}[c + d x]) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 + c + d x}\right]}{d e^2} - \frac{3 b^3 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 + c + d x}\right]}{2 d e^2}
\end{aligned}$$

Result (type 4, 248 leaves):

$$\begin{aligned} & \frac{1}{2 d e^2} \left(-\frac{2 a^3}{c+d x} - \frac{6 a^2 b \operatorname{ArcTanh}[c+d x]}{c+d x} + 6 a^2 b \operatorname{Log}[c+d x] - 3 a^2 b \operatorname{Log}[1-c^2-2 c d x-d^2 x^2] + \right. \\ & 6 a b^2 \left(\operatorname{ArcTanh}[c+d x] \left(\left(1 - \frac{1}{c+d x} \right) \operatorname{ArcTanh}[c+d x] + 2 \operatorname{Log}[1-e^{-2 \operatorname{ArcTanh}[c+d x]}] \right) - \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcTanh}[c+d x]}] \right) + \\ & 2 b^3 \left(\frac{i \pi^3}{8} - \operatorname{ArcTanh}[c+d x]^3 - \frac{\operatorname{ArcTanh}[c+d x]^3}{c+d x} + 3 \operatorname{ArcTanh}[c+d x]^2 \operatorname{Log}[1-e^{2 \operatorname{ArcTanh}[c+d x]}] + \right. \\ & \left. \left. 3 \operatorname{ArcTanh}[c+d x] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c+d x]}] - \frac{3}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c+d x]}] \right) \right) \end{aligned}$$

Problem 28: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+b \operatorname{ArcTanh}[c+d x])^3}{(c e+d e x)^4} dx$$

Optimal (type 4, 269 leaves, 16 steps):

$$\begin{aligned} & -\frac{b^2 (a+b \operatorname{ArcTanh}[c+d x])}{d e^4 (c+d x)} + \frac{b (a+b \operatorname{ArcTanh}[c+d x])^2}{2 d e^4} - \frac{b (a+b \operatorname{ArcTanh}[c+d x])^2}{2 d e^4 (c+d x)^2} + \\ & \frac{(a+b \operatorname{ArcTanh}[c+d x])^3}{3 d e^4} - \frac{(a+b \operatorname{ArcTanh}[c+d x])^3}{3 d e^4 (c+d x)^3} + \frac{b^3 \operatorname{Log}[c+d x]}{d e^4} - \frac{b^3 \operatorname{Log}[1-(c+d x)^2]}{2 d e^4} + \\ & \frac{b (a+b \operatorname{ArcTanh}[c+d x])^2 \operatorname{Log}\left[2 - \frac{2}{1+c+d x}\right]}{d e^4} - \frac{b^2 (a+b \operatorname{ArcTanh}[c+d x]) \operatorname{PolyLog}[2, -1 + \frac{2}{1+c+d x}]}{d e^4} - \frac{b^3 \operatorname{PolyLog}[3, -1 + \frac{2}{1+c+d x}]}{2 d e^4} \end{aligned}$$

Result (type 4, 393 leaves):

$$\frac{1}{6 d e^4} \left(-\frac{2 a^3}{(c+d x)^3} - \frac{3 a^2 b}{(c+d x)^2} - \frac{6 a^2 b \operatorname{ArcTanh}[c+d x]}{(c+d x)^3} + 6 a^2 b \operatorname{Log}[c+d x] - 3 a^2 b \operatorname{Log}[1-c^2-2 c d x-d^2 x^2] + 6 a b^2 \left(-\frac{(c+d x)^2 + \operatorname{ArcTanh}[c+d x]^2}{(c+d x)^3} + \operatorname{ArcTanh}[c+d x] \left(-\frac{1-(c+d x)^2}{(c+d x)^2} + \operatorname{ArcTanh}[c+d x] + 2 \operatorname{Log}[1-e^{-2 \operatorname{ArcTanh}[c+d x]}] \right) - \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcTanh}[c+d x]}] \right) + 6 b^3 \left(\frac{i \pi^3}{24} - \frac{\operatorname{ArcTanh}[c+d x]}{c+d x} - \frac{(1-(c+d x)^2) \operatorname{ArcTanh}[c+d x]^2}{2(c+d x)^2} - \frac{1}{3} \operatorname{ArcTanh}[c+d x]^3 - \frac{\operatorname{ArcTanh}[c+d x]^3}{3(c+d x)} - \frac{(1-(c+d x)^2) \operatorname{ArcTanh}[c+d x]^3}{3(c+d x)^3} + \operatorname{ArcTanh}[c+d x]^2 \operatorname{Log}[1-e^{2 \operatorname{ArcTanh}[c+d x]}] + \operatorname{Log}\left[\frac{c+d x}{\sqrt{1-(c+d x)^2}}\right] + \operatorname{ArcTanh}[c+d x] \operatorname{PolyLog}[2, e^{2 \operatorname{ArcTanh}[c+d x]}] - \frac{1}{2} \operatorname{PolyLog}[3, e^{2 \operatorname{ArcTanh}[c+d x]}] \right) \right)$$

Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcTanh}[1+x]}{2+2 x} dx$$

Optimal (type 4, 21 leaves, 3 steps):

$$-\frac{1}{4} \operatorname{PolyLog}[2, -1-x] + \frac{1}{4} \operatorname{PolyLog}[2, 1+x]$$

Result (type 4, 207 leaves):

$$\frac{1}{16} \left(-\pi^2 + 4 i \pi \operatorname{ArcTanh}[1+x] + 8 \operatorname{ArcTanh}[1+x]^2 + 8 \operatorname{ArcTanh}[1+x] \operatorname{Log}[1-e^{-2 \operatorname{ArcTanh}[1+x]}] - 4 i \pi \operatorname{Log}[1+e^{2 \operatorname{ArcTanh}[1+x]}] - 8 \operatorname{ArcTanh}[1+x] \operatorname{Log}[1+e^{2 \operatorname{ArcTanh}[1+x]}] - 8 \operatorname{ArcTanh}[1+x] \operatorname{Log}\left[\frac{1}{\sqrt{-x(2+x)}}\right] + 4 i \pi \operatorname{Log}\left[\frac{2}{\sqrt{-x(2+x)}}\right] + 8 \operatorname{ArcTanh}[1+x] \operatorname{Log}\left[\frac{2}{\sqrt{-x(2+x)}}\right] + 8 \operatorname{ArcTanh}[1+x] \operatorname{Log}\left[\frac{i(1+x)}{\sqrt{-x(2+x)}}\right] - 8 \operatorname{ArcTanh}[1+x] \operatorname{Log}\left[\frac{2 i(1+x)}{\sqrt{-x(2+x)}}\right] - 4 \operatorname{PolyLog}[2, e^{-2 \operatorname{ArcTanh}[1+x]}] - 4 \operatorname{PolyLog}[2, -e^{2 \operatorname{ArcTanh}[1+x]}] \right)$$

Problem 30: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTanh}[a + b x]}{\frac{a d}{b} + d x} dx$$

Optimal (type 4, 32 leaves, 3 steps):

$$-\frac{\text{PolyLog}[2, -a - b x]}{2 d} + \frac{\text{PolyLog}[2, a + b x]}{2 d}$$

Result (type 4, 263 leaves):

$$\begin{aligned} & -\frac{1}{8 d} \left(\pi^2 - 4 i \pi \text{ArcTanh}[a + b x] - 8 \text{ArcTanh}[a + b x]^2 - 8 \text{ArcTanh}[a + b x] \text{Log}\left[1 - e^{-2 \text{ArcTanh}[a + b x]}\right] + \right. \\ & 4 i \pi \text{Log}\left[1 + e^{2 \text{ArcTanh}[a + b x]}\right] + 8 \text{ArcTanh}[a + b x] \text{Log}\left[1 + e^{2 \text{ArcTanh}[a + b x]}\right] + 8 \text{ArcTanh}[a + b x] \text{Log}\left[\frac{1}{\sqrt{1 - (a + b x)^2}}\right] - \\ & 4 i \pi \text{Log}\left[\frac{2}{\sqrt{1 - (a + b x)^2}}\right] - 8 \text{ArcTanh}[a + b x] \text{Log}\left[\frac{2}{\sqrt{1 - (a + b x)^2}}\right] - 8 \text{ArcTanh}[a + b x] \text{Log}\left[\frac{i(a + b x)}{\sqrt{1 - (a + b x)^2}}\right] + \\ & \left. 8 \text{ArcTanh}[a + b x] \text{Log}\left[\frac{2 i(a + b x)}{\sqrt{1 - (a + b x)^2}}\right] + 4 \text{PolyLog}[2, e^{-2 \text{ArcTanh}[a + b x]}\right] + 4 \text{PolyLog}[2, -e^{2 \text{ArcTanh}[a + b x]}\right] \end{aligned}$$

Problem 35: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \text{ArcTanh}[c + d x]}{e + f x} dx$$

Optimal (type 4, 130 leaves, 5 steps):

$$\begin{aligned} & -\frac{(a + b \text{ArcTanh}[c + d x]) \text{Log}\left[\frac{2}{1 + c + d x}\right]}{f} + \frac{(a + b \text{ArcTanh}[c + d x]) \text{Log}\left[\frac{2 d (e + f x)}{(d e + f - c f) (1 + c + d x)}\right]}{f} + \\ & \frac{b \text{PolyLog}[2, 1 - \frac{2}{1 + c + d x}]}{2 f} - \frac{b \text{PolyLog}[2, 1 - \frac{2 d (e + f x)}{(d e + f - c f) (1 + c + d x)}]}{2 f} \end{aligned}$$

Result (type 4, 329 leaves):

$$\frac{1}{f} \left(a \operatorname{Log}[e + f x] + b \operatorname{ArcTanh}[c + d x] \left(-\operatorname{Log}\left[\frac{1}{\sqrt{1 - (c + d x)^2}}\right] + \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{d e - c f}{f}\right] + \operatorname{ArcTanh}[c + d x]\right]\right] \right) - \right. \\ \left. \frac{1}{2} i b \left(-\frac{1}{4} i (\pi - 2 i \operatorname{ArcTanh}[c + d x])^2 + i \left(\operatorname{ArcTanh}\left[\frac{d e - c f}{f}\right] + \operatorname{ArcTanh}[c + d x] \right)^2 + (\pi - 2 i \operatorname{ArcTanh}[c + d x]) \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[c + d x]}\right] + \right. \right. \\ \left. \left. 2 i \left(\operatorname{ArcTanh}\left[\frac{d e - c f}{f}\right] + \operatorname{ArcTanh}[c + d x] \right) \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{d e - c f}{f}\right] + \operatorname{ArcTanh}[c + d x]\right)}\right] - (\pi - 2 i \operatorname{ArcTanh}[c + d x]) \operatorname{Log}\left[\frac{2}{\sqrt{1 - (c + d x)^2}}\right] - \right. \right. \\ \left. \left. 2 i \left(\operatorname{ArcTanh}\left[\frac{d e - c f}{f}\right] + \operatorname{ArcTanh}[c + d x] \right) \operatorname{Log}\left[2 i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{d e - c f}{f}\right] + \operatorname{ArcTanh}[c + d x]\right]\right] - \right. \right. \\ \left. \left. i \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcTanh}[c + d x]}\right] - i \operatorname{PolyLog}\left[2, e^{-2 \left(\operatorname{ArcTanh}\left[\frac{d e - c f}{f}\right] + \operatorname{ArcTanh}[c + d x]\right)}\right] \right) \right) \right)$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int (e + f x)^3 (a + b \operatorname{ArcTanh}[c + d x])^2 dx$$

Optimal (type 4, 562 leaves, 20 steps):

$$\frac{b^2 f^2 (d e - c f) x}{d^3} + \frac{a b f (6 d^2 e^2 - 12 c d e f + (1 + 6 c^2) f^2) x}{2 d^3} + \frac{b^2 f^3 (c + d x)^2}{12 d^4} - \frac{b^2 f^2 (d e - c f) \operatorname{ArcTanh}[c + d x]}{d^4} + \\ \frac{b^2 f (6 d^2 e^2 - 12 c d e f + (1 + 6 c^2) f^2) (c + d x) \operatorname{ArcTanh}[c + d x]}{2 d^4} + \frac{b f^2 (d e - c f) (c + d x)^2 (a + b \operatorname{ArcTanh}[c + d x])}{d^4} + \\ \frac{b f^3 (c + d x)^3 (a + b \operatorname{ArcTanh}[c + d x])}{6 d^4} + \frac{(d e - c f) (d^2 e^2 - 2 c d e f + (1 + c^2) f^2) (a + b \operatorname{ArcTanh}[c + d x])^2}{d^4} - \\ \frac{(d^4 e^4 - 4 c d^3 e^3 f + 6 (1 + c^2) d^2 e^2 f^2 - 4 c (3 + c^2) d e f^3 + (1 + 6 c^2 + c^4) f^4) (a + b \operatorname{ArcTanh}[c + d x])^2}{4 d^4 f} + \frac{(e + f x)^4 (a + b \operatorname{ArcTanh}[c + d x])^2}{4 f} - \\ \frac{2 b (d e - c f) (d^2 e^2 - 2 c d e f + (1 + c^2) f^2) (a + b \operatorname{ArcTanh}[c + d x]) \operatorname{Log}\left[\frac{2}{1 - c - d x}\right]}{d^4} + \frac{b^2 f^3 \operatorname{Log}\left[1 - (c + d x)^2\right]}{12 d^4} + \\ \frac{b^2 f (6 d^2 e^2 - 12 c d e f + (1 + 6 c^2) f^2) \operatorname{Log}\left[1 - (c + d x)^2\right]}{4 d^4} - \frac{b^2 (d e - c f) (d^2 e^2 - 2 c d e f + (1 + c^2) f^2) \operatorname{PolyLog}\left[2, -\frac{1 + c + d x}{1 - c - d x}\right]}{d^4}$$

Result (type 4, 1215 leaves):

$$a^2 e^3 x + \frac{3}{2} a^2 e^2 f x^2 + a^2 e f^2 x^3 + \frac{1}{4} a^2 f^3 x^4 +$$

$$\begin{aligned}
& \frac{1}{12} a b \left(6 x \left(4 e^3 + 6 e^2 f x + 4 e f^2 x^2 + f^3 x^3 \right) \operatorname{ArcTanh}[c + d x] - \frac{1}{d^4} \left(-2 d f x \left(3 \left(1 + 3 c^2 \right) f^2 - 3 c d f \left(8 e + f x \right) + d^2 \left(18 e^2 + 6 e f x + f^2 x^2 \right) \right) + \right. \\
& \quad 3 \left(-1 + c \right) \left(4 d^3 e^3 - 6 \left(-1 + c \right) d^2 e^2 f + 4 \left(-1 + c \right)^2 d e f^2 - \left(-1 + c \right)^3 f^3 \right) \operatorname{Log}[1 - c - d x] + \\
& \quad \left. 3 \left(1 + c \right) \left(-4 d^3 e^3 + 6 \left(1 + c \right) d^2 e^2 f - 4 \left(1 + c \right)^2 d e f^2 + \left(1 + c \right)^3 f^3 \right) \operatorname{Log}[1 + c + d x] \right) + \frac{1}{d} \\
& b^2 e^3 \left(\operatorname{ArcTanh}[c + d x] \left(-\operatorname{ArcTanh}[c + d x] + \left(c + d x \right) \operatorname{ArcTanh}[c + d x] - 2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}\right] \right) + \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[c + d x]}\right] \right) - \\
& \frac{1}{2 d^2} \\
& 3 b^2 e^2 f \left(\left(1 - \left(c + d x \right)^2 \right) \operatorname{ArcTanh}[c + d x]^2 + 2 \left(- \left(c + d x \right) \operatorname{ArcTanh}[c + d x] - c \operatorname{ArcTanh}[c + d x]^2 + c \left(c + d x \right) \operatorname{ArcTanh}[c + d x]^2 - \right. \right. \\
& \quad \left. \left. 2 c \operatorname{ArcTanh}[c + d x] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}\right] + \operatorname{Log}\left[\frac{1}{\sqrt{1 - \left(c + d x \right)^2}}\right] \right) + 2 c \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[c + d x]}\right] \right) + \\
& \frac{1}{12 d^4} b^2 f^3 \left(3 \left(1 - \left(c + d x \right)^2 \right)^2 \operatorname{ArcTanh}[c + d x]^2 - \left(1 - \left(c + d x \right)^2 \right) \left(1 - 12 c \operatorname{ArcTanh}[c + d x] + 6 \operatorname{ArcTanh}[c + d x]^2 + 18 c^2 \operatorname{ArcTanh}[c + d x]^2 - \right. \right. \\
& \quad \left. \left. 2 \left(c + d x \right) \operatorname{ArcTanh}[c + d x] \left(-1 + 6 c \operatorname{ArcTanh}[c + d x] \right) \right) - 4 \left(-3 c \operatorname{ArcTanh}[c + d x]^2 - 3 c^3 \operatorname{ArcTanh}[c + d x]^2 + \right. \right. \\
& \quad \left. \left. \left(c + d x \right) \left(-2 \operatorname{ArcTanh}[c + d x] - 9 c^2 \operatorname{ArcTanh}[c + d x] + 3 c^3 \operatorname{ArcTanh}[c + d x]^2 + 3 c \left(1 + \operatorname{ArcTanh}[c + d x]^2 \right) \right) - 6 c \left(1 + c^2 \right) \operatorname{ArcTanh}[c + d x] \right. \right. \\
& \quad \left. \left. \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}\right] + 2 \operatorname{Log}\left[\frac{1}{\sqrt{1 - \left(c + d x \right)^2}}\right] + 9 c^2 \operatorname{Log}\left[\frac{1}{\sqrt{1 - \left(c + d x \right)^2}}\right] \right) - 12 \left(c + c^3 \right) \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[c + d x]}\right] \right) - \\
& \frac{1}{4 d^3} b^2 e f^2 \left(1 - \left(c + d x \right)^2 \right)^{3/2} \left(-\frac{c + d x}{\sqrt{1 - \left(c + d x \right)^2}} + \frac{6 c \left(c + d x \right) \operatorname{ArcTanh}[c + d x]}{\sqrt{1 - \left(c + d x \right)^2}} + \frac{3 \left(c + d x \right) \operatorname{ArcTanh}[c + d x]^2}{\sqrt{1 - \left(c + d x \right)^2}} - \right. \\
& \quad \frac{3 c^2 \left(c + d x \right) \operatorname{ArcTanh}[c + d x]^2}{\sqrt{1 - \left(c + d x \right)^2}} + \operatorname{ArcTanh}[c + d x]^2 \operatorname{Cosh}\left[3 \operatorname{ArcTanh}[c + d x]\right] + 3 c^2 \operatorname{ArcTanh}[c + d x]^2 \operatorname{Cosh}\left[3 \operatorname{ArcTanh}[c + d x]\right] + \\
& \quad 2 \operatorname{ArcTanh}[c + d x] \operatorname{Cosh}\left[3 \operatorname{ArcTanh}[c + d x]\right] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}\right] + 6 c^2 \operatorname{ArcTanh}[c + d x] \operatorname{Cosh}\left[3 \operatorname{ArcTanh}[c + d x]\right] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}\right] - \\
& \quad 6 c \operatorname{Cosh}\left[3 \operatorname{ArcTanh}[c + d x]\right] \operatorname{Log}\left[\frac{1}{\sqrt{1 - \left(c + d x \right)^2}}\right] + \frac{1}{\sqrt{1 - \left(c + d x \right)^2}} \left(\operatorname{ArcTanh}[c + d x] \left(4 + 3 \left(1 - 4 c + 3 c^2 \right) \operatorname{ArcTanh}[c + d x] \right) + \right. \\
& \quad \left. 6 \left(\operatorname{ArcTanh}[c + d x] + 3 c^2 \operatorname{ArcTanh}[c + d x] \right) \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}\right] - 18 c \operatorname{Log}\left[\frac{1}{\sqrt{1 - \left(c + d x \right)^2}}\right] \right) -
\end{aligned}$$

$$\left. \begin{aligned} & \frac{4 (1 + 3 c^2) \text{PolyLog}\left[2, -e^{-2 \text{ArcTanh}[c+dx]}\right]}{(1 - (c + dx)^2)^{3/2}} - \text{Sinh}\left[3 \text{ArcTanh}[c + dx]\right] + 6 c \text{ArcTanh}[c + dx] \text{Sinh}\left[3 \text{ArcTanh}[c + dx]\right] - \\ & \text{ArcTanh}[c + dx]^2 \text{Sinh}\left[3 \text{ArcTanh}[c + dx]\right] - 3 c^2 \text{ArcTanh}[c + dx]^2 \text{Sinh}\left[3 \text{ArcTanh}[c + dx]\right] \end{aligned} \right\}$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int (e + f x)^2 (a + b \text{ArcTanh}[c + dx])^2 dx$$

Optimal (type 4, 374 leaves, 16 steps):

$$\begin{aligned} & \frac{b^2 f^2 x}{3 d^2} + \frac{2 a b f (d e - c f) x}{d^2} - \frac{b^2 f^2 \text{ArcTanh}[c + dx]}{3 d^3} + \frac{2 b^2 f (d e - c f) (c + dx) \text{ArcTanh}[c + dx]}{d^3} + \frac{b f^2 (c + dx)^2 (a + b \text{ArcTanh}[c + dx])}{3 d^3} - \\ & \frac{(d e - c f) (d^2 e^2 - 2 c d e f + (3 + c^2) f^2) (a + b \text{ArcTanh}[c + dx])^2}{3 d^3 f} + \frac{(3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) (a + b \text{ArcTanh}[c + dx])^2}{3 d^3} + \\ & \frac{(e + f x)^3 (a + b \text{ArcTanh}[c + dx])^2}{3 f} - \frac{2 b (3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) (a + b \text{ArcTanh}[c + dx]) \text{Log}\left[\frac{2}{1 - c - dx}\right]}{3 d^3} + \\ & \frac{b^2 f (d e - c f) \text{Log}\left[1 - (c + dx)^2\right]}{d^3} - \frac{b^2 (3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) \text{PolyLog}\left[2, -\frac{1 + c + dx}{1 - c - dx}\right]}{3 d^3} \end{aligned}$$

Result (type 4, 795 leaves):

$$\begin{aligned}
& a^2 e^2 x + a^2 e f x^2 + \frac{1}{3} a^2 f^2 x^3 + \\
& \frac{1}{3} a b \left(2 x (3 e^2 + 3 e f x + f^2 x^2) \operatorname{ArcTanh}[c + d x] + \frac{1}{d^3} (d f x (6 d e - 4 c f + d f x) - (-1 + c) (3 d^2 e^2 - 3 (-1 + c) d e f + (-1 + c)^2 f^2) \operatorname{Log}[1 - c - d x] + \right. \\
& \quad \left. (1 + c) (3 d^2 e^2 - 3 (1 + c) d e f + (1 + c)^2 f^2) \operatorname{Log}[1 + c + d x] \right) + \frac{1}{d} \\
& b^2 e^2 (\operatorname{ArcTanh}[c + d x] ((-1 + c + d x) \operatorname{ArcTanh}[c + d x] - 2 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}]) + \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c + d x]}]) + \\
& \frac{1}{d^2} b^2 e f \left((-1 + 2 c - c^2 + d^2 x^2) \operatorname{ArcTanh}[c + d x]^2 + \right. \\
& \quad \left. 2 \operatorname{ArcTanh}[c + d x] (c + d x + 2 c \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}]) - 2 \operatorname{Log}\left[\frac{1}{\sqrt{1 - (c + d x)^2}}\right] - 2 c \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c + d x]}] \right) + \\
& \frac{1}{12 d^3} b^2 f^2 (1 - (c + d x)^2)^{3/2} \left(-\frac{c + d x}{\sqrt{1 - (c + d x)^2}} + \frac{6 c (c + d x) \operatorname{ArcTanh}[c + d x]}{\sqrt{1 - (c + d x)^2}} + \frac{3 (c + d x) \operatorname{ArcTanh}[c + d x]^2}{\sqrt{1 - (c + d x)^2}} - \right. \\
& \quad \frac{3 c^2 (c + d x) \operatorname{ArcTanh}[c + d x]^2}{\sqrt{1 - (c + d x)^2}} + \operatorname{ArcTanh}[c + d x]^2 \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] + \\
& \quad 3 c^2 \operatorname{ArcTanh}[c + d x]^2 \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] + 2 \operatorname{ArcTanh}[c + d x] \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}] + \\
& \quad 6 c^2 \operatorname{ArcTanh}[c + d x] \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}] - 6 c \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] \operatorname{Log}\left[\frac{1}{\sqrt{1 - (c + d x)^2}}\right] + \frac{1}{\sqrt{1 - (c + d x)^2}} \\
& \quad \left. \left(3 (1 - 4 c + 3 c^2) \operatorname{ArcTanh}[c + d x]^2 + 2 \operatorname{ArcTanh}[c + d x] (2 + (3 + 9 c^2) \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}]) - 18 c \operatorname{Log}\left[\frac{1}{\sqrt{1 - (c + d x)^2}}\right] \right) - \right. \\
& \quad \frac{4 (1 + 3 c^2) \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c + d x]}]}{(1 - (c + d x)^2)^{3/2}} - \operatorname{Sinh}[3 \operatorname{ArcTanh}[c + d x]] + 6 c \operatorname{ArcTanh}[c + d x] \operatorname{Sinh}[3 \operatorname{ArcTanh}[c + d x]] - \\
& \quad \left. \operatorname{ArcTanh}[c + d x]^2 \operatorname{Sinh}[3 \operatorname{ArcTanh}[c + d x]] - 3 c^2 \operatorname{ArcTanh}[c + d x]^2 \operatorname{Sinh}[3 \operatorname{ArcTanh}[c + d x]] \right)
\end{aligned}$$

Problem 42: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c + d x])^2}{e + f x} dx$$

Optimal (type 4, 214 leaves, 2 steps):

$$\begin{aligned}
& - \frac{(a + b \operatorname{ArcTanh}[c + d x])^2 \operatorname{Log}\left[\frac{2}{1 + c + d x}\right]}{f} + \frac{(a + b \operatorname{ArcTanh}[c + d x])^2 \operatorname{Log}\left[\frac{2 d (e + f x)}{(d e + f - c f) (1 + c + d x)}\right]}{f} + \frac{b (a + b \operatorname{ArcTanh}[c + d x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c + d x}\right]}{f} \\
& - \frac{b (a + b \operatorname{ArcTanh}[c + d x]) \operatorname{PolyLog}\left[2, 1 - \frac{2 d (e + f x)}{(d e + f - c f) (1 + c + d x)}\right]}{f} + \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 + c + d x}\right]}{2 f} - \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2 d (e + f x)}{(d e + f - c f) (1 + c + d x)}\right]}{2 f}
\end{aligned}$$

Result (type 8, 22 leaves):

$$\int \frac{(a + b \operatorname{ArcTanh}[c + d x])^2}{e + f x} dx$$

Problem 43: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c + d x])^2}{(e + f x)^2} dx$$

Optimal (type 4, 480 leaves, 24 steps):

$$\begin{aligned}
& - \frac{(a + b \operatorname{ArcTanh}[c + d x])^2}{f (e + f x)} + \frac{b^2 d \operatorname{ArcTanh}[c + d x] \operatorname{Log}\left[\frac{2}{1 - c - d x}\right]}{f (d e + f - c f)} - \frac{a b d \operatorname{Log}[1 - c - d x]}{f (d e + f - c f)} - \frac{b^2 d \operatorname{ArcTanh}[c + d x] \operatorname{Log}\left[\frac{2}{1 + c + d x}\right]}{f (d e - f - c f)} + \\
& \frac{2 b^2 d \operatorname{ArcTanh}[c + d x] \operatorname{Log}\left[\frac{2}{1 + c + d x}\right]}{(d e + f - c f) (d e - (1 + c) f)} + \frac{a b d \operatorname{Log}[1 + c + d x]}{f (d e - f - c f)} + \frac{2 a b d \operatorname{Log}[e + f x]}{f^2 - (d e - c f)^2} - \frac{2 b^2 d \operatorname{ArcTanh}[c + d x] \operatorname{Log}\left[\frac{2 d (e + f x)}{(d e + f - c f) (1 + c + d x)}\right]}{(d e + f - c f) (d e - (1 + c) f)} + \\
& \frac{b^2 d \operatorname{PolyLog}\left[2, -\frac{1 + c + d x}{1 - c - d x}\right]}{2 f (d e + f - c f)} + \frac{b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c + d x}\right]}{2 f (d e - f - c f)} - \frac{b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c + d x}\right]}{(d e + f - c f) (d e - (1 + c) f)} + \frac{b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2 d (e + f x)}{(d e + f - c f) (1 + c + d x)}\right]}{(d e + f - c f) (d e - (1 + c) f)}
\end{aligned}$$

Result (type 4, 1198 leaves):

$$\begin{aligned}
& - \frac{a^2}{f (e + f x)} + \left(2 a b (1 - (c + d x)^2) \left(\frac{d e - c f}{\sqrt{1 - (c + d x)^2}} + \frac{f (c + d x)}{\sqrt{1 - (c + d x)^2}} \right) \right. \\
& \left. \frac{\left((c + d x) \left(d e \operatorname{ArcTanh}[c + d x] - c f \operatorname{ArcTanh}[c + d x] - f \operatorname{Log}\left[\frac{d e}{\sqrt{1 - (c + d x)^2}} - \frac{c f}{\sqrt{1 - (c + d x)^2}} + \frac{f (c + d x)}{\sqrt{1 - (c + d x)^2}} \right] \right) \right)}{\sqrt{1 - (c + d x)^2}} + \right.
\end{aligned}$$

$$\left. \left. \left. \frac{f \operatorname{ArcTanh}[c + dx] + (-de + cf) \operatorname{Log}\left[\frac{de}{\sqrt{1-(c+dx)^2}} - \frac{cf}{\sqrt{1-(c+dx)^2}} + \frac{f(c+dx)}{\sqrt{1-(c+dx)^2}}\right]}{\sqrt{1-(c+dx)^2}} \right) \right) \right) /$$

$$\left(d(de + f - cf)(de - (1+c)f)(e + fx)^2 + \frac{1}{d(e + fx)^2} b^2 (1 - (c + dx)^2) \left(\frac{de - cf}{\sqrt{1 - (c + dx)^2}} + \frac{f(c + dx)}{\sqrt{1 - (c + dx)^2}} \right)^2 \right)$$

$$\left(\frac{(c + dx) \operatorname{ArcTanh}[c + dx]^2}{(de - cf) \sqrt{1 - (c + dx)^2} \left(\frac{de}{\sqrt{1 - (c + dx)^2}} - \frac{cf}{\sqrt{1 - (c + dx)^2}} + \frac{f(c + dx)}{\sqrt{1 - (c + dx)^2}} \right)} - \frac{1}{de - cf} \right)^2 \left(\frac{f \operatorname{ArcTanh}[c + dx]^2}{2(de - f - cf)(de + f - cf)} + \right.$$

$$\left. \frac{\operatorname{ArcTanh}[c + dx] \left(-f \operatorname{ArcTanh}[c + dx] + (de - cf) \operatorname{Log}\left[\frac{de}{\sqrt{1 - (c + dx)^2}} - \frac{cf}{\sqrt{1 - (c + dx)^2}} + \frac{f(c + dx)}{\sqrt{1 - (c + dx)^2}}\right]\right)}{(de + f - cf)(de - (1+c)f)} - \frac{1}{2(de + f - cf)(de - (1+c)f)} \right)$$

$$\left(-i de \pi \operatorname{ArcTanh}[c + dx] + i cf \pi \operatorname{ArcTanh}[c + dx] - f \operatorname{ArcTanh}[c + dx]^2 + e^{-\operatorname{ArcTanh}\left[\frac{de - cf}{f}\right]} \sqrt{1 - c^2 - \frac{d^2 e^2}{f^2} + \frac{2cde}{f}} f \operatorname{ArcTanh}[c + dx]^2 + \right.$$

$$i de \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[c + dx]}\right] - i cf \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[c + dx]}\right] - 2 de \operatorname{ArcTanh}[c + dx] \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{de - cf}{f}\right] + \operatorname{ArcTanh}[c + dx]\right)}\right] +$$

$$2 cf \operatorname{ArcTanh}[c + dx] \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{de - cf}{f}\right] + \operatorname{ArcTanh}[c + dx]\right)}\right] - i de \pi \operatorname{Log}\left[\frac{1}{\sqrt{1 - (c + dx)^2}}\right] + i cf \pi \operatorname{Log}\left[\frac{1}{\sqrt{1 - (c + dx)^2}}\right] +$$

$$2 de \operatorname{ArcTanh}[c + dx] \operatorname{Log}\left[\frac{de}{\sqrt{1 - (c + dx)^2}} - \frac{cf}{\sqrt{1 - (c + dx)^2}} + \frac{f(c + dx)}{\sqrt{1 - (c + dx)^2}}\right] - 2 cf \operatorname{ArcTanh}[c + dx] \operatorname{Log}\left[\frac{de}{\sqrt{1 - (c + dx)^2}} - \right.$$

$$\left. \frac{cf}{\sqrt{1 - (c + dx)^2}} + \frac{f(c + dx)}{\sqrt{1 - (c + dx)^2}}\right] - 2(de - cf) \operatorname{ArcTanh}\left[\frac{de - cf}{f}\right] \left(\operatorname{ArcTanh}[c + dx] + \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{de - cf}{f}\right] + \operatorname{ArcTanh}[c + dx]\right)}\right] \right) -$$

$$\left. \left. \left. \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{de - cf}{f}\right] + \operatorname{ArcTanh}[c + dx]\right]\right]\right) + (de - cf) \operatorname{PolyLog}\left[2, e^{-2 \left(\operatorname{ArcTanh}\left[\frac{de - cf}{f}\right] + \operatorname{ArcTanh}[c + dx]\right)}\right] \right) \right) \right)$$

Problem 44: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcTanh}[c + dx])^2}{(e + fx)^3} dx$$

Optimal (type 4, 750 leaves, 26 steps):

$$\begin{aligned} & -\frac{abd}{(f^2 - (de - cf)^2)(e + fx)} + \frac{b^2 d \operatorname{ArcTanh}[c + dx]}{(de + f - cf)(de - (1 + c)f)(e + fx)} - \frac{(a + b \operatorname{ArcTanh}[c + dx])^2}{2f(e + fx)^2} + \\ & \frac{b^2 d^2 \operatorname{ArcTanh}[c + dx] \operatorname{Log}\left[\frac{2}{1 - cx}\right]}{2f(de + f - cf)^2} - \frac{abd^2 \operatorname{Log}[1 - cx]}{2f(de + f - cf)^2} + \frac{b^2 d^2 \operatorname{Log}[1 - cx]}{2(de + f - cf)^2(de - (1 + c)f)} - \frac{b^2 d^2 \operatorname{ArcTanh}[c + dx] \operatorname{Log}\left[\frac{2}{1 + cx}\right]}{2f(de - f - cf)^2} + \\ & \frac{2b^2 d^2 (de - cf) \operatorname{ArcTanh}[c + dx] \operatorname{Log}\left[\frac{2}{1 + cx}\right]}{(de + f - cf)^2 (de - (1 + c)f)^2} + \frac{abd^2 \operatorname{Log}[1 + cx]}{2f(de - f - cf)^2} - \frac{b^2 d^2 \operatorname{Log}[1 + cx]}{2(de + f - cf)(de - (1 + c)f)^2} + \frac{b^2 d^2 f \operatorname{Log}[e + fx]}{(de + f - cf)^2 (de - (1 + c)f)^2} - \\ & \frac{2abd^2 (de - cf) \operatorname{Log}[e + fx]}{(de + f - cf)^2 (de - (1 + c)f)^2} - \frac{2b^2 d^2 (de - cf) \operatorname{ArcTanh}[c + dx] \operatorname{Log}\left[\frac{2d(e + fx)}{(de + f - cf)(1 + cx)}\right]}{(de + f - cf)^2 (de - (1 + c)f)^2} + \frac{b^2 d^2 \operatorname{PolyLog}\left[2, -\frac{1 + cx}{1 - cx}\right]}{4f(de + f - cf)^2} + \\ & \frac{b^2 d^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + cx}\right]}{4f(de - f - cf)^2} - \frac{b^2 d^2 (de - cf) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + cx}\right]}{(de + f - cf)^2 (de - (1 + c)f)^2} + \frac{b^2 d^2 (de - cf) \operatorname{PolyLog}\left[2, 1 - \frac{2d(e + fx)}{(de + f - cf)(1 + cx)}\right]}{(de + f - cf)^2 (de - (1 + c)f)^2} \end{aligned}$$

Result (type 4, 1970 leaves):

$$\begin{aligned} & -\frac{a^2}{2f(e + fx)^2} + \frac{1}{d(e + fx)^3} ab(de - cf + f(c + dx))^3 \left(\frac{f \left(2 + \frac{(de + f - cf)(de - (1 + c)f)}{\left(\frac{de - cf}{\sqrt{1 - (c + dx)^2}} + \frac{f(c + dx)}{\sqrt{1 - (c + dx)^2}} \right)^2} \right) \operatorname{ArcTanh}[c + dx]}{(de + f - cf)^2 (-de + f + cf)^2} - \right. \\ & \left. \frac{(c + dx)(f - 2de \operatorname{ArcTanh}[c + dx] + 2cf \operatorname{ArcTanh}[c + dx])}{(de - cf)(de + f - cf)(de - (1 + c)f) \sqrt{1 - (c + dx)^2} \left(\frac{de - cf}{\sqrt{1 - (c + dx)^2}} + \frac{f(c + dx)}{\sqrt{1 - (c + dx)^2}} \right)} - \frac{2(de - cf) \operatorname{Log}\left[\frac{de}{\sqrt{1 - (c + dx)^2}} - \frac{cf}{\sqrt{1 - (c + dx)^2}} + \frac{f(c + dx)}{\sqrt{1 - (c + dx)^2}}\right]}{(d^2 e^2 - 2cdef + (-1 + c^2)f^2)^2} \right) + \end{aligned}$$

$$\begin{aligned}
& \frac{1}{d(e+fx)^3} b^2 (de - cf + f(c+dx))^3 \left(\frac{f(1-(c+dx)^2)^{3/2} \left(\frac{de}{\sqrt{1-(c+dx)^2}} - \frac{cf}{\sqrt{1-(c+dx)^2}} + \frac{f(c+dx)}{\sqrt{1-(c+dx)^2}} \right)^3 \operatorname{ArcTanh}[c+dx]^2}{2(de-f-cf)(de+f-cf)(de-cf+f(c+dx))^3 \left(-\frac{de}{\sqrt{1-(c+dx)^2}} + \frac{cf}{\sqrt{1-(c+dx)^2}} - \frac{f(c+dx)}{\sqrt{1-(c+dx)^2}} \right)^2} \right)^2 + \\
& \left((1-(c+dx)^2)^{3/2} \left(\frac{de}{\sqrt{1-(c+dx)^2}} - \frac{cf}{\sqrt{1-(c+dx)^2}} + \frac{f(c+dx)}{\sqrt{1-(c+dx)^2}} \right)^3 \right. \\
& \left. \left(\frac{f(c+dx) \operatorname{ArcTanh}[c+dx]}{\sqrt{1-(c+dx)^2}} - \frac{de(c+dx) \operatorname{ArcTanh}[c+dx]^2}{\sqrt{1-(c+dx)^2}} + \frac{cf(c+dx) \operatorname{ArcTanh}[c+dx]^2}{\sqrt{1-(c+dx)^2}} \right) \right) / \\
& \left((de-cf)(de-f-cf)(de+f-cf)(de-cf+f(c+dx))^3 \left(-\frac{de}{\sqrt{1-(c+dx)^2}} + \frac{cf}{\sqrt{1-(c+dx)^2}} - \frac{f(c+dx)}{\sqrt{1-(c+dx)^2}} \right) \right) + \\
& \left(f(1-(c+dx)^2)^{3/2} \left(\frac{de}{\sqrt{1-(c+dx)^2}} - \frac{cf}{\sqrt{1-(c+dx)^2}} + \frac{f(c+dx)}{\sqrt{1-(c+dx)^2}} \right)^3 \right. \\
& \left. \left(-f \operatorname{ArcTanh}[c+dx] + (de-cf) \operatorname{Log} \left[\frac{de-cf}{\sqrt{1-(c+dx)^2}} + \frac{f(c+dx)}{\sqrt{1-(c+dx)^2}} \right] \right) \right) / \\
& \left((de-cf)(de-f-cf)(de+f-cf)(-f^2+(de-cf)^2)(de-cf+f(c+dx))^3 \right) - \\
& \left(c(1-(c+dx)^2)^{3/2} \left(\frac{de}{\sqrt{1-(c+dx)^2}} - \frac{cf}{\sqrt{1-(c+dx)^2}} + \frac{f(c+dx)}{\sqrt{1-(c+dx)^2}} \right)^3 \left(-e^{-\operatorname{ArcTanh}\left[\frac{de-cf}{f}\right]} \operatorname{ArcTanh}[c+dx]^2 + \right. \right. \\
& \left. \left. \frac{1}{f \sqrt{1-\frac{(de-cf)^2}{f^2}}} \operatorname{Im}(de-cf) \left(-\left(-\pi + 2 \operatorname{Im} \operatorname{ArcTanh}\left[\frac{de-cf}{f}\right] \right) \operatorname{ArcTanh}[c+dx] - 2 \left(\operatorname{Im} \operatorname{ArcTanh}\left[\frac{de-cf}{f}\right] + \operatorname{Im} \operatorname{ArcTanh}[c+dx] \right) \operatorname{Log} \left[\right. \right. \right. \right. \\
& \left. \left. \left. \left. 1 - e^{2 \operatorname{Im} \left(\operatorname{Im} \operatorname{ArcTanh}\left[\frac{de-cf}{f}\right] + \operatorname{Im} \operatorname{ArcTanh}[c+dx] \right)} \right] - \pi \operatorname{Log} \left[1 + e^{2 \operatorname{ArcTanh}[c+dx]} \right] + \pi \operatorname{Log} \left[\frac{1}{\sqrt{1-(c+dx)^2}} \right] + 2 \operatorname{Im} \operatorname{ArcTanh} \left[\right. \right. \right. \right.
\end{aligned}$$

$$\left. \left. \left. \frac{de - cf}{f} \right] \text{Log} \left[i \text{Sinh} \left[\text{ArcTanh} \left[\frac{de - cf}{f} \right] + \text{ArcTanh} [c + dx] \right] \right] + i \text{PolyLog} \left[2, e^{2i \left(i \text{ArcTanh} \left[\frac{de - cf}{f} \right] + i \text{ArcTanh} [c + dx] \right)} \right] \right] \right) \right) \Bigg/$$

$$\left((de - cf) (de - f - cf) (de + f - cf) \sqrt{\frac{f^2 - (de - cf)^2}{f^2}} (de - cf + f(c + dx))^3 \right) +$$

$$\left(de (1 - (c + dx)^2)^{3/2} \left(\frac{de}{\sqrt{1 - (c + dx)^2}} - \frac{cf}{\sqrt{1 - (c + dx)^2}} + \frac{f(c + dx)}{\sqrt{1 - (c + dx)^2}} \right)^3 \right)$$

$$\left(-e^{-\text{ArcTanh} \left[\frac{de - cf}{f} \right]} \text{ArcTanh} [c + dx]^2 + \frac{1}{f \sqrt{1 - \frac{(de - cf)^2}{f^2}}} i (de - cf) \left(- \left(-\pi + 2i \text{ArcTanh} \left[\frac{de - cf}{f} \right] \right) \text{ArcTanh} [c + dx] - 2 \left(i \text{ArcTanh} \left[\frac{de - cf}{f} \right] + i \text{ArcTanh} [c + dx] \right) \text{Log} \left[1 - e^{2i \left(i \text{ArcTanh} \left[\frac{de - cf}{f} \right] + i \text{ArcTanh} [c + dx] \right)} \right] - \pi \text{Log} [1 + e^{2 \text{ArcTanh} [c + dx]}] + \pi \text{Log} \left[\frac{1}{\sqrt{1 - (c + dx)^2}} \right] + \right. \right.$$

$$\left. \left. \left. 2i \text{ArcTanh} \left[\frac{de - cf}{f} \right] \text{Log} \left[i \text{Sinh} \left[\text{ArcTanh} \left[\frac{de - cf}{f} \right] + \text{ArcTanh} [c + dx] \right] \right] + i \text{PolyLog} \left[2, e^{2i \left(i \text{ArcTanh} \left[\frac{de - cf}{f} \right] + i \text{ArcTanh} [c + dx] \right)} \right] \right] \right) \right) \Bigg/$$

$$\left(f (de - cf) (de - f - cf) (de + f - cf) \sqrt{\frac{f^2 - (de - cf)^2}{f^2}} (de - cf + f(c + dx))^3 \right)$$

Problem 45: Result more than twice size of optimal antiderivative.

$$\int (e + fx)^2 (a + b \text{ArcTanh} [c + dx])^3 dx$$

Optimal (type 4, 546 leaves, 21 steps):

$$\begin{aligned}
& \frac{a b^2 f^2 x}{d^2} + \frac{b^3 f^2 (c + d x) \operatorname{ArcTanh}[c + d x]}{d^3} - \frac{b f^2 (a + b \operatorname{ArcTanh}[c + d x])^2}{2 d^3} + \frac{3 b f (d e - c f) (a + b \operatorname{ArcTanh}[c + d x])^2}{d^3} + \\
& \frac{3 b f (d e - c f) (c + d x) (a + b \operatorname{ArcTanh}[c + d x])^2}{d^3} + \frac{b f^2 (c + d x)^2 (a + b \operatorname{ArcTanh}[c + d x])^2}{2 d^3} - \\
& \frac{(d e - c f) (d^2 e^2 - 2 c d e f + (3 + c^2) f^2) (a + b \operatorname{ArcTanh}[c + d x])^3}{3 d^3 f} + \frac{(3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) (a + b \operatorname{ArcTanh}[c + d x])^3}{3 d^3} + \\
& \frac{(e + f x)^3 (a + b \operatorname{ArcTanh}[c + d x])^3}{3 f} - \frac{6 b^2 f (d e - c f) (a + b \operatorname{ArcTanh}[c + d x]) \operatorname{Log}\left[\frac{2}{1 - c - d x}\right]}{d^3} - \\
& \frac{b (3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) (a + b \operatorname{ArcTanh}[c + d x])^2 \operatorname{Log}\left[\frac{2}{1 - c - d x}\right]}{d^3} + \frac{b^3 f^2 \operatorname{Log}\left[1 - (c + d x)^2\right]}{2 d^3} - \frac{3 b^3 f (d e - c f) \operatorname{PolyLog}\left[2, -\frac{1 + c + d x}{1 - c - d x}\right]}{d^3} - \\
& \frac{b^2 (3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) (a + b \operatorname{ArcTanh}[c + d x]) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - c - d x}\right]}{d^3} + \frac{b^3 (3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - c - d x}\right]}{2 d^3}
\end{aligned}$$

Result (type 4, 1868 leaves):

$$\begin{aligned}
& \frac{a^2 (a d^2 e^2 + 3 b d e f - 2 b c f^2) x}{d^2} + \frac{a^2 f (2 a d e + b f) x^2}{2 d} + \frac{1}{3} a^3 f^2 x^3 + a^2 b x (3 e^2 + 3 e f x + f^2 x^2) \operatorname{ArcTanh}[c + d x] + \frac{1}{2 d^3} \\
& (3 a^2 b d^2 e^2 - 3 a^2 b c d^2 e^2 + 3 a^2 b d e f - 6 a^2 b c d e f + 3 a^2 b c^2 d e f + a^2 b f^2 - 3 a^2 b c f^2 + 3 a^2 b c^2 f^2 - a^2 b c^3 f^2) \operatorname{Log}[1 - c - d x] + \frac{1}{2 d^3} \\
& (3 a^2 b d^2 e^2 + 3 a^2 b c d^2 e^2 - 3 a^2 b d e f - 6 a^2 b c d e f - 3 a^2 b c^2 d e f + a^2 b f^2 + 3 a^2 b c f^2 + 3 a^2 b c^2 f^2 + a^2 b c^3 f^2) \operatorname{Log}[1 + c + d x] + \frac{1}{d} \\
& 3 a b^2 e^2 (\operatorname{ArcTanh}[c + d x] (-\operatorname{ArcTanh}[c + d x] + (c + d x) \operatorname{ArcTanh}[c + d x] - 2 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}]) + \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c + d x]}]) - \\
& \frac{1}{d^2} 3 a b^2 e f \left((1 - (c + d x)^2) \operatorname{ArcTanh}[c + d x]^2 + 2 \left(- (c + d x) \operatorname{ArcTanh}[c + d x] - c \operatorname{ArcTanh}[c + d x]^2 + c (c + d x) \operatorname{ArcTanh}[c + d x]^2 - \right. \right. \\
& \left. \left. 2 c \operatorname{ArcTanh}[c + d x] \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}] + \operatorname{Log}\left[\frac{1}{\sqrt{1 - (c + d x)^2}}\right] \right) + 2 c \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c + d x]}] \right) + \\
& \frac{1}{d} b^3 e^2 \left(\operatorname{ArcTanh}[c + d x]^2 (-\operatorname{ArcTanh}[c + d x] + (c + d x) \operatorname{ArcTanh}[c + d x] - 3 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}]) \right) + \\
& 3 \operatorname{ArcTanh}[c + d x] \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c + d x]}] + \frac{3}{2} \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c + d x]}] \Big) + \\
& \frac{1}{d^2} b^3 e f \left(-\operatorname{ArcTanh}[c + d x] \left(3 \operatorname{ArcTanh}[c + d x] - 2 c \operatorname{ArcTanh}[c + d x]^2 + (1 - (c + d x)^2) \operatorname{ArcTanh}[c + d x]^2 + \right. \right. \\
& \left. \left. (c + d x) \operatorname{ArcTanh}[c + d x] (-3 + 2 c \operatorname{ArcTanh}[c + d x]) + 6 \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}] - 6 c \operatorname{ArcTanh}[c + d x] \operatorname{Log}[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}] \right) \right) + \\
& (3 - 6 c \operatorname{ArcTanh}[c + d x]) \operatorname{PolyLog}[2, -e^{-2 \operatorname{ArcTanh}[c + d x]}] - 3 c \operatorname{PolyLog}[3, -e^{-2 \operatorname{ArcTanh}[c + d x]}] \Big) -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4 d^3} a b^2 f^2 \left(1 - (c + d x)^2\right)^{3/2} \left(-\frac{c + d x}{\sqrt{1 - (c + d x)^2}} + \frac{6 c (c + d x) \operatorname{ArcTanh}[c + d x]}{\sqrt{1 - (c + d x)^2}} + \frac{3 (c + d x) \operatorname{ArcTanh}[c + d x]^2}{\sqrt{1 - (c + d x)^2}} - \right. \\
& \frac{3 c^2 (c + d x) \operatorname{ArcTanh}[c + d x]^2}{\sqrt{1 - (c + d x)^2}} + \operatorname{ArcTanh}[c + d x]^2 \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] + 3 c^2 \operatorname{ArcTanh}[c + d x]^2 \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] + \\
& 2 \operatorname{ArcTanh}[c + d x] \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}\right] + 6 c^2 \operatorname{ArcTanh}[c + d x] \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}\right] - \\
& 6 c \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] \operatorname{Log}\left[\frac{1}{\sqrt{1 - (c + d x)^2}}\right] + \frac{1}{\sqrt{1 - (c + d x)^2}} \left(\operatorname{ArcTanh}[c + d x] (4 + 3 (1 - 4 c + 3 c^2) \operatorname{ArcTanh}[c + d x]) + \right. \\
& \left. 6 (\operatorname{ArcTanh}[c + d x] + 3 c^2 \operatorname{ArcTanh}[c + d x]) \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}\right] - 18 c \operatorname{Log}\left[\frac{1}{\sqrt{1 - (c + d x)^2}}\right] \right) - \\
& \frac{4 (1 + 3 c^2) \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[c + d x]}\right]}{(1 - (c + d x)^2)^{3/2}} - \operatorname{Sinh}[3 \operatorname{ArcTanh}[c + d x]] + 6 c \operatorname{ArcTanh}[c + d x] \operatorname{Sinh}[3 \operatorname{ArcTanh}[c + d x]] - \\
& \left. \operatorname{ArcTanh}[c + d x]^2 \operatorname{Sinh}[3 \operatorname{ArcTanh}[c + d x]] - 3 c^2 \operatorname{ArcTanh}[c + d x]^2 \operatorname{Sinh}[3 \operatorname{ArcTanh}[c + d x]] \right) + \\
& \frac{1}{d^3} b^3 f^2 \left((-3 c + \operatorname{ArcTanh}[c + d x] + 3 c^2 \operatorname{ArcTanh}[c + d x]) \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[c + d x]}\right] - \right. \\
& \frac{1}{12} \left(1 - (c + d x)^2\right)^{3/2} \left(-\frac{3 (c + d x) \operatorname{ArcTanh}[c + d x]}{\sqrt{1 - (c + d x)^2}} + \frac{9 c (c + d x) \operatorname{ArcTanh}[c + d x]^2}{\sqrt{1 - (c + d x)^2}} + \frac{3 (c + d x) \operatorname{ArcTanh}[c + d x]^3}{\sqrt{1 - (c + d x)^2}} - \right. \\
& \frac{3 c^2 (c + d x) \operatorname{ArcTanh}[c + d x]^3}{\sqrt{1 - (c + d x)^2}} - 9 c \operatorname{ArcTanh}[c + d x]^2 \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] + \operatorname{ArcTanh}[c + d x]^3 \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] + \\
& 3 c^2 \operatorname{ArcTanh}[c + d x]^3 \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] - 18 c \operatorname{ArcTanh}[c + d x] \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}\right] + \\
& 3 \operatorname{ArcTanh}[c + d x]^2 \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}\right] + \\
& 9 c^2 \operatorname{ArcTanh}[c + d x]^2 \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}\right] + 3 \operatorname{Cosh}[3 \operatorname{ArcTanh}[c + d x]] \operatorname{Log}\left[\frac{1}{\sqrt{1 - (c + d x)^2}}\right] + \\
& \left. \frac{1}{\sqrt{1 - (c + d x)^2}} 3 \left(\operatorname{ArcTanh}[c + d x]^2 (2 - 9 c + \operatorname{ArcTanh}[c + d x]) - 4 c \operatorname{ArcTanh}[c + d x] + 3 c^2 \operatorname{ArcTanh}[c + d x] \right) + \right.
\end{aligned}$$

$$\left. \begin{aligned} & 3 \operatorname{ArcTanh}[c + d x] \left(-6 c + \operatorname{ArcTanh}[c + d x] + 3 c^2 \operatorname{ArcTanh}[c + d x] \right) \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[c + d x]}\right] + 3 \operatorname{Log}\left[\frac{1}{\sqrt{1 - (c + d x)^2}}\right] \Bigg) - \\ & \frac{6(1 + 3 c^2) \operatorname{PolyLog}\left[3, -e^{-2 \operatorname{ArcTanh}[c + d x]}\right]}{\left(1 - (c + d x)^2\right)^{3/2}} - 3 \operatorname{ArcTanh}[c + d x] \operatorname{Sinh}\left[3 \operatorname{ArcTanh}[c + d x]\right] + 9 c \operatorname{ArcTanh}[c + d x]^2 \operatorname{Sinh}\left[3 \operatorname{ArcTanh}[c + d x]\right] - \\ & \operatorname{ArcTanh}[c + d x]^3 \operatorname{Sinh}\left[3 \operatorname{ArcTanh}[c + d x]\right] - 3 c^2 \operatorname{ArcTanh}[c + d x]^3 \operatorname{Sinh}\left[3 \operatorname{ArcTanh}[c + d x]\right] \Bigg) \Bigg) \end{aligned}$$

Problem 48: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcTanh}[c + d x])^3}{e + f x} dx$$

Optimal (type 4, 308 leaves, 2 steps):

$$\begin{aligned} & - \frac{(a + b \operatorname{ArcTanh}[c + d x])^3 \operatorname{Log}\left[\frac{2}{1 + c + d x}\right]}{f} + \frac{(a + b \operatorname{ArcTanh}[c + d x])^3 \operatorname{Log}\left[\frac{2 d (e + f x)}{(d e + f - c f) (1 + c + d x)}\right]}{f} + \frac{3 b (a + b \operatorname{ArcTanh}[c + d x])^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + c + d x}\right]}{2 f} - \\ & \frac{3 b (a + b \operatorname{ArcTanh}[c + d x])^2 \operatorname{PolyLog}\left[2, 1 - \frac{2 d (e + f x)}{(d e + f - c f) (1 + c + d x)}\right]}{2 f} + \frac{3 b^2 (a + b \operatorname{ArcTanh}[c + d x]) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 + c + d x}\right]}{2 f} - \\ & \frac{3 b^2 (a + b \operatorname{ArcTanh}[c + d x]) \operatorname{PolyLog}\left[3, 1 - \frac{2 d (e + f x)}{(d e + f - c f) (1 + c + d x)}\right]}{2 f} + \frac{3 b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2}{1 + c + d x}\right]}{4 f} - \frac{3 b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2 d (e + f x)}{(d e + f - c f) (1 + c + d x)}\right]}{4 f} \end{aligned}$$

Result (type 8, 22 leaves):

$$\int \frac{(a + b \operatorname{ArcTanh}[c + d x])^3}{e + f x} dx$$

Problem 49: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \operatorname{ArcTanh}[c + d x])^3}{(e + f x)^2} dx$$

Optimal (type 4, 1089 leaves, 33 steps):

$$\begin{aligned}
& - \frac{(a + b \operatorname{ArcTanh}[c + d x])^3}{f (e + f x)} + \frac{3 a b^2 d \operatorname{ArcTanh}[c + d x] \operatorname{Log}\left[\frac{2}{1-c-d x}\right]}{f (d e + f - c f)} + \frac{3 b^3 d \operatorname{ArcTanh}[c + d x]^2 \operatorname{Log}\left[\frac{2}{1-c-d x}\right]}{2 f (d e + f - c f)} - \\
& \frac{3 a^2 b d \operatorname{Log}[1 - c - d x]}{2 f (d e + f - c f)} - \frac{3 a b^2 d \operatorname{ArcTanh}[c + d x] \operatorname{Log}\left[\frac{2}{1+c+d x}\right]}{f (d e - f - c f)} + \frac{6 a b^2 d \operatorname{ArcTanh}[c + d x] \operatorname{Log}\left[\frac{2}{1+c+d x}\right]}{(d e + f - c f) (d e - (1 + c) f)} - \\
& \frac{3 b^3 d \operatorname{ArcTanh}[c + d x]^2 \operatorname{Log}\left[\frac{2}{1+c+d x}\right]}{2 f (d e - f - c f)} + \frac{3 b^3 d \operatorname{ArcTanh}[c + d x]^2 \operatorname{Log}\left[\frac{2}{1+c+d x}\right]}{(d e + f - c f) (d e - (1 + c) f)} + \frac{3 a^2 b d \operatorname{Log}[1 + c + d x]}{2 f (d e - f - c f)} + \\
& \frac{3 a^2 b d \operatorname{Log}[e + f x]}{f^2 - (d e - c f)^2} - \frac{6 a b^2 d \operatorname{ArcTanh}[c + d x] \operatorname{Log}\left[\frac{2 d (e+f x)}{(d e+f-c f) (1+c+d x)}\right]}{(d e + f - c f) (d e - (1 + c) f)} - \frac{3 b^3 d \operatorname{ArcTanh}[c + d x]^2 \operatorname{Log}\left[\frac{2 d (e+f x)}{(d e+f-c f) (1+c+d x)}\right]}{(d e + f - c f) (d e - (1 + c) f)} + \\
& \frac{3 a b^2 d \operatorname{PolyLog}\left[2, -\frac{1+c+d x}{1-c-d x}\right]}{2 f (d e + f - c f)} + \frac{3 b^3 d \operatorname{ArcTanh}[c + d x] \operatorname{PolyLog}\left[2, 1 - \frac{2}{1-c-d x}\right]}{2 f (d e + f - c f)} + \frac{3 a b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c+d x}\right]}{2 f (d e - f - c f)} - \\
& \frac{3 a b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c+d x}\right]}{(d e + f - c f) (d e - (1 + c) f)} + \frac{3 b^3 d \operatorname{ArcTanh}[c + d x] \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c+d x}\right]}{2 f (d e - f - c f)} - \frac{3 b^3 d \operatorname{ArcTanh}[c + d x] \operatorname{PolyLog}\left[2, 1 - \frac{2}{1+c+d x}\right]}{(d e + f - c f) (d e - (1 + c) f)} + \\
& \frac{3 a b^2 d \operatorname{PolyLog}\left[2, 1 - \frac{2 d (e+f x)}{(d e+f-c f) (1+c+d x)}\right]}{(d e + f - c f) (d e - (1 + c) f)} + \frac{3 b^3 d \operatorname{ArcTanh}[c + d x] \operatorname{PolyLog}\left[2, 1 - \frac{2 d (e+f x)}{(d e+f-c f) (1+c+d x)}\right]}{(d e + f - c f) (d e - (1 + c) f)} - \frac{3 b^3 d \operatorname{PolyLog}\left[3, 1 - \frac{2}{1-c-d x}\right]}{4 f (d e + f - c f)} + \\
& \frac{3 b^3 d \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+c+d x}\right]}{4 f (d e - f - c f)} - \frac{3 b^3 d \operatorname{PolyLog}\left[3, 1 - \frac{2}{1+c+d x}\right]}{2 (d e + f - c f) (d e - (1 + c) f)} + \frac{3 b^3 d \operatorname{PolyLog}\left[3, 1 - \frac{2 d (e+f x)}{(d e+f-c f) (1+c+d x)}\right]}{2 (d e + f - c f) (d e - (1 + c) f)}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 52: Unable to integrate problem.

$$\int (e + f x)^m (a + b \operatorname{ArcTanh}[c + d x]) dx$$

Optimal (type 5, 162 leaves, 6 steps):

$$\begin{aligned}
& \frac{(e + f x)^{1+m} (a + b \operatorname{ArcTanh}[c + d x])}{f (1 + m)} + \frac{b d (e + f x)^{2+m} \operatorname{Hypergeometric2F1}\left[1, 2 + m, 3 + m, \frac{d (e+f x)}{d e - f - c f}\right]}{2 f (d e - (1 + c) f) (1 + m) (2 + m)} - \\
& \frac{b d (e + f x)^{2+m} \operatorname{Hypergeometric2F1}\left[1, 2 + m, 3 + m, \frac{d (e+f x)}{d e + f - c f}\right]}{2 f (d e + f - c f) (1 + m) (2 + m)}
\end{aligned}$$

Result (type 8, 20 leaves):

$$\int (e + f x)^m (a + b \operatorname{ArcTanh}[c + d x]) dx$$

Problem 53: Result is not expressed in closed-form.

$$\int \frac{\operatorname{ArcTanh}[a + b x]}{c + d x^3} dx$$

Optimal (type 4, 780 leaves, 23 steps):

$$\begin{aligned} & - \frac{\operatorname{Log}[1 - a - b x] \operatorname{Log}\left[\frac{b(c^{1/3} + d^{1/3} x)}{b c^{1/3} + (1-a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}} + \frac{\operatorname{Log}[1 + a + b x] \operatorname{Log}\left[\frac{b(c^{1/3} + d^{1/3} x)}{b c^{1/3} - (1+a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}} - \\ & \frac{(-1)^{2/3} \operatorname{Log}[1 - a - b x] \operatorname{Log}\left[\frac{b(c^{1/3} - (-1)^{1/3} d^{1/3} x)}{b c^{1/3} - (-1)^{1/3} (1-a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}} + \frac{(-1)^{2/3} \operatorname{Log}[1 + a + b x] \operatorname{Log}\left[\frac{b(c^{1/3} - (-1)^{1/3} d^{1/3} x)}{b c^{1/3} + (-1)^{1/3} (1+a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}} + \\ & \frac{(-1)^{1/3} \operatorname{Log}[1 - a - b x] \operatorname{Log}\left[\frac{b(c^{1/3} + (-1)^{2/3} d^{1/3} x)}{b c^{1/3} + (-1)^{2/3} (1-a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}} - \frac{(-1)^{1/3} \operatorname{Log}[1 + a + b x] \operatorname{Log}\left[\frac{b(c^{1/3} + (-1)^{2/3} d^{1/3} x)}{b c^{1/3} - (-1)^{2/3} (1+a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}} - \\ & \frac{\operatorname{PolyLog}\left[2, \frac{d^{1/3} (1-a-b x)}{b c^{1/3} + (1-a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}} - \frac{(-1)^{2/3} \operatorname{PolyLog}\left[2, -\frac{(-1)^{1/3} d^{1/3} (1-a-b x)}{b c^{1/3} - (-1)^{1/3} (1-a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}} + \frac{(-1)^{1/3} \operatorname{PolyLog}\left[2, \frac{(-1)^{2/3} d^{1/3} (1-a-b x)}{b c^{1/3} + (-1)^{2/3} (1-a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}} + \\ & \frac{\operatorname{PolyLog}\left[2, -\frac{d^{1/3} (1+a+b x)}{b c^{1/3} - (1+a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}} + \frac{(-1)^{2/3} \operatorname{PolyLog}\left[2, \frac{(-1)^{1/3} d^{1/3} (1+a+b x)}{b c^{1/3} + (-1)^{1/3} (1+a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}} - \frac{(-1)^{1/3} \operatorname{PolyLog}\left[2, -\frac{(-1)^{2/3} d^{1/3} (1+a+b x)}{b c^{1/3} - (-1)^{2/3} (1+a) d^{1/3}}\right]}{6 c^{2/3} d^{1/3}} \end{aligned}$$

Result (type 7, 881 leaves):

$$\begin{aligned}
& \frac{1}{6} b^2 \text{RootSum}\left[b^3 c - d - 3 a d - 3 a^2 d - a^3 d + 3 b^3 c \#1 + 3 d \#1 + 3 a d \#1 - 3 a^2 d \#1 - \right. \\
& \quad \left. 3 a^3 d \#1 + 3 b^3 c \#1^2 - 3 d \#1^2 + 3 a d \#1^2 + 3 a^2 d \#1^2 - 3 a^3 d \#1^2 + b^3 c \#1^3 + d \#1^3 - 3 a d \#1^3 + 3 a^2 d \#1^3 - a^3 d \#1^3 \ \&, \right. \\
& \quad \left. \left(i \pi \text{ArcTanh}[a + b x] + 2 \text{ArcTanh}[a + b x]^2 + 2 \text{ArcTanh}[a + b x] \text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right] - i \pi \text{Log}\left[1 + e^{2 \text{ArcTanh}[a + b x]}\right] + \right. \right. \\
& \quad \left. \left. 2 \text{ArcTanh}[a + b x] \text{Log}\left[1 - e^{-2 \left(\text{ArcTanh}[a + b x] + \text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right]}\right)}\right] + 2 \text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right] \text{Log}\left[1 - e^{-2 \left(\text{ArcTanh}[a + b x] + \text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right]}\right)}\right] + \right. \\
& \quad \left. i \pi \text{Log}\left[\frac{1}{\sqrt{1 - (a + b x)^2}}\right] - 2 \text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcTanh}[a + b x] + \text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right]\right]\right] - \text{PolyLog}\left[2, e^{-2 \left(\text{ArcTanh}[a + b x] + \text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right]}\right)}\right] \right) + \\
& \quad 2 \text{ArcTanh}[a + b x]^2 \#1 - i \pi \text{ArcTanh}[a + b x] \#1^2 - 2 \text{ArcTanh}[a + b x] \text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right] \#1^2 + i \pi \text{Log}\left[1 + e^{2 \text{ArcTanh}[a + b x]}\right] \#1^2 - \\
& \quad 2 \text{ArcTanh}[a + b x] \text{Log}\left[1 - e^{-2 \left(\text{ArcTanh}[a + b x] + \text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right]}\right)}\right] \#1^2 - 2 \text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right] \text{Log}\left[1 - e^{-2 \left(\text{ArcTanh}[a + b x] + \text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right]}\right)}\right] \#1^2 - \\
& \quad i \pi \text{Log}\left[\frac{1}{\sqrt{1 - (a + b x)^2}}\right] \#1^2 + 2 \text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcTanh}[a + b x] + \text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right]\right]\right] \#1^2 + \\
& \quad \text{PolyLog}\left[2, e^{-2 \left(\text{ArcTanh}[a + b x] + \text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right]}\right)}\right] \#1^2 - 2 e^{-\text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right]} \text{ArcTanh}[a + b x]^2 \sqrt{\frac{\#1}{(1 + \#1)^2}} - \\
& \quad \left. 4 e^{-\text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right]} \text{ArcTanh}[a + b x]^2 \#1 \sqrt{\frac{\#1}{(1 + \#1)^2}} - 2 e^{-\text{ArcTanh}\left[\frac{1 - \#1}{1 + \#1}\right]} \text{ArcTanh}[a + b x]^2 \#1^2 \sqrt{\frac{\#1}{(1 + \#1)^2}}\right) / \\
& \quad \left. (b^3 c - a d - 2 a^2 d - a^3 d + 2 b^3 c \#1 + 2 a d \#1 - 2 a^3 d \#1 + b^3 c \#1^2 - a d \#1^2 + 2 a^2 d \#1^2 - a^3 d \#1^2) \ \& \right]
\end{aligned}$$

Problem 54: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTanh}[a + b x]}{c + d x^2} dx$$

Optimal (type 4, 481 leaves, 17 steps):

$$\begin{aligned}
& - \frac{\text{Log}[1 - a - b x] \text{Log}\left[\frac{b(\sqrt{-c} - \sqrt{d} x)}{b\sqrt{-c} - (1-a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} + \frac{\text{Log}[1 + a + b x] \text{Log}\left[\frac{b(\sqrt{-c} - \sqrt{d} x)}{b\sqrt{-c} + (1+a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} + \\
& \frac{\text{Log}[1 - a - b x] \text{Log}\left[\frac{b(\sqrt{-c} + \sqrt{d} x)}{b\sqrt{-c} + (1+a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} - \frac{\text{Log}[1 + a + b x] \text{Log}\left[\frac{b(\sqrt{-c} + \sqrt{d} x)}{b\sqrt{-c} - (1+a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} - \frac{\text{PolyLog}\left[2, -\frac{\sqrt{d}(1-a-bx)}{b\sqrt{-c} - (1-a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} + \\
& \frac{\text{PolyLog}\left[2, \frac{\sqrt{d}(1-a-bx)}{b\sqrt{-c} + (1+a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} - \frac{\text{PolyLog}\left[2, -\frac{\sqrt{d}(1+a+bx)}{b\sqrt{-c} - (1+a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} + \frac{\text{PolyLog}\left[2, \frac{\sqrt{d}(1+a+bx)}{b\sqrt{-c} + (1+a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}}
\end{aligned}$$

Result (type 4, 1419 leaves):

$$\begin{aligned}
& \frac{1}{4(1-a^2)\sqrt{c}d} \\
& \left(2i\sqrt{d} \text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] - 2ia^2\sqrt{d} \text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] - 2i\sqrt{d} \text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] + \right. \\
& 2ia^2\sqrt{d} \text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] - 2b\sqrt{c} \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 + b\sqrt{c} \sqrt{\frac{b^2c + (-1+a)^2d}{b^2c}} e^{-i \text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]} \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 + \\
& ab\sqrt{c} \sqrt{\frac{b^2c + (-1+a)^2d}{b^2c}} e^{-i \text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right]} \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 + b\sqrt{c} \sqrt{\frac{b^2c + (1+a)^2d}{b^2c}} e^{-i \text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]} \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 - \\
& ab\sqrt{c} \sqrt{\frac{b^2c + (1+a)^2d}{b^2c}} e^{-i \text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right]} \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]^2 - 4(-1+a^2)\sqrt{d} \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] \text{ArcTanh}[a+bx] + \\
& 2\sqrt{d} \text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] \text{Log}\left[1 - e^{-2i\left(\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] + \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] - 2a^2\sqrt{d} \text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] \text{Log}\left[1 - e^{-2i\left(\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] + \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] + \\
& 2\sqrt{d} \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] \text{Log}\left[1 - e^{-2i\left(\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] + \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] - 2a^2\sqrt{d} \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] \text{Log}\left[1 - e^{-2i\left(\text{ArcTan}\left[\frac{(-1+a)\sqrt{d}}{b\sqrt{c}}\right] + \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] - \\
& 2\sqrt{d} \text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] \text{Log}\left[1 - e^{-2i\left(\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] + \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] + 2a^2\sqrt{d} \text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] \text{Log}\left[1 - e^{-2i\left(\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] + \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] - \\
& 2\sqrt{d} \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] \text{Log}\left[1 - e^{-2i\left(\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] + \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] + 2a^2\sqrt{d} \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] \text{Log}\left[1 - e^{-2i\left(\text{ArcTan}\left[\frac{(1+a)\sqrt{d}}{b\sqrt{c}}\right] + \text{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right]\right)}\right] -
\end{aligned}$$

$$\begin{aligned}
& 2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(-1+a) \sqrt{d}}{b \sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(-1+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right]\right] + \\
& 2 a^2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(-1+a) \sqrt{d}}{b \sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(-1+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right]\right] + \\
& 2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(1+a) \sqrt{d}}{b \sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(1+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right]\right] - \\
& 2 a^2 \sqrt{d} \operatorname{ArcTan}\left[\frac{(1+a) \sqrt{d}}{b \sqrt{c}}\right] \operatorname{Log}\left[-\operatorname{Sin}\left[\operatorname{ArcTan}\left[\frac{(1+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right]\right] - \\
& \left. i (-1+a^2) \sqrt{d} \operatorname{PolyLog}\left[2, e^{-2 i \left(\operatorname{ArcTan}\left[\frac{(-1+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right] + i (-1+a^2) \sqrt{d} \operatorname{PolyLog}\left[2, e^{-2 i \left(\operatorname{ArcTan}\left[\frac{(1+a) \sqrt{d}}{b \sqrt{c}}\right] + \operatorname{ArcTan}\left[\frac{\sqrt{d} x}{\sqrt{c}}\right]\right)}\right]\right)
\end{aligned}$$

Problem 55: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcTanh}[a + b x]}{c + d x} dx$$

Optimal (type 4, 120 leaves, 5 steps):

$$-\frac{\operatorname{ArcTanh}[a + b x] \operatorname{Log}\left[\frac{2}{1+a+b x}\right]}{d} + \frac{\operatorname{ArcTanh}[a + b x] \operatorname{Log}\left[\frac{2 b (c+d x)}{(b c+d-a d)(1+a+b x)}\right]}{d} + \frac{\operatorname{PolyLog}\left[2, 1 - \frac{2}{1+a+b x}\right]}{2 d} - \frac{\operatorname{PolyLog}\left[2, 1 - \frac{2 b (c+d x)}{(b c+d-a d)(1+a+b x)}\right]}{2 d}$$

Result (type 4, 304 leaves):

$$\begin{aligned}
& -\frac{1}{2 d} \left(\frac{1}{4} (\pi - 2 i \operatorname{ArcTanh}[a + b x])^2 - \left(\operatorname{ArcTanh}\left[\frac{b c - a d}{d}\right] + \operatorname{ArcTanh}[a + b x] \right)^2 + (i \pi + 2 \operatorname{ArcTanh}[a + b x]) \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[a + b x]}\right] - \right. \\
& 2 \left(\operatorname{ArcTanh}\left[\frac{b c - a d}{d}\right] + \operatorname{ArcTanh}[a + b x] \right) \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{b c - a d}{d}\right] + \operatorname{ArcTanh}[a + b x]\right)}\right] - (i \pi + 2 \operatorname{ArcTanh}[a + b x]) \operatorname{Log}\left[\frac{2}{\sqrt{1 - (a + b x)^2}}\right] + \\
& 2 \operatorname{ArcTanh}[a + b x] \left(\operatorname{Log}\left[\frac{1}{\sqrt{1 - (a + b x)^2}}\right] - \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{b c - a d}{d}\right] + \operatorname{ArcTanh}[a + b x]\right]\right] \right) + \\
& 2 \left(\operatorname{ArcTanh}\left[\frac{b c - a d}{d}\right] + \operatorname{ArcTanh}[a + b x] \right) \operatorname{Log}\left[2 i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{b c - a d}{d}\right] + \operatorname{ArcTanh}[a + b x]\right]\right] + \\
& \left. \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcTanh}[a + b x]}\right] + \operatorname{PolyLog}\left[2, e^{-2 \left(\operatorname{ArcTanh}\left[\frac{b c - a d}{d}\right] + \operatorname{ArcTanh}[a + b x]\right)}\right] \right)
\end{aligned}$$

Problem 56: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTanh}[a + b x]}{c + \frac{d}{x}} dx$$

Optimal (type 4, 186 leaves, 15 steps):

$$\frac{(1 - a - b x) \text{Log}[1 - a - b x]}{2 b c} + \frac{(1 + a + b x) \text{Log}[1 + a + b x]}{2 b c} - \frac{d \text{Log}[1 + a + b x] \text{Log}\left[-\frac{b(d + c x)}{c + a c - b d}\right]}{2 c^2} +$$

$$\frac{d \text{Log}[1 - a - b x] \text{Log}\left[\frac{b(d + c x)}{c - a c + b d}\right]}{2 c^2} + \frac{d \text{PolyLog}\left[2, \frac{c(1 - a - b x)}{c - a c + b d}\right]}{2 c^2} - \frac{d \text{PolyLog}\left[2, \frac{c(1 + a + b x)}{c + a c - b d}\right]}{2 c^2}$$

Result (type 4, 759 leaves):

$$\frac{1}{2 b c^2 (-a c + b d)} \left(-2 a^2 c^2 \operatorname{ArcTanh}[a + b x] + 2 a b c d \operatorname{ArcTanh}[a + b x] + i a b c d \pi \operatorname{ArcTanh}[a + b x] - i b^2 d^2 \pi \operatorname{ArcTanh}[a + b x] - 2 a b c^2 x \operatorname{ArcTanh}[a + b x] + 2 b^2 c d x \operatorname{ArcTanh}[a + b x] - 2 a b c d \operatorname{ArcTanh}\left[a - \frac{b d}{c}\right] \operatorname{ArcTanh}[a + b x] + 2 b^2 d^2 \operatorname{ArcTanh}\left[a - \frac{b d}{c}\right] \operatorname{ArcTanh}[a + b x] - b c d \operatorname{ArcTanh}[a + b x]^2 - a b c d \operatorname{ArcTanh}[a + b x]^2 + b^2 d^2 \operatorname{ArcTanh}[a + b x]^2 + b c d \sqrt{1 - a^2 + \frac{2 a b d}{c} - \frac{b^2 d^2}{c^2}} e^{\operatorname{ArcTanh}\left[a - \frac{b d}{c}\right]} \operatorname{ArcTanh}[a + b x]^2 - 2 a b c d \operatorname{ArcTanh}\left[a - \frac{b d}{c}\right] \operatorname{Log}\left[1 - e^{2 \left(\operatorname{ArcTanh}\left[a - \frac{b d}{c}\right] - \operatorname{ArcTanh}[a + b x]\right)}\right] + 2 b^2 d^2 \operatorname{ArcTanh}\left[a - \frac{b d}{c}\right] \operatorname{Log}\left[1 - e^{2 \left(\operatorname{ArcTanh}\left[a - \frac{b d}{c}\right] - \operatorname{ArcTanh}[a + b x]\right)}\right] + 2 a b c d \operatorname{ArcTanh}[a + b x] \operatorname{Log}\left[1 - e^{2 \left(\operatorname{ArcTanh}\left[a - \frac{b d}{c}\right] - \operatorname{ArcTanh}[a + b x]\right)}\right] - 2 b^2 d^2 \operatorname{ArcTanh}[a + b x] \operatorname{Log}\left[1 - e^{2 \left(\operatorname{ArcTanh}\left[a - \frac{b d}{c}\right] - \operatorname{ArcTanh}[a + b x]\right)}\right] - 2 a b c d \operatorname{ArcTanh}[a + b x] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[a + b x]}\right] + 2 b^2 d^2 \operatorname{ArcTanh}[a + b x] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[a + b x]}\right] - i a b c d \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[a + b x]}\right] + i b^2 d^2 \pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[a + b x]}\right] + 2 a c^2 \operatorname{Log}\left[\frac{1}{\sqrt{1 - (a + b x)^2}}\right] - 2 b c d \operatorname{Log}\left[\frac{1}{\sqrt{1 - (a + b x)^2}}\right] + i a b c d \pi \operatorname{Log}\left[\frac{1}{\sqrt{1 - (a + b x)^2}}\right] - i b^2 d^2 \pi \operatorname{Log}\left[\frac{1}{\sqrt{1 - (a + b x)^2}}\right] + 2 a b c d \operatorname{ArcTanh}\left[a - \frac{b d}{c}\right] \operatorname{Log}\left[-i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[a - \frac{b d}{c}\right] - \operatorname{ArcTanh}[a + b x]\right]\right] - 2 b^2 d^2 \operatorname{ArcTanh}\left[a - \frac{b d}{c}\right] \operatorname{Log}\left[-i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[a - \frac{b d}{c}\right] - \operatorname{ArcTanh}[a + b x]\right]\right] + b d (-a c + b d) \operatorname{PolyLog}\left[2, e^{2 \left(\operatorname{ArcTanh}\left[a - \frac{b d}{c}\right] - \operatorname{ArcTanh}[a + b x]\right)}\right] + b d (a c - b d) \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcTanh}[a + b x]}\right] \right)$$

Problem 57: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcTanh}[a + b x]}{c + \frac{d}{x^2}} dx$$

Optimal (type 4, 545 leaves, 25 steps):

$$\begin{aligned}
& \frac{(1-a-bx) \operatorname{Log}[1-a-bx]}{2bc} + \frac{(1+a+bx) \operatorname{Log}[1+a+bx]}{2bc} + \frac{\sqrt{d} \operatorname{Log}[1-a-bx] \operatorname{Log}\left[-\frac{b(\sqrt{d}-\sqrt{-c}x)}{(1-a)\sqrt{-c}-b\sqrt{d}}\right]}{4(-c)^{3/2}} - \\
& \frac{\sqrt{d} \operatorname{Log}[1+a+bx] \operatorname{Log}\left[\frac{b(\sqrt{d}-\sqrt{-c}x)}{(1+a)\sqrt{-c}+b\sqrt{d}}\right]}{4(-c)^{3/2}} + \frac{\sqrt{d} \operatorname{Log}[1+a+bx] \operatorname{Log}\left[-\frac{b(\sqrt{d}+\sqrt{-c}x)}{(1+a)\sqrt{-c}-b\sqrt{d}}\right]}{4(-c)^{3/2}} - \frac{\sqrt{d} \operatorname{Log}[1-a-bx] \operatorname{Log}\left[\frac{b(\sqrt{d}+\sqrt{-c}x)}{(1-a)\sqrt{-c}+b\sqrt{d}}\right]}{4(-c)^{3/2}} + \\
& \frac{\sqrt{d} \operatorname{PolyLog}\left[2, \frac{\sqrt{-c}(1-a-bx)}{\sqrt{-c}-a\sqrt{-c}-b\sqrt{d}}\right]}{4(-c)^{3/2}} - \frac{\sqrt{d} \operatorname{PolyLog}\left[2, \frac{\sqrt{-c}(1-a-bx)}{(1-a)\sqrt{-c}+b\sqrt{d}}\right]}{4(-c)^{3/2}} + \frac{\sqrt{d} \operatorname{PolyLog}\left[2, \frac{\sqrt{-c}(1+a+bx)}{(1+a)\sqrt{-c}-b\sqrt{d}}\right]}{4(-c)^{3/2}} - \frac{\sqrt{d} \operatorname{PolyLog}\left[2, \frac{\sqrt{-c}(1+a+bx)}{(1+a)\sqrt{-c}+b\sqrt{d}}\right]}{4(-c)^{3/2}}
\end{aligned}$$

Result (type 4, 1458 leaves):

Problem 58: Result is not expressed in closed-form.

$$\int \frac{\text{ArcTanh}[a + b x]}{c + \frac{d}{x^3}} dx$$

Optimal (type 4, 832 leaves, 31 steps):

$$\begin{aligned} & \frac{(1-a-bx) \text{Log}[1-a-bx]}{2bc} + \frac{(1+a+bx) \text{Log}[1+a+bx]}{2bc} - \frac{d^{1/3} \text{Log}[1+a+bx] \text{Log}\left[-\frac{b(d^{1/3}+c^{1/3}x)}{(1+a)c^{1/3}-bd^{1/3}}\right]}{6c^{4/3}} + \frac{d^{1/3} \text{Log}[1-a-bx] \text{Log}\left[\frac{b(d^{1/3}+c^{1/3}x)}{(1-a)c^{1/3}+bd^{1/3}}\right]}{6c^{4/3}} + \\ & \frac{(-1)^{2/3} d^{1/3} \text{Log}[1-a-bx] \text{Log}\left[-\frac{b(d^{1/3}-(-1)^{1/3}c^{1/3}x)}{(-1)^{1/3}(1-a)c^{1/3}-bd^{1/3}}\right]}{6c^{4/3}} - \frac{(-1)^{2/3} d^{1/3} \text{Log}[1+a+bx] \text{Log}\left[\frac{b(d^{1/3}-(-1)^{1/3}c^{1/3}x)}{(-1)^{1/3}(1+a)c^{1/3}+bd^{1/3}}\right]}{6c^{4/3}} + \\ & \frac{(-1)^{1/3} d^{1/3} \text{Log}[1+a+bx] \text{Log}\left[-\frac{b(d^{1/3}+(-1)^{2/3}c^{1/3}x)}{(-1)^{2/3}(1+a)c^{1/3}-bd^{1/3}}\right]}{6c^{4/3}} - \frac{(-1)^{1/3} d^{1/3} \text{Log}[1-a-bx] \text{Log}\left[\frac{b(d^{1/3}+(-1)^{2/3}c^{1/3}x)}{(-1)^{2/3}(1-a)c^{1/3}+bd^{1/3}}\right]}{6c^{4/3}} + \\ & \frac{(-1)^{2/3} d^{1/3} \text{PolyLog}\left[2, \frac{(-1)^{1/3}c^{1/3}(1-a-bx)}{(-1)^{1/3}(1-a)c^{1/3}-bd^{1/3}}\right]}{6c^{4/3}} + \frac{d^{1/3} \text{PolyLog}\left[2, \frac{c^{1/3}(1-a-bx)}{(1-a)c^{1/3}+bd^{1/3}}\right]}{6c^{4/3}} - \frac{(-1)^{1/3} d^{1/3} \text{PolyLog}\left[2, \frac{(-1)^{2/3}c^{1/3}(1-a-bx)}{(-1)^{2/3}(1-a)c^{1/3}+bd^{1/3}}\right]}{6c^{4/3}} - \\ & \frac{d^{1/3} \text{PolyLog}\left[2, \frac{c^{1/3}(1+a+bx)}{(1+a)c^{1/3}-bd^{1/3}}\right]}{6c^{4/3}} + \frac{(-1)^{1/3} d^{1/3} \text{PolyLog}\left[2, \frac{(-1)^{2/3}c^{1/3}(1+a+bx)}{(-1)^{2/3}(1+a)c^{1/3}-bd^{1/3}}\right]}{6c^{4/3}} - \frac{(-1)^{2/3} d^{1/3} \text{PolyLog}\left[2, \frac{(-1)^{1/3}c^{1/3}(1+a+bx)}{(-1)^{1/3}(1+a)c^{1/3}+bd^{1/3}}\right]}{6c^{4/3}} \end{aligned}$$

Result (type 7, 917 leaves):

$$\begin{aligned}
& -\frac{1}{6bc} \\
& \left(-6(a+bx) \operatorname{ArcTanh}[a+bx] + 6 \operatorname{Log}\left[\frac{1}{\sqrt{1-(a+bx)^2}}\right] + b^3 d \operatorname{RootSum}\left[c + 3ac + 3a^2c + a^3c - b^3d - 3c\#1 - 3ac\#1 + 3a^2c\#1 + 3a^3c\#1 - 3b^3d\#1 + \right. \right. \\
& \quad \left. \left. 3c\#1^2 - 3ac\#1^2 - 3a^2c\#1^2 + 3a^3c\#1^2 - 3b^3d\#1^2 - c\#1^3 + 3ac\#1^3 - 3a^2c\#1^3 + a^3c\#1^3 - b^3d\#1^3 \&, \right. \right. \\
& \quad \left. \left. -i\pi \operatorname{ArcTanh}[a+bx] - 2 \operatorname{ArcTanh}[a+bx]^2 - 2 \operatorname{ArcTanh}[a+bx] \operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right] + i\pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[a+bx]}\right] - 2 \operatorname{ArcTanh}[a+bx] \right. \right. \\
& \quad \left. \left. \operatorname{Log}\left[1 - e^{-2(\operatorname{ArcTanh}[a+bx] + \operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right])}\right] - 2 \operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right] \operatorname{Log}\left[1 - e^{-2(\operatorname{ArcTanh}[a+bx] + \operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right])}\right] - i\pi \operatorname{Log}\left[\frac{1}{\sqrt{1-(a+bx)^2}}\right] + \right. \right. \\
& \quad \left. \left. 2 \operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}[a+bx] + \operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right]\right]\right] + \operatorname{PolyLog}\left[2, e^{-2(\operatorname{ArcTanh}[a+bx] + \operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right])}\right] - \right. \right. \\
& \quad \left. \left. 2 \operatorname{ArcTanh}[a+bx]^2 \#1 + i\pi \operatorname{ArcTanh}[a+bx] \#1^2 + 2 \operatorname{ArcTanh}[a+bx] \operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right] \#1^2 - i\pi \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[a+bx]}\right] \#1^2 + \right. \right. \\
& \quad \left. \left. 2 \operatorname{ArcTanh}[a+bx] \operatorname{Log}\left[1 - e^{-2(\operatorname{ArcTanh}[a+bx] + \operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right])}\right] \#1^2 + 2 \operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right] \operatorname{Log}\left[1 - e^{-2(\operatorname{ArcTanh}[a+bx] + \operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right])}\right] \#1^2 + \right. \right. \\
& \quad \left. \left. i\pi \operatorname{Log}\left[\frac{1}{\sqrt{1-(a+bx)^2}}\right] \#1^2 - 2 \operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}[a+bx] + \operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right]\right]\right] \#1^2 - \right. \right. \\
& \quad \left. \left. \operatorname{PolyLog}\left[2, e^{-2(\operatorname{ArcTanh}[a+bx] + \operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right])}\right] \#1^2 + 2 e^{-\operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right]} \operatorname{ArcTanh}[a+bx]^2 \sqrt{\frac{\#1}{(1+\#1)^2}} + \right. \right. \\
& \quad \left. \left. 4 e^{-\operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right]} \operatorname{ArcTanh}[a+bx]^2 \#1 \sqrt{\frac{\#1}{(1+\#1)^2}} + 2 e^{-\operatorname{ArcTanh}\left[\frac{1-\#1}{1+\#1}\right]} \operatorname{ArcTanh}[a+bx]^2 \#1^2 \sqrt{\frac{\#1}{(1+\#1)^2}} \right) / \right. \\
& \quad \left. (ac + 2a^2c + a^3c - b^3d - 2ac\#1 + 2a^3c\#1 - 2b^3d\#1 + ac\#1^2 - 2a^2c\#1^2 + a^3c\#1^2 - b^3d\#1^2) \& \right)
\end{aligned}$$

Problem 59: Unable to integrate problem.

$$\int \frac{\operatorname{ArcTanh}[a+bx]}{c+d\sqrt{x}} dx$$

Optimal (type 4, 585 leaves, 31 steps):

$$\begin{aligned}
& \frac{2\sqrt{1+a} \operatorname{ArcTan}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right]}{\sqrt{b}d} - \frac{2\sqrt{1-a} \operatorname{ArcTanh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right]}{\sqrt{b}d} + \frac{c \operatorname{Log}\left[\frac{d(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{b}c+\sqrt{-1-a}d}\right] \operatorname{Log}[c+d\sqrt{x}]}{d^2} - \\
& \frac{c \operatorname{Log}\left[\frac{d(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{b}c+\sqrt{-1-a}d}\right] \operatorname{Log}[c+d\sqrt{x}]}{d^2} + \frac{c \operatorname{Log}\left[-\frac{d(\sqrt{-1-a}+\sqrt{b}\sqrt{x})}{\sqrt{b}c-\sqrt{-1-a}d}\right] \operatorname{Log}[c+d\sqrt{x}]}{d^2} - \frac{c \operatorname{Log}\left[-\frac{d(\sqrt{-1-a}+\sqrt{b}\sqrt{x})}{\sqrt{b}c-\sqrt{-1-a}d}\right] \operatorname{Log}[c+d\sqrt{x}]}{d^2} - \\
& \frac{\sqrt{x} \operatorname{Log}[1-a-bx]}{d} + \frac{c \operatorname{Log}[c+d\sqrt{x}] \operatorname{Log}[1-a-bx]}{d^2} + \frac{\sqrt{x} \operatorname{Log}[1+a+bx]}{d} - \frac{c \operatorname{Log}[c+d\sqrt{x}] \operatorname{Log}[1+a+bx]}{d^2} + \\
& \frac{c \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-1-a}d}\right]}{d^2} + \frac{c \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-1-a}d}\right]}{d^2} - \frac{c \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-1-a}d}\right]}{d^2} - \frac{c \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-1-a}d}\right]}{d^2}
\end{aligned}$$

Result (type 8, 20 leaves):

$$\int \frac{\operatorname{ArcTanh}[a+bx]}{c+d\sqrt{x}} dx$$

Problem 60: Attempted integration timed out after 120 seconds.

$$\int \frac{\operatorname{ArcTanh}[a+bx]}{c+\frac{d}{\sqrt{x}}} dx$$

Optimal (type 4, 661 leaves, 37 steps):

$$\begin{aligned}
& -\frac{2\sqrt{1+a}d \operatorname{ArcTan}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right]}{\sqrt{b}c^2} + \frac{2\sqrt{1-a}d \operatorname{ArcTanh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right]}{\sqrt{b}c^2} - \frac{d^2 \operatorname{Log}\left[\frac{c(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right] \operatorname{Log}[d+c\sqrt{x}]}{c^3} + \frac{d^2 \operatorname{Log}\left[\frac{c(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right] \operatorname{Log}[d+c\sqrt{x}]}{c^3} - \\
& \frac{d^2 \operatorname{Log}\left[\frac{c(\sqrt{-1-a}+\sqrt{b}\sqrt{x})}{\sqrt{-1-a}c-\sqrt{b}d}\right] \operatorname{Log}[d+c\sqrt{x}]}{c^3} + \frac{d^2 \operatorname{Log}\left[\frac{c(\sqrt{-1-a}+\sqrt{b}\sqrt{x})}{\sqrt{-1-a}c-\sqrt{b}d}\right] \operatorname{Log}[d+c\sqrt{x}]}{c^3} + \frac{d\sqrt{x} \operatorname{Log}[1-a-bx]}{c^2} + \frac{(1-a-bx) \operatorname{Log}[1-a-bx]}{2bc} - \\
& \frac{d^2 \operatorname{Log}[d+c\sqrt{x}] \operatorname{Log}[1-a-bx]}{c^3} - \frac{d\sqrt{x} \operatorname{Log}[1+a+bx]}{c^2} + \frac{(1+a+bx) \operatorname{Log}[1+a+bx]}{2bc} + \frac{d^2 \operatorname{Log}[d+c\sqrt{x}] \operatorname{Log}[1+a+bx]}{c^3} - \\
& \frac{d^2 \operatorname{PolyLog}\left[2, -\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-1-a}c-\sqrt{b}d}\right]}{c^3} + \frac{d^2 \operatorname{PolyLog}\left[2, -\frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-1-a}c-\sqrt{b}d}\right]}{c^3} - \frac{d^2 \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right]}{c^3} + \frac{d^2 \operatorname{PolyLog}\left[2, \frac{\sqrt{b}(d+c\sqrt{x})}{\sqrt{-1-a}c+\sqrt{b}d}\right]}{c^3}
\end{aligned}$$

Result (type 1, 1 leaves):

???

Problem 61: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTanh}[d + e x]}{a + b x + c x^2} dx$$

Optimal (type 4, 335 leaves, 12 steps):

$$\frac{\text{ArcTanh}[d + e x] \text{Log}\left[\frac{2 e (b - \sqrt{b^2 - 4 a c} + 2 c x)}{(2 c (1 - d) + (b - \sqrt{b^2 - 4 a c}) e) (1 + d + e x)}\right] - \text{ArcTanh}[d + e x] \text{Log}\left[\frac{2 e (b + \sqrt{b^2 - 4 a c} + 2 c x)}{(2 c (1 - d) + (b + \sqrt{b^2 - 4 a c}) e) (1 + d + e x)}\right]}{\sqrt{b^2 - 4 a c}} - \frac{\text{PolyLog}\left[2, 1 + \frac{2 (2 c d - (b - \sqrt{b^2 - 4 a c}) e - 2 c (d + e x))}{(2 c - 2 c d + b e - \sqrt{b^2 - 4 a c}) e} (1 + d + e x)\right]}{2 \sqrt{b^2 - 4 a c}} + \frac{\text{PolyLog}\left[2, 1 + \frac{2 (2 c d - (b + \sqrt{b^2 - 4 a c}) e - 2 c (d + e x))}{(2 c (1 - d) + (b + \sqrt{b^2 - 4 a c}) e) (1 + d + e x)}\right]}{2 \sqrt{b^2 - 4 a c}}$$

Result (type 4, 8801 leaves):

$$\frac{1}{e (a + b x + c x^2)} (a e + b e x + c e x^2)$$

$$\left(-\frac{2 \text{ArcTanh}[d + e x] \text{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c} e}\right]}{\sqrt{b^2 - 4 a c}} - \frac{1}{c (-1 + (d + e x)^2)} e \left(-1 + \frac{(2 c d - b e + \sqrt{b^2 - 4 a c} e \left(\frac{b}{\sqrt{b^2 - 4 a c}} - \frac{2 c d}{\sqrt{b^2 - 4 a c} e} + \frac{2 c (d + e x)}{\sqrt{b^2 - 4 a c} e} \right))^2}{4 c^2} \right) \right)$$

$$\left(\frac{2 c^2 \text{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c} e}\right]^2}{4 c^2 (-1 + d^2) - 4 b c d e + b^2 e^2} + \frac{1}{(b^2 - 4 a c) (2 c - 2 c d + b e) \sqrt{\frac{(b^2 - 4 a c) e^2 - (2 c (-1 + d) - b e)^2}{(b^2 - 4 a c) e^2}}}$$

$$2 a c^2 \left(-e^{-\text{ArcTanh}\left[\frac{2 c (-1 + d) - b e}{\sqrt{b^2 - 4 a c} e}\right]} \text{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c} e}\right]^2 + \frac{1}{\sqrt{b^2 - 4 a c} e \sqrt{1 - \frac{(2 c (-1 + d) - b e)^2}{(b^2 - 4 a c) e^2}}} i (2 c (-1 + d) - b e) \right)$$

$$\begin{aligned}
& \left(- \left(-\pi + 2 i \operatorname{ArcTanh} \left[\frac{2 c (-1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] \right) \operatorname{ArcTanh} \left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] - \pi \operatorname{Log} \left[1 + e^{\frac{2 \operatorname{ArcTanh} \left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right]}{\sqrt{b^2 - 4 a c e}}} \right] - \right. \\
& 2 \left(i \operatorname{ArcTanh} \left[\frac{2 c (-1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + i \operatorname{ArcTanh} \left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right) \operatorname{Log} \left[1 - e^{-2 \left(\operatorname{ArcTanh} \left[\frac{2 c (-1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)} \right] + \\
& \pi \operatorname{Log} \left[\frac{1}{\sqrt{1 - \left(\frac{b}{\sqrt{b^2 - 4 a c}} - \frac{2 c d}{\sqrt{b^2 - 4 a c e}} + \frac{2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right)^2}} \right] + 2 i \operatorname{ArcTanh} \left[\frac{2 c (-1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] \operatorname{Log} \left[i \operatorname{Sinh} \left[\operatorname{ArcTanh} \left[\frac{2 c (-1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] \right] + \right. \\
& \left. \left. \operatorname{ArcTanh} \left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right] \right] + i \operatorname{PolyLog} \left[2, e^{-2 \left(\operatorname{ArcTanh} \left[\frac{2 c (-1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + \operatorname{ArcTanh} \left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)} \right] \right) + \\
& \frac{1}{(b^2 - 4 a c) e^2 (2 c - 2 c d + b e) \sqrt{\frac{(b^2 - 4 a c) e^2 - (2 c (-1+d) - b e)^2}{(b^2 - 4 a c) e^2}}} 2 c^3 \left(- e^{-\operatorname{ArcTanh} \left[\frac{2 c (-1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right]} \operatorname{ArcTanh} \left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right)^2 + \\
& \frac{1}{\sqrt{b^2 - 4 a c e} \sqrt{1 - \frac{(2 c (-1+d) - b e)^2}{(b^2 - 4 a c) e^2}}} i (2 c (-1+d) - b e) \left(- \left(-\pi + 2 i \operatorname{ArcTanh} \left[\frac{2 c (-1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] \right) \operatorname{ArcTanh} \left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] - \right. \\
& \left. \pi \operatorname{Log} \left[1 + e^{\frac{2 \operatorname{ArcTanh} \left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right]}{\sqrt{b^2 - 4 a c e}}} \right] - 2 \left(i \operatorname{ArcTanh} \left[\frac{2 c (-1+d) - b e}{\sqrt{b^2 - 4 a c e}} \right] + i \operatorname{ArcTanh} \left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
 & \text{Log}\left[1 - e^{-2\left(\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}e}\right] + \text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}e}\right]\right)}\right] + \pi \text{Log}\left[\frac{1}{\sqrt{1 - \left(\frac{b}{\sqrt{b^2-4ac}} - \frac{2cd}{\sqrt{b^2-4ac}e} + \frac{2c(d+ex)}{\sqrt{b^2-4ac}e}\right)^2}}\right] + \\
 & 2i \text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}e}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}e}\right] + \text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}e}\right]\right]\right] + \\
 & i \text{PolyLog}\left[2, e^{-2\left(\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}e}\right] + \text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}e}\right]\right)}\right] - \\
 & \frac{1}{(b^2-4ac)e^2(2c-2cd+be)\sqrt{\frac{(b^2-4ac)e^2-(2c(-1+d)-be)^2}{(b^2-4ac)e^2}}} 4c^3d \left(-e^{-\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}e}\right]} \text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}e}\right] \right)^2 + \\
 & \frac{1}{\sqrt{b^2-4ac}e\sqrt{1 - \frac{(2c(-1+d)-be)^2}{(b^2-4ac)e^2}}} i(2c(-1+d)-be) \left(-\left(-\pi + 2i \text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}e}\right]\right) \text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}e}\right] \right) - \\
 & \pi \text{Log}\left[1 + e^{2\text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}e}\right]}\right] - 2\left(i \text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}e}\right] + i \text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}e}\right]\right) \\
 & \text{Log}\left[1 - e^{-2\left(\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}e}\right] + \text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}e}\right]\right)}\right] + \pi \text{Log}\left[\frac{1}{\sqrt{1 - \left(\frac{b}{\sqrt{b^2-4ac}} - \frac{2cd}{\sqrt{b^2-4ac}e} + \frac{2c(d+ex)}{\sqrt{b^2-4ac}e}\right)^2}}\right] + \\
 & 2i \text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}e}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcTanh}\left[\frac{2c(-1+d)-be}{\sqrt{b^2-4ac}e}\right] + \text{ArcTanh}\left[\frac{-2cd+be+2c(d+ex)}{\sqrt{b^2-4ac}e}\right]\right]\right] +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{(b^2 - 4ac)e(2c - 2cd + be) \sqrt{\frac{(b^2 - 4ac)e^2 - (2c(-1+d) - be)^2}{(b^2 - 4ac)e^2}}} 2bc^2 \left(-e^{-\text{ArcTanh}\left[\frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e}\right]} \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right]^2 + \right. \\
 & \frac{1}{\sqrt{b^2 - 4ac}e} \sqrt{1 - \frac{(2c(-1+d) - be)^2}{(b^2 - 4ac)e^2}} i(2c(-1+d) - be) \left(-\left(-\pi + 2i \text{ArcTanh}\left[\frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e}\right]\right) \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right] - \right. \\
 & \left. \pi \text{Log}\left[1 + e^{\frac{2 \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right]}{\sqrt{b^2 - 4ac}e}}\right] - 2 \left(i \text{ArcTanh}\left[\frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e}\right] + i \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right] \right) \right) \\
 & \left. \text{Log}\left[1 - e^{-2 \left(\text{ArcTanh}\left[\frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e}\right] + \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right] \right)}\right] + \pi \text{Log}\left[\frac{1}{\sqrt{1 - \left(\frac{b}{\sqrt{b^2 - 4ac}} - \frac{2cd}{\sqrt{b^2 - 4ac}e} + \frac{2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right)^2}}\right] + \right. \\
 & \left. 2i \text{ArcTanh}\left[\frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcTanh}\left[\frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e}\right] + \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right]\right]\right] + \right. \\
 & \left. i \text{PolyLog}\left[2, e^{-2 \left(\text{ArcTanh}\left[\frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e}\right] + \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right] \right)}\right] \right) \left. \right) \\
 & \frac{1}{(b^2 - 4ac)e(2c - 2cd + be) \sqrt{\frac{(b^2 - 4ac)e^2 - (2c(-1+d) - be)^2}{(b^2 - 4ac)e^2}}} 2bc^2 d \left(-e^{-\text{ArcTanh}\left[\frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac}e}\right]} \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right]^2 + \right.
 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{b^2 - 4ac} e \sqrt{1 - \frac{(2c(-1+d) - be)^2}{(b^2 - 4ac)e^2}}} i (2c(-1+d) - be) \left(- \left(-\pi + 2i \operatorname{ArcTanh} \left[\frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac} e} \right] \right) \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right] - \right. \\
& \left. \pi \operatorname{Log} \left[1 + e^{\frac{2 \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right]}{\sqrt{b^2 - 4ac} e}} \right] - 2 \left(i \operatorname{ArcTanh} \left[\frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac} e} \right] + i \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right] \right) \right. \\
& \left. \operatorname{Log} \left[1 - e^{-2 \left(\operatorname{ArcTanh} \left[\frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac} e} \right] + \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right] \right)} \right] + \pi \operatorname{Log} \left[\frac{1}{\sqrt{1 - \left(\frac{b}{\sqrt{b^2 - 4ac}} - \frac{2cd}{\sqrt{b^2 - 4ac} e} + \frac{2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right)^2}} \right] + \right. \\
& \left. 2i \operatorname{ArcTanh} \left[\frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac} e} \right] \operatorname{Log} \left[i \operatorname{Sinh} \left[\operatorname{ArcTanh} \left[\frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac} e} \right] + \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right] \right] \right] + \right. \\
& \left. i \operatorname{PolyLog} \left[2, e^{-2 \left(\operatorname{ArcTanh} \left[\frac{2c(-1+d) - be}{\sqrt{b^2 - 4ac} e} \right] + \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right] \right)} \right] \right) \left. \right) - \\
& \frac{1}{(b^2 - 4ac)(-2c - 2cd + be) \sqrt{\frac{(b^2 - 4ac)e^2 - (2c(1+d) - be)^2}{(b^2 - 4ac)e^2}}} 2ac^2 \left(-e^{-\operatorname{ArcTanh} \left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac} e} \right]} \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right] \right)^2 + \\
& \frac{1}{\sqrt{b^2 - 4ac} e \sqrt{1 - \frac{(2c(1+d) - be)^2}{(b^2 - 4ac)e^2}}} i (2c(1+d) - be) \left(- \left(-\pi + 2i \operatorname{ArcTanh} \left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac} e} \right] \right) \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right] - \right.
\end{aligned}$$

$$\begin{aligned}
& \pi \operatorname{Log}\left[1 + e^{\frac{2 \operatorname{ArcTanh}\left[\frac{-2 c d - b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}}\right]}{1}}\right] - 2 \left(i \operatorname{ArcTanh}\left[\frac{2 c (1 + d) - b e}{\sqrt{b^2 - 4 a c e}}\right] + i \operatorname{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}}\right] \right) \\
& \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{2 c (1 + d) - b e}{\sqrt{b^2 - 4 a c e}}\right] + \operatorname{ArcTanh}\left[\frac{-2 c d - b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}}\right] \right)}\right] + \pi \operatorname{Log}\left[\frac{1}{\sqrt{1 - \left(\frac{b}{\sqrt{b^2 - 4 a c}} - \frac{2 c d}{\sqrt{b^2 - 4 a c e}} + \frac{2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right)^2}}\right] + \\
& 2 i \operatorname{ArcTanh}\left[\frac{2 c (1 + d) - b e}{\sqrt{b^2 - 4 a c e}}\right] \operatorname{Log}\left[i \operatorname{Sinh}\left[\operatorname{ArcTanh}\left[\frac{2 c (1 + d) - b e}{\sqrt{b^2 - 4 a c e}}\right] + \operatorname{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}}\right] \right] \right] + \\
& i \operatorname{PolyLog}\left[2, e^{-2 \left(\operatorname{ArcTanh}\left[\frac{2 c (1 + d) - b e}{\sqrt{b^2 - 4 a c e}}\right] + \operatorname{ArcTanh}\left[\frac{-2 c d - b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}}\right] \right)}\right] \left. \right) - \\
& \frac{1}{(b^2 - 4 a c) e^2 (-2 c - 2 c d + b e) \sqrt{\frac{(b^2 - 4 a c) e^2 - (2 c (1 + d) - b e)^2}{(b^2 - 4 a c) e^2}}} 2 c^3 \left(-e^{-\operatorname{ArcTanh}\left[\frac{2 c (1 + d) - b e}{\sqrt{b^2 - 4 a c e}}\right]} \operatorname{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}}\right]^2 + \right. \\
& \left. \frac{1}{\sqrt{b^2 - 4 a c e} \sqrt{1 - \frac{(2 c (1 + d) - b e)^2}{(b^2 - 4 a c) e^2}}} i (2 c (1 + d) - b e) \left(-\left(-\pi + 2 i \operatorname{ArcTanh}\left[\frac{2 c (1 + d) - b e}{\sqrt{b^2 - 4 a c e}}\right] \right) \operatorname{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}}\right] - \right. \right. \\
& \left. \left. \pi \operatorname{Log}\left[1 + e^{\frac{2 \operatorname{ArcTanh}\left[\frac{-2 c d - b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}}\right]}{1}}\right] - 2 \left(i \operatorname{ArcTanh}\left[\frac{2 c (1 + d) - b e}{\sqrt{b^2 - 4 a c e}}\right] + i \operatorname{ArcTanh}\left[\frac{-2 c d + b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}}\right] \right) \right. \right. \\
& \left. \left. \operatorname{Log}\left[1 - e^{-2 \left(\operatorname{ArcTanh}\left[\frac{2 c (1 + d) - b e}{\sqrt{b^2 - 4 a c e}}\right] + \operatorname{ArcTanh}\left[\frac{-2 c d - b e + 2 c (d + e x)}{\sqrt{b^2 - 4 a c e}}\right] \right)}\right] + \pi \operatorname{Log}\left[\frac{1}{\sqrt{1 - \left(\frac{b}{\sqrt{b^2 - 4 a c}} - \frac{2 c d}{\sqrt{b^2 - 4 a c e}} + \frac{2 c (d + e x)}{\sqrt{b^2 - 4 a c e}} \right)^2}}\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \operatorname{Im} \operatorname{ArcTanh} \left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac} e} \right] \operatorname{Log} \left[\operatorname{Im} \operatorname{Sinh} \left[\operatorname{ArcTanh} \left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac} e} \right] + \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right] \right] \right] + \\
& \operatorname{Im} \operatorname{PolyLog} \left[2, e^{-2 \left(\operatorname{ArcTanh} \left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac} e} \right] + \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right] \right)} \right] \right) - \\
& \frac{1}{(b^2 - 4ac) e^2 (-2c - 2cd + be) \sqrt{\frac{(b^2 - 4ac) e^2 - (2c(1+d) - be)^2}{(b^2 - 4ac) e^2}}} 4c^3 d \left(-e^{-\operatorname{ArcTanh} \left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac} e} \right]} \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right] \right)^2 + \\
& \frac{1}{\sqrt{b^2 - 4ac} e} \sqrt{1 - \frac{(2c(1+d) - be)^2}{(b^2 - 4ac) e^2}} \operatorname{Im} (2c(1+d) - be) \left(-\left(-\pi + 2 \operatorname{Im} \operatorname{ArcTanh} \left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac} e} \right] \right) \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right] - \right. \\
& \left. \pi \operatorname{Log} \left[1 + e^{2 \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right]} \right] - 2 \left(\operatorname{Im} \operatorname{ArcTanh} \left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac} e} \right] + \operatorname{Im} \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right] \right) \right) \\
& \operatorname{Log} \left[1 - e^{-2 \left(\operatorname{ArcTanh} \left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac} e} \right] + \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right] \right)} \right] + \pi \operatorname{Log} \left[\frac{1}{\sqrt{1 - \left(\frac{b}{\sqrt{b^2 - 4ac}} - \frac{2cd}{\sqrt{b^2 - 4ac} e} + \frac{2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right)^2}} \right] + \\
& 2 \operatorname{Im} \operatorname{ArcTanh} \left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac} e} \right] \operatorname{Log} \left[\operatorname{Im} \operatorname{Sinh} \left[\operatorname{ArcTanh} \left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac} e} \right] + \operatorname{ArcTanh} \left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac} e} \right] \right] \right] +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{(b^2 - 4ac)e(-2c - 2cd + be)\sqrt{\frac{(b^2 - 4ac)e^2 - (2c(1+d) - be)^2}{(b^2 - 4ac)e^2}}} 2bc^2 \left(-e^{-\text{ArcTanh}\left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e}\right]} \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right]^2 + \right. \\
& \frac{1}{\sqrt{b^2 - 4ac}e}\sqrt{1 - \frac{(2c(1+d) - be)^2}{(b^2 - 4ac)e^2}} i(2c(1+d) - be) \left(-\left(-\pi + 2i \text{ArcTanh}\left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e}\right]\right) \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right] - \right. \\
& \pi \text{Log}\left[1 + e^{\frac{2 \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right]}{2}}\right] - 2 \left(i \text{ArcTanh}\left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e}\right] + i \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right] \right) \\
& \text{Log}\left[1 - e^{-2 \left(\text{ArcTanh}\left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e}\right] + \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right] \right)}\right] + \pi \text{Log}\left[\frac{1}{\sqrt{1 - \left(\frac{b}{\sqrt{b^2 - 4ac}} - \frac{2cd}{\sqrt{b^2 - 4ac}e} + \frac{2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right)^2}}\right] + \\
& 2i \text{ArcTanh}\left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e}\right] \text{Log}\left[i \text{Sinh}\left[\text{ArcTanh}\left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e}\right] + \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right]\right]\right] + \\
& i \text{PolyLog}\left[2, e^{-2 \left(\text{ArcTanh}\left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e}\right] + \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right] \right)}\right] \left. \right) + \\
& \frac{1}{(b^2 - 4ac)e(-2c - 2cd + be)\sqrt{\frac{(b^2 - 4ac)e^2 - (2c(1+d) - be)^2}{(b^2 - 4ac)e^2}}} 2bc^2 d \left(-e^{-\text{ArcTanh}\left[\frac{2c(1+d) - be}{\sqrt{b^2 - 4ac}e}\right]} \text{ArcTanh}\left[\frac{-2cd + be + 2c(d+ex)}{\sqrt{b^2 - 4ac}e}\right]^2 + \right.
\end{aligned}$$

Problem 47: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{-2 \operatorname{ArcTanh}[a x]}}{x} dx$$

Optimal (type 3, 11 leaves, 3 steps):

$$\operatorname{Log}[x] - 2 \operatorname{Log}[1 + a x]$$

Result (type 3, 25 leaves):

$$\operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[a x]}\right] + \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[a x]}\right]$$

Problem 60: Unable to integrate problem.

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} x^m dx$$

Optimal (type 6, 31 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, \frac{1}{4}, -\frac{1}{4}, 2+m, a x, -a x\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} x^m dx$$

Problem 61: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} x^2 dx$$

Optimal (type 3, 282 leaves, 15 steps):

$$\begin{aligned} & -\frac{3(1-ax)^{3/4}(1+ax)^{1/4}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} + \\ & \frac{3 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^3} - \frac{3 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^3} - \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{16\sqrt{2}a^3} + \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{16\sqrt{2}a^3} \end{aligned}$$

Result (type 7, 93 leaves):

$$\frac{-\frac{8 e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} (9+6 e^{2 \operatorname{ArcTanh}[a x]}+29 e^{4 \operatorname{ArcTanh}[a x]})}{(1+e^{2 \operatorname{ArcTanh}[a x]})^3} - 9 \operatorname{RootSum}\left[1+\#1^4 \&, \frac{\operatorname{ArcTanh}[a x]-2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}-\#1\right]}{\#1^3} \&\right]}{96 a^3}$$

Problem 62: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} x \, dx$$

Optimal (type 3, 255 leaves, 14 steps):

$$\begin{aligned} & -\frac{(1-ax)^{3/4}(1+ax)^{1/4}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{4\sqrt{2}a^2} - \\ & \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{4\sqrt{2}a^2} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^2} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^2} \end{aligned}$$

Result (type 7, 83 leaves):

$$\frac{-\frac{8 e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} (1+5 e^{2 \operatorname{ArcTanh}[a x]})}{(1+e^{2 \operatorname{ArcTanh}[a x]})^2} + \operatorname{RootSum}\left[1+\#1^4 \&, \frac{-\operatorname{ArcTanh}[a x]+2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}-\#1\right]}{\#1^3} \&\right]}{16 a^2}$$

Problem 63: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} \, dx$$

Optimal (type 3, 222 leaves, 13 steps):

$$\begin{aligned} & -\frac{(1-ax)^{3/4}(1+ax)^{1/4}}{a} + \frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}a} - \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}a} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{2\sqrt{2}a} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{2\sqrt{2}a} \end{aligned}$$

Result (type 7, 71 leaves):

$$\frac{-\frac{8 e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}}{1+e^{2 \operatorname{ArcTanh}[a x]}} + \operatorname{RootSum}\left[1+\#1^4 \&, \frac{-\operatorname{ArcTanh}[a x]+2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}-\#1\right]}{\#1^3} \&\right]}{4 a}$$

Problem 64: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}}{x} dx$$

Optimal (type 3, 227 leaves, 17 steps):

$$\begin{aligned} & -2 \operatorname{ArcTan}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right] - \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right] - \\ & 2 \operatorname{ArcTanh}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}} \end{aligned}$$

Result (type 7, 87 leaves):

$$-2 \operatorname{ArcTan}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] + \operatorname{Log}\left[1 - e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] - \operatorname{Log}\left[1 + e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] + \frac{1}{2} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{-\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right]$$

Problem 70: Unable to integrate problem.

$$\int e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} x^m dx$$

Optimal (type 6, 31 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, \frac{3}{4}, -\frac{3}{4}, 2+m, a x, -a x\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} x^m dx$$

Problem 71: Result is not expressed in closed-form.

$$\int e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} x^3 dx$$

Optimal (type 3, 290 leaves, 15 steps):

$$\begin{aligned}
& - \frac{41 (1 - a x)^{1/4} (1 + a x)^{3/4}}{64 a^4} - \frac{x^2 (1 - a x)^{1/4} (1 + a x)^{7/4}}{4 a^2} - \frac{(1 - a x)^{1/4} (1 + a x)^{7/4} (11 + 4 a x)}{32 a^4} + \\
& \frac{123 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a^4} - \frac{123 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a^4} + \frac{123 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a^4} - \frac{123 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a^4}
\end{aligned}$$

Result (type 7, 103 leaves):

$$\begin{aligned}
& \frac{1}{256 a^4} \\
& \left(- \frac{8 e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} (41 + 183 e^{2 \operatorname{ArcTanh}[a x]} + 147 e^{4 \operatorname{ArcTanh}[a x]} + 133 e^{6 \operatorname{ArcTanh}[a x]})}{(1 + e^{2 \operatorname{ArcTanh}[a x]})^4} - 123 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \& \right)
\end{aligned}$$

Problem 72: Result is not expressed in closed-form.

$$\int e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} x^2 dx$$

Optimal (type 3, 282 leaves, 15 steps):

$$\begin{aligned}
& - \frac{17 (1 - a x)^{1/4} (1 + a x)^{3/4}}{24 a^3} - \frac{(1 - a x)^{1/4} (1 + a x)^{7/4}}{4 a^3} - \frac{x (1 - a x)^{1/4} (1 + a x)^{7/4}}{3 a^2} + \frac{17 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^3} - \\
& \frac{17 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^3} + \frac{17 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{16 \sqrt{2} a^3} - \frac{17 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{16 \sqrt{2} a^3}
\end{aligned}$$

Result (type 7, 93 leaves):

$$\begin{aligned}
& - \frac{8 e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} (17 + 30 e^{2 \operatorname{ArcTanh}[a x]} + 45 e^{4 \operatorname{ArcTanh}[a x]})}{(1 + e^{2 \operatorname{ArcTanh}[a x]})^3} - 51 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \& \right) \\
& \frac{1}{96 a^3}
\end{aligned}$$

Problem 73: Result is not expressed in closed-form.

$$\int e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} x dx$$

Optimal (type 3, 255 leaves, 14 steps):

$$\begin{aligned}
& - \frac{3(1-ax)^{1/4}(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{1/4}(1+ax)^{7/4}}{2a^2} + \frac{9 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{4\sqrt{2}a^2} \\
& \frac{9 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{4\sqrt{2}a^2} + \frac{9 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^2} - \frac{9 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^2}
\end{aligned}$$

Result (type 7, 84 leaves):

$$\frac{-\frac{e^{\frac{3}{2}\operatorname{ArcTanh}[ax]}(3+7e^{2\operatorname{ArcTanh}[ax]})}{2(1+e^{2\operatorname{ArcTanh}[ax]})^2} - \frac{9}{16} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[ax] - 2 \operatorname{Log}\left[e^{\frac{1}{2}\operatorname{ArcTanh}[ax]} - \#1\right]}{\#1}\right] \&]}{a^2}$$

Problem 74: Result is not expressed in closed-form.

$$\int e^{\frac{3}{2}\operatorname{ArcTanh}[ax]} dx$$

Optimal (type 3, 223 leaves, 13 steps):

$$\begin{aligned}
& - \frac{(1-ax)^{1/4}(1+ax)^{3/4}}{a} + \frac{3 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}a} - \frac{3 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}a} + \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{2\sqrt{2}a} - \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{2\sqrt{2}a}
\end{aligned}$$

Result (type 7, 72 leaves):

$$\frac{-\frac{2e^{\frac{3}{2}\operatorname{ArcTanh}[ax]}}{a(1+e^{2\operatorname{ArcTanh}[ax]})} - \frac{3 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[ax] - 2 \operatorname{Log}\left[e^{\frac{1}{2}\operatorname{ArcTanh}[ax]} - \#1\right]}{\#1}\right] \&]}{4a}$$

Problem 75: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{3}{2}\operatorname{ArcTanh}[ax]}}{x} dx$$

Optimal (type 3, 227 leaves, 17 steps):

$$\begin{aligned}
& 2 \operatorname{ArcTan}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right] - \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right] - \\
& 2 \operatorname{ArcTanh}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}}
\end{aligned}$$

Result (type 7, 87 leaves):

$$2 \operatorname{ArcTan}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] + \operatorname{Log}\left[1 - e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] - \operatorname{Log}\left[1 + e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] + \frac{1}{2} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{-\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1}\right]$$

Problem 80: Unable to integrate problem.

$$\int e^{\frac{5}{2} \operatorname{ArcTanh}[a x]} x^m dx$$

Optimal (type 6, 31 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, \frac{5}{4}, -\frac{5}{4}, 2+m, a x, -a x\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{\frac{5}{2} \operatorname{ArcTanh}[a x]} x^m dx$$

Problem 81: Result is not expressed in closed-form.

$$\int e^{\frac{5}{2} \operatorname{ArcTanh}[a x]} x^3 dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$\frac{475 (1-a x)^{3/4} (1+a x)^{1/4}}{64 a^4} + \frac{4 x^3 (1+a x)^{5/4}}{a (1-a x)^{1/4}} + \frac{17 x^2 (1-a x)^{3/4} (1+a x)^{5/4}}{4 a^2} + \frac{(1-a x)^{3/4} (1+a x)^{5/4} (521+452 a x)}{96 a^4} -$$

$$\frac{475 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{64 \sqrt{2} a^4} + \frac{475 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{64 \sqrt{2} a^4} + \frac{475 \operatorname{Log}\left[1 + \frac{\sqrt{1-a x}}{\sqrt{1+a x}} - \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{128 \sqrt{2} a^4} - \frac{475 \operatorname{Log}\left[1 + \frac{\sqrt{1-a x}}{\sqrt{1+a x}} + \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{128 \sqrt{2} a^4}$$

Result (type 7, 114 leaves):

$$\frac{1}{a^4} \left(\frac{e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} (1425 + 5415 e^{2 \operatorname{ArcTanh}[a x]} + 7483 e^{4 \operatorname{ArcTanh}[a x]} + 4645 e^{6 \operatorname{ArcTanh}[a x]} + 768 e^{8 \operatorname{ArcTanh}[a x]})}{96 (1 + e^{2 \operatorname{ArcTanh}[a x]})^4} + \right.$$

$$\left. \frac{475}{256} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3}\right] \&\right)$$

Problem 82: Result is not expressed in closed-form.

$$\int e^{\frac{5}{2} \operatorname{ArcTanh}[a x]} x^2 dx$$

Optimal (type 3, 305 leaves, 16 steps):

$$\frac{55 (1 - a x)^{3/4} (1 + a x)^{1/4}}{8 a^3} + \frac{11 (1 - a x)^{3/4} (1 + a x)^{5/4}}{4 a^3} + \frac{2 (1 + a x)^{9/4}}{a^3 (1 - a x)^{1/4}} + \frac{(1 - a x)^{3/4} (1 + a x)^{9/4}}{3 a^3} -$$

$$\frac{55 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^3} + \frac{55 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^3} + \frac{55 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{16 \sqrt{2} a^3} - \frac{55 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{16 \sqrt{2} a^3}$$

Result (type 7, 104 leaves):

$$\frac{1}{a^3} \left(\frac{e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} (165 + 462 e^{2 \operatorname{ArcTanh}[a x]} + 425 e^{4 \operatorname{ArcTanh}[a x]} + 96 e^{6 \operatorname{ArcTanh}[a x]})}{12 (1 + e^{2 \operatorname{ArcTanh}[a x]})^3} + \frac{55}{32} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right] \right)$$

Problem 83: Result is not expressed in closed-form.

$$\int e^{\frac{5}{2} \operatorname{ArcTanh}[a x]} x dx$$

Optimal (type 3, 279 leaves, 15 steps):

$$\frac{25 (1 - a x)^{3/4} (1 + a x)^{1/4}}{4 a^2} + \frac{5 (1 - a x)^{3/4} (1 + a x)^{5/4}}{2 a^2} + \frac{2 (1 + a x)^{9/4}}{a^2 (1 - a x)^{1/4}} - \frac{25 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a^2} +$$

$$\frac{25 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a^2} + \frac{25 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^2} - \frac{25 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^2}$$

Result (type 7, 94 leaves):

$$\frac{e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} (25 + 45 e^{2 \operatorname{ArcTanh}[a x]} + 16 e^{4 \operatorname{ArcTanh}[a x]})}{2 (1 + e^{2 \operatorname{ArcTanh}[a x]})^2} + \frac{25}{16} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right]$$

a^2

Problem 84: Result is not expressed in closed-form.

$$\int e^{\frac{5}{2} \operatorname{ArcTanh}[a x]} dx$$

Optimal (type 3, 247 leaves, 14 steps):

$$\frac{5(1-ax)^{3/4}(1+ax)^{1/4}}{a} + \frac{4(1+ax)^{5/4}}{a(1-ax)^{1/4}} - \frac{5 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}a} +$$

$$\frac{5 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}a} + \frac{5 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{2\sqrt{2}a} - \frac{5 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{2\sqrt{2}a}$$

Result (type 7, 83 leaves):

$$\frac{8 e^{\frac{1}{2} \operatorname{ArcTanh}[ax]} (5 + 4 e^{2 \operatorname{ArcTanh}[ax]})}{1 + e^{2 \operatorname{ArcTanh}[ax]}} + 5 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[ax] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[ax]} - \#1\right]}{\#1^3} \&\right]$$

4 a

Problem 85: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{5}{2} \operatorname{ArcTanh}[ax]}}{x} dx$$

Optimal (type 3, 248 leaves, 19 steps):

$$\frac{8(1+ax)^{1/4}}{(1-ax)^{1/4}} - 2 \operatorname{ArcTan}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] - \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right] +$$

$$\sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right] - 2 \operatorname{ArcTanh}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 97 leaves):

$$8 e^{\frac{1}{2} \operatorname{ArcTanh}[ax]} - 2 \operatorname{ArcTan}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[ax]}\right] + \operatorname{Log}\left[1 - e^{\frac{1}{2} \operatorname{ArcTanh}[ax]}\right] -$$

$$\operatorname{Log}\left[1 + e^{\frac{1}{2} \operatorname{ArcTanh}[ax]}\right] + \frac{1}{2} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[ax] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[ax]} - \#1\right]}{\#1^3} \&\right]$$

Problem 90: Unable to integrate problem.

$$\int e^{-\frac{1}{2} \operatorname{ArcTanh}[ax]} x^m dx$$

Optimal (type 6, 31 leaves, 2 steps):

$$\frac{x^{1+m} \text{AppellF1}\left[1+m, -\frac{1}{4}, \frac{1}{4}, 2+m, ax, -ax\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{-\frac{1}{2} \text{ArcTanh}[ax]} x^m dx$$

Problem 91: Result is not expressed in closed-form.

$$\int e^{-\frac{1}{2} \text{ArcTanh}[ax]} x^3 dx$$

Optimal (type 3, 290 leaves, 15 steps):

$$\begin{aligned} & -\frac{11(1-ax)^{1/4}(1+ax)^{3/4}}{64a^4} - \frac{x^2(1-ax)^{5/4}(1+ax)^{3/4}}{4a^2} - \frac{(25-4ax)(1-ax)^{5/4}(1+ax)^{3/4}}{96a^4} \\ & - \frac{11 \text{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{64\sqrt{2}a^4} + \frac{11 \text{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{64\sqrt{2}a^4} \\ & - \frac{11 \text{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{128\sqrt{2}a^4} + \frac{11 \text{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{128\sqrt{2}a^4} \end{aligned}$$

Result (type 7, 103 leaves):

$$\begin{aligned} & \frac{1}{768a^4} \\ & \left(-\frac{8e^{\frac{3}{2}\text{ArcTanh}[ax]}(245 + 107e^{2\text{ArcTanh}[ax]} + 279e^{4\text{ArcTanh}[ax]} + 33e^{6\text{ArcTanh}[ax]})}{(1 + e^{2\text{ArcTanh}[ax]})^4} + 33 \text{RootSum}\left[1 + \#1^4, \frac{\text{ArcTanh}[ax] + 2 \text{Log}\left[e^{-\frac{1}{2}\text{ArcTanh}[ax]} - \#1\right]}{\#1^3}\right] \& \right) \end{aligned}$$

Problem 92: Result is not expressed in closed-form.

$$\int e^{-\frac{1}{2} \text{ArcTanh}[ax]} x^2 dx$$

Optimal (type 3, 282 leaves, 15 steps):

$$\begin{aligned} & \frac{3(1-ax)^{1/4}(1+ax)^{3/4}}{8a^3} + \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{12a^3} - \frac{x(1-ax)^{5/4}(1+ax)^{3/4}}{3a^2} + \\ & \frac{3 \text{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^3} - \frac{3 \text{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^3} \\ & + \frac{3 \text{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{16\sqrt{2}a^3} - \frac{3 \text{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{16\sqrt{2}a^3} \end{aligned}$$

Result (type 7, 93 leaves):

$$\frac{8 e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} (29 + 6 e^{2 \operatorname{ArcTanh}[a x]} + 9 e^{4 \operatorname{ArcTanh}[a x]}) - 9 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right]}{(1 + e^{2 \operatorname{ArcTanh}[a x]})^3} \quad 96 a^3$$

Problem 93: Result is not expressed in closed-form.

$$\int e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} x \, dx$$

Optimal (type 3, 255 leaves, 14 steps):

$$\begin{aligned} & - \frac{(1 - a x)^{1/4} (1 + a x)^{3/4}}{4 a^2} - \frac{(1 - a x)^{5/4} (1 + a x)^{3/4}}{2 a^2} - \frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a^2} + \\ & \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a^2} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^2} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^2} \end{aligned}$$

Result (type 7, 79 leaves):

$$\frac{8 e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} (5 + e^{2 \operatorname{ArcTanh}[a x]}) - \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right]}{(1 + e^{2 \operatorname{ArcTanh}[a x]})^2} \quad 16 a^2$$

Problem 94: Result is not expressed in closed-form.

$$\int e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} \, dx$$

Optimal (type 3, 221 leaves, 13 steps):

$$\begin{aligned} & \frac{(1 - a x)^{1/4} (1 + a x)^{3/4}}{a} + \frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2} a} - \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2} a} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{2 \sqrt{2} a} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{2 \sqrt{2} a} \end{aligned}$$

Result (type 7, 69 leaves):

$$\frac{8 e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} + \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right]}{1 + e^{2 \operatorname{ArcTanh}[a x]}} \quad 4 a$$

Problem 95: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]}}{x} dx$$

Optimal (type 3, 227 leaves, 17 steps):

$$2 \operatorname{ArcTan}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] - \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right] -$$

$$2 \operatorname{ArcTanh}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 85 leaves):

$$-2 \operatorname{ArcTan}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] + \operatorname{Log}\left[1 - e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] - \operatorname{Log}\left[1 + e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] + \frac{1}{2} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right]$$

Problem 100: Unable to integrate problem.

$$\int e^{-\frac{3}{2} \operatorname{ArcTanh}[a x]} x^m dx$$

Optimal (type 6, 31 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, -\frac{3}{4}, \frac{3}{4}, 2+m, a x, -a x\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{-\frac{3}{2} \operatorname{ArcTanh}[a x]} x^m dx$$

Problem 101: Result is not expressed in closed-form.

$$\int e^{-\frac{3}{2} \operatorname{ArcTanh}[a x]} x^3 dx$$

Optimal (type 3, 290 leaves, 15 steps):

$$\begin{aligned}
& - \frac{41 (1 - a x)^{3/4} (1 + a x)^{1/4}}{64 a^4} - \frac{x^2 (1 - a x)^{7/4} (1 + a x)^{1/4}}{4 a^2} - \frac{(11 - 4 a x) (1 - a x)^{7/4} (1 + a x)^{1/4}}{32 a^4} - \\
& \frac{123 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a^4} + \frac{123 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a^4} + \frac{123 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a^4} - \frac{123 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a^4}
\end{aligned}$$

Result (type 7, 103 leaves):

$$\begin{aligned}
& \frac{1}{256 a^4} \\
& \left(- \frac{8 e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} (133 + 147 e^{2 \operatorname{ArcTanh}[a x]} + 183 e^{4 \operatorname{ArcTanh}[a x]} + 41 e^{6 \operatorname{ArcTanh}[a x]})}{(1 + e^{2 \operatorname{ArcTanh}[a x]})^4} + 123 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \& \right)
\end{aligned}$$

Problem 102: Result is not expressed in closed-form.

$$\int e^{-\frac{3}{2} \operatorname{ArcTanh}[a x]} x^2 dx$$

Optimal (type 3, 282 leaves, 15 steps):

$$\begin{aligned}
& \frac{17 (1 - a x)^{3/4} (1 + a x)^{1/4}}{24 a^3} + \frac{(1 - a x)^{7/4} (1 + a x)^{1/4}}{4 a^3} - \frac{x (1 - a x)^{7/4} (1 + a x)^{1/4}}{3 a^2} + \frac{17 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^3} - \\
& \frac{17 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^3} - \frac{17 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{16 \sqrt{2} a^3} + \frac{17 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{16 \sqrt{2} a^3}
\end{aligned}$$

Result (type 7, 93 leaves):

$$\frac{8 e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} (45 + 30 e^{2 \operatorname{ArcTanh}[a x]} + 17 e^{4 \operatorname{ArcTanh}[a x]})}{(1 + e^{2 \operatorname{ArcTanh}[a x]})^3} - 51 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \& \right]$$

96 a³

Problem 103: Result is not expressed in closed-form.

$$\int e^{-\frac{3}{2} \operatorname{ArcTanh}[a x]} x dx$$

Optimal (type 3, 255 leaves, 14 steps):

$$\begin{aligned}
& - \frac{3 (1 - a x)^{3/4} (1 + a x)^{1/4}}{4 a^2} - \frac{(1 - a x)^{7/4} (1 + a x)^{1/4}}{2 a^2} - \frac{9 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a^2} + \\
& \frac{9 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a^2} + \frac{9 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^2} - \frac{9 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^2}
\end{aligned}$$

Result (type 7, 84 leaves):

$$\frac{-\frac{e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} (7 + 3 e^{2 \operatorname{ArcTanh}[a x]})}{2 (1 + e^{2 \operatorname{ArcTanh}[a x]})^2} + \frac{9}{16} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1} \&\right]}{a^2}$$

Problem 104: Result is not expressed in closed-form.

$$\int e^{-\frac{3}{2} \operatorname{ArcTanh}[a x]} dx$$

Optimal (type 3, 222 leaves, 13 steps):

$$\frac{(1 - a x)^{3/4} (1 + a x)^{1/4}}{a} + \frac{3 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2} a} - \frac{3 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2} a} - \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{2 \sqrt{2} a} + \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{2 \sqrt{2} a}$$

Result (type 7, 72 leaves):

$$\frac{2 e^{-\frac{3}{2} \operatorname{ArcTanh}[a x]}}{a (1 + e^{-2 \operatorname{ArcTanh}[a x]})} - \frac{3 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1} \&\right]}{4 a}$$

Problem 105: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{3}{2} \operatorname{ArcTanh}[a x]}}{x} dx$$

Optimal (type 3, 227 leaves, 17 steps):

$$\begin{aligned}
& -2 \operatorname{ArcTan}\left[\frac{(1 + a x)^{1/4}}{(1 - a x)^{1/4}}\right] - \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right] - \\
& 2 \operatorname{ArcTanh}\left[\frac{(1 + a x)^{1/4}}{(1 - a x)^{1/4}}\right] + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2}} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2}}
\end{aligned}$$

Result (type 7, 85 leaves):

$$2 \operatorname{ArcTan}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] + \operatorname{Log}\left[1 - e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] - \operatorname{Log}\left[1 + e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] + \frac{1}{2} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \&]$$

Problem 110: Unable to integrate problem.

$$\int e^{-\frac{5}{2} \operatorname{ArcTanh}[a x]} x^m dx$$

Optimal (type 6, 31 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, -\frac{5}{4}, \frac{5}{4}, 2+m, a x, -a x\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{-\frac{5}{2} \operatorname{ArcTanh}[a x]} x^m dx$$

Problem 111: Result is not expressed in closed-form.

$$\int e^{-\frac{5}{2} \operatorname{ArcTanh}[a x]} x^3 dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$\begin{aligned} & -\frac{4 x^3 (1-a x)^{5/4}}{a (1+a x)^{1/4}} + \frac{475 (1-a x)^{1/4} (1+a x)^{3/4}}{64 a^4} + \frac{17 x^2 (1-a x)^{5/4} (1+a x)^{3/4}}{4 a^2} + \frac{(521-452 a x) (1-a x)^{5/4} (1+a x)^{3/4}}{96 a^4} + \\ & \frac{475 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{64 \sqrt{2} a^4} - \frac{475 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{64 \sqrt{2} a^4} + \frac{475 \operatorname{Log}\left[1 + \frac{\sqrt{1-a x}}{\sqrt{1+a x}} - \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{128 \sqrt{2} a^4} - \frac{475 \operatorname{Log}\left[1 + \frac{\sqrt{1-a x}}{\sqrt{1+a x}} + \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{128 \sqrt{2} a^4} \end{aligned}$$

Result (type 7, 114 leaves):

$$\frac{1}{a^4} \left(\frac{e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} (768 + 4645 e^{2 \operatorname{ArcTanh}[a x]} + 7483 e^{4 \operatorname{ArcTanh}[a x]} + 5415 e^{6 \operatorname{ArcTanh}[a x]} + 1425 e^{8 \operatorname{ArcTanh}[a x]})}{96 (1 + e^{2 \operatorname{ArcTanh}[a x]})^4} - \frac{475 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3}\right] \&}{256} \right)$$

Problem 112: Result is not expressed in closed-form.

$$\int e^{-\frac{5}{2} \operatorname{ArcTanh}[a x]} x^2 dx$$

Optimal (type 3, 305 leaves, 16 steps):

$$\begin{aligned} & -\frac{2(1-ax)^{9/4}}{a^3(1+ax)^{1/4}} - \frac{55(1-ax)^{1/4}(1+ax)^{3/4}}{8a^3} - \frac{11(1-ax)^{5/4}(1+ax)^{3/4}}{4a^3} - \frac{(1-ax)^{9/4}(1+ax)^{3/4}}{3a^3} \\ & + \frac{55 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^3} + \frac{55 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^3} - \frac{55 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{16\sqrt{2}a^3} + \frac{55 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{16\sqrt{2}a^3} \end{aligned}$$

Result (type 7, 104 leaves):

$$\frac{1}{a^3} \left(-\frac{e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} (96 + 425 e^{2 \operatorname{ArcTanh}[a x]} + 462 e^{4 \operatorname{ArcTanh}[a x]} + 165 e^{6 \operatorname{ArcTanh}[a x]})}{12 (1 + e^{2 \operatorname{ArcTanh}[a x]})^3} + \frac{55}{32} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right] \right)$$

Problem 113: Result is not expressed in closed-form.

$$\int e^{-\frac{5}{2} \operatorname{ArcTanh}[a x]} x dx$$

Optimal (type 3, 279 leaves, 15 steps):

$$\begin{aligned} & \frac{2(1-ax)^{9/4}}{a^2(1+ax)^{1/4}} + \frac{25(1-ax)^{1/4}(1+ax)^{3/4}}{4a^2} + \frac{5(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} + \frac{25 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{4\sqrt{2}a^2} \\ & + \frac{25 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{4\sqrt{2}a^2} + \frac{25 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^2} - \frac{25 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^2} \end{aligned}$$

Result (type 7, 94 leaves):

$$\frac{e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} (16 + 45 e^{2 \operatorname{ArcTanh}[a x]} + 25 e^{4 \operatorname{ArcTanh}[a x]})}{2 (1 + e^{2 \operatorname{ArcTanh}[a x]})^2} - \frac{25}{16} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right]}{a^2}$$

Problem 114: Result is not expressed in closed-form.

$$\int e^{-\frac{5}{2} \operatorname{ArcTanh}[a x]} dx$$

Optimal (type 3, 247 leaves, 14 steps):

$$\begin{aligned} & -\frac{4(1-ax)^{5/4}}{a(1+ax)^{1/4}} - \frac{5(1-ax)^{1/4}(1+ax)^{3/4}}{a} - \frac{5 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}a} + \\ & \frac{5 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}a} - \frac{5 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{2\sqrt{2}a} + \frac{5 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{2\sqrt{2}a} \end{aligned}$$

Result (type 7, 83 leaves):

$$\frac{-\frac{8e^{-\frac{1}{2}\operatorname{ArcTanh}[ax]}(4+5e^{2\operatorname{ArcTanh}[ax]})}{1+e^{2\operatorname{ArcTanh}[ax]}} + 5 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[ax] + 2 \operatorname{Log}\left[e^{-\frac{1}{2}\operatorname{ArcTanh}[ax]} - \#1\right]}{\#1^3} \&\right]}{4a}$$

Problem 115: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{5}{2}\operatorname{ArcTanh}[ax]}}{x} dx$$

Optimal (type 3, 248 leaves, 19 steps):

$$\begin{aligned} & \frac{8(1-ax)^{1/4}}{(1+ax)^{1/4}} + 2 \operatorname{ArcTan}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right] - \\ & \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right] - 2 \operatorname{ArcTanh}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}} \end{aligned}$$

Result (type 7, 99 leaves):

$$\begin{aligned} & 8e^{-\frac{1}{2}\operatorname{ArcTanh}[ax]} - 2 \operatorname{ArcTan}\left[e^{-\frac{1}{2}\operatorname{ArcTanh}[ax]}\right] + \operatorname{Log}\left[1 - e^{-\frac{1}{2}\operatorname{ArcTanh}[ax]}\right] - \\ & \operatorname{Log}\left[1 + e^{-\frac{1}{2}\operatorname{ArcTanh}[ax]}\right] + \frac{1}{2} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{-\operatorname{ArcTanh}[ax] - 2 \operatorname{Log}\left[e^{-\frac{1}{2}\operatorname{ArcTanh}[ax]} - \#1\right]}{\#1^3} \&\right] \end{aligned}$$

Problem 120: Unable to integrate problem.

$$\int e^{\frac{\operatorname{ArcTanh}[x]}{3}} x^m dx$$

Optimal (type 6, 28 leaves, 2 steps):

$$\frac{x^{1+m} \text{AppellF1}\left[1+m, \frac{1}{6}, -\frac{1}{6}, 2+m, x, -x\right]}{1+m}$$

Result (type 8, 14 leaves):

$$\int e^{\frac{\text{ArcTanh}[x]}{3}} x^m dx$$

Problem 121: Result is not expressed in closed-form.

$$\int e^{\frac{\text{ArcTanh}[x]}{3}} x^2 dx$$

Optimal (type 3, 245 leaves, 16 steps):

$$\begin{aligned} & -\frac{19}{54} (1-x)^{5/6} (1+x)^{1/6} - \frac{1}{18} (1-x)^{5/6} (1+x)^{7/6} - \frac{1}{3} (1-x)^{5/6} x (1+x)^{7/6} - \frac{19}{81} \text{ArcTan}\left[\frac{(1-x)^{1/6}}{(1+x)^{1/6}}\right] + \\ & \frac{19}{162} \text{ArcTan}\left[\sqrt{3} - \frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right] - \frac{19}{162} \text{ArcTan}\left[\sqrt{3} + \frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right] - \frac{19 \text{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} - \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{108\sqrt{3}} + \frac{19 \text{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} + \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{108\sqrt{3}} \end{aligned}$$

Result (type 7, 133 leaves):

$$\begin{aligned} & \frac{1}{486} \left(-\frac{18 e^{\frac{\text{ArcTanh}[x]}{3}} (19 + 8 e^{2 \text{ArcTanh}[x]} + 61 e^{4 \text{ArcTanh}[x]})}{(1 + e^{2 \text{ArcTanh}[x]})^3} + 114 \text{ArcTan}\left[e^{\frac{\text{ArcTanh}[x]}{3}}\right] + \right. \\ & \left. 19 \text{RootSum}\left[1 - \#1^2 + \#1^4, \frac{-2 \text{ArcTanh}[x] + 6 \text{Log}\left[e^{\frac{\text{ArcTanh}[x]}{3}} - \#1\right] + \text{ArcTanh}[x] \#1^2 - 3 \text{Log}\left[e^{\frac{\text{ArcTanh}[x]}{3}} - \#1\right] \#1^2}{-\#1 + 2 \#1^3} \&\right] \right) \end{aligned}$$

Problem 122: Result is not expressed in closed-form.

$$\int e^{\frac{\text{ArcTanh}[x]}{3}} x dx$$

Optimal (type 3, 224 leaves, 15 steps):

$$\begin{aligned} & -\frac{1}{6} (1-x)^{5/6} (1+x)^{1/6} - \frac{1}{2} (1-x)^{5/6} (1+x)^{7/6} - \frac{1}{9} \text{ArcTan}\left[\frac{(1-x)^{1/6}}{(1+x)^{1/6}}\right] + \frac{1}{18} \text{ArcTan}\left[\sqrt{3} - \frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right] - \\ & \frac{1}{18} \text{ArcTan}\left[\sqrt{3} + \frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right] - \frac{\text{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} - \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{12\sqrt{3}} + \frac{\text{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} + \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{12\sqrt{3}} \end{aligned}$$

Result (type 7, 127 leaves):

$$\frac{1}{9} \left(-\frac{3 e^{\frac{\text{ArcTanh}[x]}{3}} (1 + 7 e^{2 \text{ArcTanh}[x]})}{(1 + e^{2 \text{ArcTanh}[x]})^2} + \text{ArcTan} \left[e^{\frac{\text{ArcTanh}[x]}{3}} \right] \right) -$$

$$\frac{1}{54} \text{RootSum} \left[1 - \#1^2 + \#1^4 \&, \frac{2 \text{ArcTanh}[x] - 6 \text{Log} \left[e^{\frac{\text{ArcTanh}[x]}{3}} - \#1 \right] - \text{ArcTanh}[x] \#1^2 + 3 \text{Log} \left[e^{\frac{\text{ArcTanh}[x]}{3}} - \#1 \right] \#1^2}{-\#1 + 2 \#1^3} \& \right]$$

Problem 123: Result is not expressed in closed-form.

$$\int e^{\frac{\text{ArcTanh}[x]}{3}} dx$$

Optimal (type 3, 202 leaves, 14 steps):

$$-(1-x)^{5/6} (1+x)^{1/6} - \frac{2}{3} \text{ArcTan} \left[\frac{(1-x)^{1/6}}{(1+x)^{1/6}} \right] + \frac{1}{3} \text{ArcTan} \left[\sqrt{3} - \frac{2(1-x)^{1/6}}{(1+x)^{1/6}} \right] -$$

$$\frac{1}{3} \text{ArcTan} \left[\sqrt{3} + \frac{2(1-x)^{1/6}}{(1+x)^{1/6}} \right] - \frac{\text{Log} \left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} - \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}} \right]}{2\sqrt{3}} + \frac{\text{Log} \left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} + \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}} \right]}{2\sqrt{3}}$$

Result (type 7, 116 leaves):

$$-\frac{2 e^{\frac{\text{ArcTanh}[x]}{3}}}{1 + e^{2 \text{ArcTanh}[x]}} + \frac{2}{3} \text{ArcTan} \left[e^{\frac{\text{ArcTanh}[x]}{3}} \right] - \frac{1}{9} \text{RootSum} \left[1 - \#1^2 + \#1^4 \&, \frac{2 \text{ArcTanh}[x] - 6 \text{Log} \left[e^{\frac{\text{ArcTanh}[x]}{3}} - \#1 \right] - \text{ArcTanh}[x] \#1^2 + 3 \text{Log} \left[e^{\frac{\text{ArcTanh}[x]}{3}} - \#1 \right] \#1^2}{-\#1 + 2 \#1^3} \& \right]$$

Problem 124: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{\text{ArcTanh}[x]}{3}}}{x} dx$$

Optimal (type 3, 346 leaves, 25 steps):

$$\begin{aligned}
& -2 \operatorname{ArcTan}\left[\frac{(1-x)^{1/6}}{(1+x)^{1/6}}\right] + \operatorname{ArcTan}\left[\sqrt{3} - \frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right] - \operatorname{ArcTan}\left[\sqrt{3} + \frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right] + \sqrt{3} \operatorname{ArcTan}\left[\frac{1 - \frac{2(1+x)^{1/6}}{(1-x)^{1/6}}}{\sqrt{3}}\right] - \\
& \sqrt{3} \operatorname{ArcTan}\left[\frac{1 + \frac{2(1+x)^{1/6}}{(1-x)^{1/6}}}{\sqrt{3}}\right] - 2 \operatorname{ArcTanh}\left[\frac{(1+x)^{1/6}}{(1-x)^{1/6}}\right] - \frac{1}{2} \sqrt{3} \operatorname{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} - \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right] + \\
& \frac{1}{2} \sqrt{3} \operatorname{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} + \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right] + \frac{1}{2} \operatorname{Log}\left[1 - \frac{(1+x)^{1/6}}{(1-x)^{1/6}} + \frac{(1+x)^{1/3}}{(1-x)^{1/3}}\right] - \frac{1}{2} \operatorname{Log}\left[1 + \frac{(1+x)^{1/6}}{(1-x)^{1/6}} + \frac{(1+x)^{1/3}}{(1-x)^{1/3}}\right]
\end{aligned}$$

Result (type 7, 220 leaves):

$$\begin{aligned}
& 2 \operatorname{ArcTan}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}}\right] - \sqrt{3} \operatorname{ArcTan}\left[\frac{-1 + 2e^{\frac{\operatorname{ArcTanh}[x]}{3}}}{\sqrt{3}}\right] - \sqrt{3} \operatorname{ArcTan}\left[\frac{1 + 2e^{\frac{\operatorname{ArcTanh}[x]}{3}}}{\sqrt{3}}\right] + \operatorname{Log}\left[1 - e^{\frac{\operatorname{ArcTanh}[x]}{3}}\right] - \operatorname{Log}\left[1 + e^{\frac{\operatorname{ArcTanh}[x]}{3}}\right] + \frac{1}{2} \operatorname{Log}\left[1 - e^{\frac{\operatorname{ArcTanh}[x]}{3}} + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}\right] - \\
& \frac{1}{2} \operatorname{Log}\left[1 + e^{\frac{\operatorname{ArcTanh}[x]}{3}} + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}\right] - \frac{1}{3} \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \frac{2 \operatorname{ArcTanh}[x] - 6 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] - \operatorname{ArcTanh}[x] \#1^2 + 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] \#1^2}{-\#1 + 2 \#1^3} \&\right]
\end{aligned}$$

Problem 127: Unable to integrate problem.

$$\int e^{\frac{2 \operatorname{ArcTanh}[x]}{3}} x^m dx$$

Optimal (type 6, 28 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, \frac{1}{3}, -\frac{1}{3}, 2+m, x, -x\right]}{1+m}$$

Result (type 8, 14 leaves):

$$\int e^{\frac{2 \operatorname{ArcTanh}[x]}{3}} x^m dx$$

Problem 128: Result is not expressed in closed-form.

$$\int e^{\frac{2 \operatorname{ArcTanh}[x]}{3}} x^2 dx$$

Optimal (type 3, 133 leaves, 5 steps):

$$-\frac{11}{27} (1-x)^{2/3} (1+x)^{1/3} - \frac{1}{9} (1-x)^{2/3} (1+x)^{4/3} - \frac{1}{3} (1-x)^{2/3} x (1+x)^{4/3} + \frac{22 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2(1-x)^{1/3}}{\sqrt{3}(1+x)^{1/3}}\right]}{27\sqrt{3}} + \frac{11}{81} \operatorname{Log}[1+x] + \frac{11}{27} \operatorname{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}}\right]$$

Result (type 7, 154 leaves):

$$\frac{2}{243} \left(-\frac{324 e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}}{(1 + e^{2 \operatorname{ArcTanh}[x]})^3} + \frac{540 e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}}{(1 + e^{2 \operatorname{ArcTanh}[x]})^2} - \frac{315 e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}}{1 + e^{2 \operatorname{ArcTanh}[x]}} - 22 \operatorname{ArcTanh}[x] + 33 \operatorname{Log}\left[1 + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}\right] - \right. \\ \left. 11 \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \frac{\operatorname{ArcTanh}[x] - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] + \operatorname{ArcTanh}[x] \#1^2 - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] \#1^2}{-2 + \#1^2} \&\right] \right)$$

Problem 129: Result is not expressed in closed-form.

$$\int e^{\frac{2 \operatorname{ArcTanh}[x]}{3}} x \, dx$$

Optimal (type 3, 112 leaves, 4 steps):

$$-\frac{1}{3} (1-x)^{2/3} (1+x)^{1/3} - \frac{1}{2} (1-x)^{2/3} (1+x)^{4/3} + \frac{2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2(1-x)^{1/3}}{\sqrt{3}(1+x)^{1/3}}\right]}{3\sqrt{3}} + \frac{1}{9} \operatorname{Log}[1+x] + \frac{1}{3} \operatorname{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}}\right]$$

Result (type 7, 124 leaves):

$$\frac{2}{27} \left(-\frac{9 e^{\frac{2 \operatorname{ArcTanh}[x]}{3}} (1 + 4 e^{2 \operatorname{ArcTanh}[x]})}{(1 + e^{2 \operatorname{ArcTanh}[x]})^2} - 2 \operatorname{ArcTanh}[x] + 3 \operatorname{Log}\left[1 + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}\right] - \right. \\ \left. \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \frac{\operatorname{ArcTanh}[x] - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] + \operatorname{ArcTanh}[x] \#1^2 - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] \#1^2}{-2 + \#1^2} \&\right] \right)$$

Problem 130: Result is not expressed in closed-form.

$$\int e^{\frac{2 \operatorname{ArcTanh}[x]}{3}} \, dx$$

Optimal (type 3, 84 leaves, 3 steps):

$$-(1-x)^{2/3} (1+x)^{1/3} + \frac{2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2(1-x)^{1/3}}{\sqrt{3}(1+x)^{1/3}}\right]}{\sqrt{3}} + \frac{1}{3} \operatorname{Log}[1+x] + \operatorname{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}}\right]$$

Result (type 7, 116 leaves):

$$-\frac{2 e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}}{1 + e^{2 \operatorname{ArcTanh}[x]}} - \frac{4 \operatorname{ArcTanh}[x]}{9} + \frac{2}{3} \operatorname{Log}\left[1 + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}\right] -$$

$$\frac{2}{9} \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \frac{\operatorname{ArcTanh}[x] - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] + \operatorname{ArcTanh}[x] \#1^2 - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] \#1^2}{-2 + \#1^2} \&\right]$$

Problem 131: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}}{x} dx$$

Optimal (type 3, 135 leaves, 4 steps):

$$\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2(1-x)^{1/3}}{\sqrt{3}(1+x)^{1/3}}\right] + \sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(1-x)^{1/3}}{\sqrt{3}(1+x)^{1/3}}\right] -$$

$$\frac{\operatorname{Log}[x]}{2} + \frac{1}{2} \operatorname{Log}[1+x] + \frac{3}{2} \operatorname{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}}\right] + \frac{3}{2} \operatorname{Log}\left[(1-x)^{1/3} - (1+x)^{1/3}\right]$$

Result (type 7, 215 leaves):

$$-\sqrt{3} \operatorname{ArcTan}\left[\frac{-1 + 2 e^{\frac{\operatorname{ArcTanh}[x]}{3}}}{\sqrt{3}}\right] + \sqrt{3} \operatorname{ArcTan}\left[\frac{1 + 2 e^{\frac{\operatorname{ArcTanh}[x]}{3}}}{\sqrt{3}}\right] - \frac{2 \operatorname{ArcTanh}[x]}{3} + \operatorname{Log}\left[1 - e^{\frac{\operatorname{ArcTanh}[x]}{3}}\right] +$$

$$\operatorname{Log}\left[1 + e^{\frac{\operatorname{ArcTanh}[x]}{3}}\right] + \operatorname{Log}\left[1 + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}\right] - \frac{1}{2} \operatorname{Log}\left[1 - e^{\frac{\operatorname{ArcTanh}[x]}{3}} + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}\right] - \frac{1}{2} \operatorname{Log}\left[1 + e^{\frac{\operatorname{ArcTanh}[x]}{3}} + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}\right] -$$

$$\frac{1}{3} \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \frac{\operatorname{ArcTanh}[x] - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] + \operatorname{ArcTanh}[x] \#1^2 - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] \#1^2}{-2 + \#1^2} \&\right]$$

Problem 134: Unable to integrate problem.

$$\int e^{\frac{1}{4} \operatorname{ArcTanh}[a x]} x^m dx$$

Optimal (type 6, 31 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, \frac{1}{8}, -\frac{1}{8}, 2+m, a x, -a x\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{\frac{1}{4} \operatorname{ArcTanh}[a x]} x^m dx$$

Problem 135: Result is not expressed in closed-form.

$$\int e^{\frac{1}{4} \operatorname{ArcTanh}[a x]} x^2 dx$$

Optimal (type 3, 646 leaves, 27 steps):

$$\begin{aligned} & -\frac{11(1-ax)^{7/8}(1+ax)^{1/8}}{32a^3} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{24a^3} - \frac{x(1-ax)^{7/8}(1+ax)^{9/8}}{3a^2} + \frac{11\sqrt{2+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} - \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]}{128a^3} + \\ & \frac{11\sqrt{2-\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} - \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]}{128a^3} - \frac{11\sqrt{2+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} + \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]}{128a^3} - \frac{11\sqrt{2-\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} + \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]}{128a^3} - \\ & \frac{11\sqrt{2-\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} - \frac{\sqrt{2-\sqrt{2}}(1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{256a^3} + \frac{11\sqrt{2-\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} + \frac{\sqrt{2-\sqrt{2}}(1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{256a^3} - \\ & \frac{11\sqrt{2+\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} - \frac{\sqrt{2+\sqrt{2}}(1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{256a^3} + \frac{11\sqrt{2+\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} + \frac{\sqrt{2+\sqrt{2}}(1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{256a^3} \end{aligned}$$

Result (type 7, 94 leaves):

$$-\frac{e^{\frac{1}{4} \operatorname{ArcTanh}[a x]} (33 + 10 e^{2 \operatorname{ArcTanh}[a x]} + 105 e^{4 \operatorname{ArcTanh}[a x]})}{48 (1 + e^{2 \operatorname{ArcTanh}[a x]})^3} - \frac{11}{512} \operatorname{RootSum}\left[1 + \#1^8 \&, \frac{\operatorname{ArcTanh}[a x] - 4 \operatorname{Log}\left[e^{\frac{1}{4} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^7} \&\right] a^3$$

Problem 136: Result is not expressed in closed-form.

$$\int e^{\frac{1}{4} \operatorname{ArcTanh}[a x]} x dx$$

Optimal (type 3, 619 leaves, 26 steps):

$$\begin{aligned}
& - \frac{(1-ax)^{7/8} (1+ax)^{1/8}}{8a^2} - \frac{(1-ax)^{7/8} (1+ax)^{9/8}}{2a^2} + \frac{\sqrt{2+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} - \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]}{32a^2} + \frac{\sqrt{2-\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} - \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]}{32a^2} \\
& - \frac{\sqrt{2+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} + \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]}{32a^2} - \frac{\sqrt{2-\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} + \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]}{32a^2} - \frac{\sqrt{2-\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} - \frac{\sqrt{2-\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{64a^2} + \\
& - \frac{\sqrt{2-\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} + \frac{\sqrt{2-\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{64a^2} - \frac{\sqrt{2+\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} - \frac{\sqrt{2+\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{64a^2} + \frac{\sqrt{2+\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} + \frac{\sqrt{2+\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{64a^2}
\end{aligned}$$

Result (type 7, 83 leaves):

$$\frac{-\frac{32 e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} (1+9 e^{2 \operatorname{ArcTanh}[ax]})}{(1+e^{2 \operatorname{ArcTanh}[ax]})^2} + \operatorname{RootSum}\left[1 + \#1^8 \&, \frac{-\operatorname{ArcTanh}[ax] + 4 \operatorname{Log}\left[e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} - \#1\right]}{\#1^7}\right] \&]}{128 a^2}$$

Problem 137: Result is not expressed in closed-form.

$$\int e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} dx$$

Optimal (type 3, 591 leaves, 25 steps):

$$\begin{aligned}
& - \frac{(1-ax)^{7/8} (1+ax)^{1/8}}{a} + \frac{\sqrt{2+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} - \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]}{4a} + \frac{\sqrt{2-\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} - \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]}{4a} \\
& - \frac{\sqrt{2+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} + \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]}{4a} - \frac{\sqrt{2-\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} + \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]}{4a} - \frac{\sqrt{2-\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} - \frac{\sqrt{2-\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{8a} + \\
& - \frac{\sqrt{2-\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} + \frac{\sqrt{2-\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{8a} - \frac{\sqrt{2+\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} - \frac{\sqrt{2+\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{8a} + \frac{\sqrt{2+\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} + \frac{\sqrt{2+\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{8a}
\end{aligned}$$

Result (type 7, 71 leaves):

$$\frac{-\frac{32 e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}}{1+e^{2 \operatorname{ArcTanh}[ax]}} + \operatorname{RootSum}\left[1 + \#1^8 \&, \frac{-\operatorname{ArcTanh}[ax] + 4 \operatorname{Log}\left[e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} - \#1\right]}{\#1^7}\right] \&]}{16 a}$$

Problem 138: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{4} \operatorname{ArcTanh}[a x]}}{x} dx$$

Optimal (type 3, 759 leaves, 39 steps):

$$\begin{aligned} & -2 \operatorname{ArcTan}\left[\frac{(1+ax)^{1/8}}{(1-ax)^{1/8}}\right] + \sqrt{2+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} - \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right] + \sqrt{2-\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} - \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right] - \\ & \sqrt{2+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} + \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right] - \sqrt{2-\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} + \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}}\right] - \\ & \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}}\right] - 2 \operatorname{ArcTanh}\left[\frac{(1+ax)^{1/8}}{(1-ax)^{1/8}}\right] - \frac{1}{2} \sqrt{2-\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} - \frac{\sqrt{2-\sqrt{2}}(1-ax)^{1/8}}{(1+ax)^{1/8}}\right] + \\ & \frac{1}{2} \sqrt{2-\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} + \frac{\sqrt{2-\sqrt{2}}(1-ax)^{1/8}}{(1+ax)^{1/8}}\right] - \frac{1}{2} \sqrt{2+\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} - \frac{\sqrt{2+\sqrt{2}}(1-ax)^{1/8}}{(1+ax)^{1/8}}\right] + \\ & \frac{1}{2} \sqrt{2+\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} + \frac{\sqrt{2+\sqrt{2}}(1-ax)^{1/8}}{(1+ax)^{1/8}}\right] + \frac{\operatorname{Log}\left[1 - \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}} + \frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right]}{\sqrt{2}} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}} + \frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right]}{\sqrt{2}} \end{aligned}$$

Result (type 7, 128 leaves):

$$\begin{aligned} & -2 \operatorname{ArcTan}\left[e^{\frac{1}{4} \operatorname{ArcTanh}[a x]}\right] + \operatorname{Log}\left[1 - e^{\frac{1}{4} \operatorname{ArcTanh}[a x]}\right] - \operatorname{Log}\left[1 + e^{\frac{1}{4} \operatorname{ArcTanh}[a x]}\right] + \\ & \frac{1}{4} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 4 \operatorname{Log}\left[e^{\frac{1}{4} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right] + \frac{1}{4} \operatorname{RootSum}\left[1 + \#1^8 \&, \frac{-\operatorname{ArcTanh}[a x] + 4 \operatorname{Log}\left[e^{\frac{1}{4} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^7} \&\right] \end{aligned}$$

Problem 139: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{4} \operatorname{ArcTanh}[a x]}}{x^2} dx$$

Optimal (type 3, 271 leaves, 16 steps):

$$\begin{aligned}
& - \frac{(1-ax)^{7/8} (1+ax)^{1/8}}{x} - \frac{1}{2} a \operatorname{ArcTan} \left[\frac{(1+ax)^{1/8}}{(1-ax)^{1/8}} \right] + \frac{a \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (1+ax)^{1/8}}{(1-ax)^{1/8}} \right]}{2\sqrt{2}} - \frac{a \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (1+ax)^{1/8}}{(1-ax)^{1/8}} \right]}{2\sqrt{2}} - \\
& \frac{1}{2} a \operatorname{ArcTanh} \left[\frac{(1+ax)^{1/8}}{(1-ax)^{1/8}} \right] + \frac{a \operatorname{Log} \left[1 - \frac{\sqrt{2} (1+ax)^{1/8}}{(1-ax)^{1/8}} + \frac{(1+ax)^{1/4}}{(1-ax)^{1/4}} \right]}{4\sqrt{2}} - \frac{a \operatorname{Log} \left[1 + \frac{\sqrt{2} (1+ax)^{1/8}}{(1-ax)^{1/8}} + \frac{(1+ax)^{1/4}}{(1-ax)^{1/4}} \right]}{4\sqrt{2}}
\end{aligned}$$

Result (type 7, 113 leaves):

$$\begin{aligned}
& \frac{1}{16} a \left(4 \left(- \frac{8 e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}}{-1 + e^{2 \operatorname{ArcTanh}[ax]}} - 2 \operatorname{ArcTan} \left[e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} \right] + \operatorname{Log} \left[1 - e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} \right] - \operatorname{Log} \left[1 + e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} \right] \right) + \right. \\
& \left. \operatorname{RootSum} \left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[ax] - 4 \operatorname{Log} \left[e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} - \#1 \right]}{\#1^3} \& \right] \right)
\end{aligned}$$

Problem 140: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}}{x^3} dx$$

Optimal (type 3, 312 leaves, 17 steps):

$$\begin{aligned}
& - \frac{a (1-ax)^{7/8} (1+ax)^{1/8}}{8x} - \frac{(1-ax)^{7/8} (1+ax)^{9/8}}{2x^2} - \frac{1}{16} a^2 \operatorname{ArcTan} \left[\frac{(1+ax)^{1/8}}{(1-ax)^{1/8}} \right] + \frac{a^2 \operatorname{ArcTan} \left[1 - \frac{\sqrt{2} (1+ax)^{1/8}}{(1-ax)^{1/8}} \right]}{16\sqrt{2}} - \\
& \frac{a^2 \operatorname{ArcTan} \left[1 + \frac{\sqrt{2} (1+ax)^{1/8}}{(1-ax)^{1/8}} \right]}{16\sqrt{2}} - \frac{1}{16} a^2 \operatorname{ArcTanh} \left[\frac{(1+ax)^{1/8}}{(1-ax)^{1/8}} \right] + \frac{a^2 \operatorname{Log} \left[1 - \frac{\sqrt{2} (1+ax)^{1/8}}{(1-ax)^{1/8}} + \frac{(1+ax)^{1/4}}{(1-ax)^{1/4}} \right]}{32\sqrt{2}} - \frac{a^2 \operatorname{Log} \left[1 + \frac{\sqrt{2} (1+ax)^{1/8}}{(1-ax)^{1/8}} + \frac{(1+ax)^{1/4}}{(1-ax)^{1/4}} \right]}{32\sqrt{2}}
\end{aligned}$$

Result (type 7, 139 leaves):

$$\begin{aligned}
& \frac{1}{128} a^2 \left(4 \left(- \frac{64 e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}}{(-1 + e^{2 \operatorname{ArcTanh}[ax]})^2} - \frac{72 e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}}{-1 + e^{2 \operatorname{ArcTanh}[ax]}} - 2 \operatorname{ArcTan} \left[e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} \right] + \operatorname{Log} \left[1 - e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} \right] - \operatorname{Log} \left[1 + e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} \right] \right) + \right. \\
& \left. \operatorname{RootSum} \left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[ax] - 4 \operatorname{Log} \left[e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} - \#1 \right]}{\#1^3} \& \right] \right)
\end{aligned}$$

Problem 142: Result unnecessarily involves higher level functions.

$$\int e^{3 \operatorname{ArcTanh}[a x]} x^m dx$$

Optimal (type 5, 151 leaves, 9 steps):

$$\frac{3 x^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{1+m} - \frac{a x^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{2+m} + \frac{4 x^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{1+m} + \frac{4 a x^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{2+m}$$

Result (type 6, 265 leaves):

$$\frac{1}{(1+m) (-1+ax)^{3/2}} 2(2+m) x^{1+m} \sqrt{-1-ax} \left(\left(2 \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, -ax, ax\right] \right) / \left(2(2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, -ax, ax\right] + ax \right. \right. \\ \left. \left. \left(3 \operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{5}{2}, 3+m, -ax, ax\right] + \operatorname{AppellF1}\left[2+m, \frac{1}{2}, \frac{3}{2}, 3+m, -ax, ax\right] \right) \right) - \right. \\ \left. \left(\sqrt{1-ax} \sqrt{1-a^2x^2} \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -ax, ax\right] \right) / \left(\sqrt{1+ax} \left(2(2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -ax, ax\right] + \right. \right. \right. \\ \left. \left. \left. ax \left(\operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{3}{2}, 3+m, -ax, ax\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2x^2\right]\right) \right) \right) \right)$$

Problem 144: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int e^{\operatorname{ArcTanh}[a x]} x^m dx$$

Optimal (type 5, 74 leaves, 4 steps):

$$\frac{x^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{1+m} + \frac{a x^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{2+m}$$

Result (type 6, 166 leaves):

$$\left(2 (2+m) x^{1+m} \sqrt{-1-ax} \sqrt{1-ax} \sqrt{1-a^2x^2} \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -ax, ax\right] \right) /$$

$$\left((1+m) (-1+ax)^{3/2} \sqrt{1+ax} \left(2 (2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -ax, ax\right] + \right.$$

$$\left. ax \left(\operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{3}{2}, 3+m, -ax, ax\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2x^2\right] \right) \right) \right)$$

Problem 145: Result unnecessarily involves higher level functions.

$$\int e^{-\operatorname{ArcTanh}[ax]} x^m dx$$

Optimal (type 5, 75 leaves, 4 steps):

$$\frac{x^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2x^2\right]}{1+m} - \frac{ax^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2x^2\right]}{2+m}$$

Result (type 6, 134 leaves):

$$\left(2 (2+m) x^{1+m} \sqrt{1-ax} \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, ax, -ax\right] \right) / \left((1+m) \sqrt{1+ax} \left(2 (2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, ax, -ax\right] - \right.$$

$$\left. ax \left(\operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{3}{2}, 3+m, ax, -ax\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2x^2\right] \right) \right) \right)$$

Problem 147: Result unnecessarily involves higher level functions.

$$\int e^{-3 \operatorname{ArcTanh}[ax]} x^m dx$$

Optimal (type 5, 150 leaves, 9 steps):

$$-\frac{3x^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2x^2\right]}{1+m} + \frac{ax^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2x^2\right]}{2+m} +$$

$$\frac{4x^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2x^2\right]}{1+m} - \frac{4ax^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2x^2\right]}{2+m}$$

Result (type 6, 237 leaves):

$$\frac{1}{(1+m)(1+ax)^{3/2}} 2(2+m)x^{1+m}\sqrt{1-ax} \left(\left(2 \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, ax, -ax\right] \right) / \left(2(2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, ax, -ax\right] - \right. \right. \\ \left. \left. ax \left(3 \operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{5}{2}, 3+m, ax, -ax\right] + \operatorname{AppellF1}\left[2+m, \frac{1}{2}, \frac{3}{2}, 3+m, ax, -ax\right] \right) \right) + \right. \\ \left. \left((1+ax) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, ax, -ax\right] \right) / \left(-2(2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, ax, -ax\right] + \right. \right. \\ \left. \left. ax \left(\operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{3}{2}, 3+m, ax, -ax\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2x^2\right] \right) \right) \right)$$

Problem 148: Unable to integrate problem.

$$\int e^{n \operatorname{ArcTanh}[ax]} x^m dx$$

Optimal (type 6, 35 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, \frac{n}{2}, -\frac{n}{2}, 2+m, ax, -ax\right]}{1+m}$$

Result (type 8, 14 leaves):

$$\int e^{n \operatorname{ArcTanh}[ax]} x^m dx$$

Problem 211: Unable to integrate problem.

$$\int \frac{e^{-2 \operatorname{ArcTanh}[ax]}}{c - acx} dx$$

Optimal (type 3, 13 leaves, 2 steps):

$$\frac{\operatorname{Log}[1+ax]}{ac}$$

Result (type 8, 20 leaves):

$$\int \frac{e^{-2 \operatorname{ArcTanh}[ax]}}{c - acx} dx$$

Problem 212: Unable to integrate problem.

$$\int \frac{e^{-2 \operatorname{ArcTanh}[ax]}}{(c - acx)^2} dx$$

Optimal (type 3, 11 leaves, 3 steps):

$$\frac{\text{ArcTanh}[a x]}{a c^2}$$

Result (type 8, 20 leaves):

$$\int \frac{e^{-2 \text{ArcTanh}[a x]}}{(c - a c x)^2} dx$$

Problem 278: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^{7/2} dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$\frac{2^{1+\frac{n}{2}} (1 - a x)^{-n/2} (c - a c x)^{9/2} \text{Hypergeometric2F1}\left[\frac{9-n}{2}, -\frac{n}{2}, \frac{11-n}{2}, \frac{1}{2} (1 - a x)\right]}{a c (9 - n)}$$

Result (type 8, 22 leaves):

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^{7/2} dx$$

Problem 279: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^{5/2} dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$\frac{2^{1+\frac{n}{2}} (1 - a x)^{-n/2} (c - a c x)^{7/2} \text{Hypergeometric2F1}\left[\frac{7-n}{2}, -\frac{n}{2}, \frac{9-n}{2}, \frac{1}{2} (1 - a x)\right]}{a c (7 - n)}$$

Result (type 8, 22 leaves):

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^{5/2} dx$$

Problem 280: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^{3/2} dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$\frac{2^{1+\frac{n}{2}} (1-ax)^{-n/2} (c-ax)^{5/2} \text{Hypergeometric2F1}\left[\frac{5-n}{2}, -\frac{n}{2}, \frac{7-n}{2}, \frac{1}{2}(1-ax)\right]}{ac(5-n)}$$

Result (type 8, 22 leaves):

$$\int e^{n \text{ArcTanh}[ax]} (c-ax)^{3/2} dx$$

Problem 281: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[ax]} \sqrt{c-ax} dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$\frac{2^{1+\frac{n}{2}} (1-ax)^{-n/2} (c-ax)^{3/2} \text{Hypergeometric2F1}\left[\frac{3-n}{2}, -\frac{n}{2}, \frac{5-n}{2}, \frac{1}{2}(1-ax)\right]}{ac(3-n)}$$

Result (type 8, 22 leaves):

$$\int e^{n \text{ArcTanh}[ax]} \sqrt{c-ax} dx$$

Problem 282: Unable to integrate problem.

$$\int \frac{e^{n \text{ArcTanh}[ax]}}{\sqrt{c-ax}} dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$\frac{2^{1+\frac{n}{2}} (1-ax)^{-n/2} \sqrt{c-ax} \text{Hypergeometric2F1}\left[\frac{1-n}{2}, -\frac{n}{2}, \frac{3-n}{2}, \frac{1}{2}(1-ax)\right]}{ac(1-n)}$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n \text{ArcTanh}[ax]}}{\sqrt{c-ax}} dx$$

Problem 283: Unable to integrate problem.

$$\int \frac{e^{n \text{ArcTanh}[ax]}}{(c-ax)^{3/2}} dx$$

Optimal (type 5, 78 leaves, 3 steps):

$$\frac{2^{1+\frac{n}{2}} (1-ax)^{-n/2} \text{Hypergeometric2F1}\left[\frac{1}{2}(-1-n), -\frac{n}{2}, \frac{1-n}{2}, \frac{1}{2}(1-ax)\right]}{ac(1+n)\sqrt{c-ax}}$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n \text{ArcTanh}[ax]}}{(c-ax)^{3/2}} dx$$

Problem 284: Unable to integrate problem.

$$\int \frac{e^{n \text{ArcTanh}[ax]}}{(c-ax)^{5/2}} dx$$

Optimal (type 5, 78 leaves, 3 steps):

$$\frac{2^{1+\frac{n}{2}} (1-ax)^{-n/2} \text{Hypergeometric2F1}\left[\frac{1}{2}(-3-n), -\frac{n}{2}, \frac{1}{2}(-1-n), \frac{1}{2}(1-ax)\right]}{ac(3+n)(c-ax)^{3/2}}$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n \text{ArcTanh}[ax]}}{(c-ax)^{5/2}} dx$$

Problem 285: Unable to integrate problem.

$$\int \frac{e^{n \text{ArcTanh}[ax]}}{(c-ax)^{7/2}} dx$$

Optimal (type 5, 78 leaves, 3 steps):

$$\frac{2^{1+\frac{n}{2}} (1-ax)^{-n/2} \text{Hypergeometric2F1}\left[\frac{1}{2}(-5-n), -\frac{n}{2}, \frac{1}{2}(-3-n), \frac{1}{2}(1-ax)\right]}{ac(5+n)(c-ax)^{5/2}}$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n \text{ArcTanh}[ax]}}{(c-ax)^{7/2}} dx$$

Problem 378: Result more than twice size of optimal antiderivative.

$$\int e^{\text{ArcTanh}[x]} \sqrt{1-x} dx$$

Optimal (type 2, 11 leaves, 3 steps):

$$\frac{2}{3} (1+x)^{3/2}$$

Result (type 2, 27 leaves):

$$\frac{2(1+x)\sqrt{1-x^2}}{3\sqrt{1-x}}$$

Problem 387: Unable to integrate problem.

$$\int e^{\text{ArcTanh}[a x]} x^m \sqrt{c - a c x} dx$$

Optimal (type 5, 64 leaves, 4 steps):

$$\frac{2 c x^m (-a x)^{-m} (1+a x) \sqrt{1-a^2 x^2} \text{Hypergeometric2F1}\left[\frac{3}{2}, -m, \frac{5}{2}, 1+a x\right]}{3 a \sqrt{c - a c x}}$$

Result (type 8, 23 leaves):

$$\int e^{\text{ArcTanh}[a x]} x^m \sqrt{c - a c x} dx$$

Problem 411: Unable to integrate problem.

$$\int e^{-\text{ArcTanh}[a x]} x^m \sqrt{c - a c x} dx$$

Optimal (type 5, 114 leaves, 5 steps):

$$-\frac{2 c x^{1+m} \sqrt{1-a^2 x^2}}{(3+2 m) \sqrt{c - a c x}} + \frac{2(5+4 m) x^m (-a x)^{-m} (1+a x) \sqrt{c - a c x} \text{Hypergeometric2F1}\left[\frac{1}{2}, -m, \frac{3}{2}, 1+a x\right]}{a(3+2 m) \sqrt{1-a^2 x^2}}$$

Result (type 8, 25 leaves):

$$\int e^{-\text{ArcTanh}[a x]} x^m \sqrt{c - a c x} dx$$

Problem 437: Unable to integrate problem.

$$\int e^{-2 p \text{ArcTanh}[a x]} (c - a c x)^p dx$$

Optimal (type 5, 61 leaves, 3 steps):

$$\frac{2^{-p} (1 - a x)^p (c - a c x)^{1+p} \text{Hypergeometric2F1}\left[p, 1 + 2 p, 2 (1 + p), \frac{1}{2} (1 - a x)\right]}{a c (1 + 2 p)}$$

Result (type 8, 21 leaves):

$$\int e^{-2 p \text{ArcTanh}[a x]} (c - a c x)^p dx$$

Problem 439: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^p dx$$

Optimal (type 5, 82 leaves, 3 steps):

$$\frac{2^{1+\frac{n}{2}} (1 - a x)^{-n/2} (c - a c x)^{1+p} \text{Hypergeometric2F1}\left[-\frac{n}{2}, 1 - \frac{n}{2} + p, 2 - \frac{n}{2} + p, \frac{1}{2} (1 - a x)\right]}{a c (2 - n + 2 p)}$$

Result (type 8, 20 leaves):

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^p dx$$

Problem 440: Result more than twice size of optimal antiderivative.

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^3 dx$$

Optimal (type 5, 68 leaves, 2 steps):

$$\frac{2^{1+\frac{n}{2}} c^3 (1 - a x)^{4-\frac{n}{2}} \text{Hypergeometric2F1}\left[4 - \frac{n}{2}, -\frac{n}{2}, 5 - \frac{n}{2}, \frac{1}{2} (1 - a x)\right]}{a (8 - n)}$$

Result (type 5, 195 leaves):

$$\begin{aligned} & -\frac{1}{24 a (2 + n)} c^3 e^{n \text{ArcTanh}[a x]} \left(-e^{2 \text{ArcTanh}[a x]} n (-48 + 44 n - 12 n^2 + n^3) \text{Hypergeometric2F1}\left[1, 1 + \frac{n}{2}, 2 + \frac{n}{2}, -e^{2 \text{ArcTanh}[a x]}\right] + \right. \\ & (2 + n) \left(a n^3 x + n^2 (-1 - 12 a x + a^2 x^2) + 2 n (6 + 21 a x - 6 a^2 x^2 + a^3 x^3) + \right. \\ & \left. \left. 6 (-7 - 4 a x + 6 a^2 x^2 - 4 a^3 x^3 + a^4 x^4) + (-48 + 44 n - 12 n^2 + n^3) \text{Hypergeometric2F1}\left[1, \frac{n}{2}, 1 + \frac{n}{2}, -e^{2 \text{ArcTanh}[a x]}\right] \right) \right) \end{aligned}$$

Problem 441: Result more than twice size of optimal antiderivative.

$$\int e^{n \operatorname{ArcTanh}[a x]} (c - a c x)^2 dx$$

Optimal (type 5, 68 leaves, 2 steps):

$$\frac{2^{1+\frac{n}{2}} c^2 (1 - a x)^{3-\frac{n}{2}} \operatorname{Hypergeometric2F1}\left[3 - \frac{n}{2}, -\frac{n}{2}, 4 - \frac{n}{2}, \frac{1}{2} (1 - a x)\right]}{a (6 - n)}$$

Result (type 5, 149 leaves):

$$\frac{1}{6 a (2 + n)} c^2 e^{n \operatorname{ArcTanh}[a x]} \left(-e^{2 \operatorname{ArcTanh}[a x]} n (8 - 6 n + n^2) \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{n}{2}, 2 + \frac{n}{2}, -e^{2 \operatorname{ArcTanh}[a x]}\right] + (2 + n) \left(6 + 6 a x + a n^2 x - 6 a^2 x^2 + 2 a^3 x^3 + n (-1 - 6 a x + a^2 x^2) + (8 - 6 n + n^2) \operatorname{Hypergeometric2F1}\left[1, \frac{n}{2}, 1 + \frac{n}{2}, -e^{2 \operatorname{ArcTanh}[a x]}\right] \right) \right)$$

Problem 447: Unable to integrate problem.

$$\int e^{\operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^p dx$$

Optimal (type 6, 60 leaves, 3 steps):

$$\frac{\left(c - \frac{c}{a x} \right)^p x (1 - a x)^{-p} \operatorname{AppellF1}\left[1 - p, \frac{1}{2} - p, -\frac{1}{2}, 2 - p, a x, -a x\right]}{1 - p}$$

Result (type 8, 22 leaves):

$$\int e^{\operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^p dx$$

Problem 456: Unable to integrate problem.

$$\int e^{2 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^p dx$$

Optimal (type 5, 59 leaves, 6 steps):

$$-\left(c - \frac{c}{a x} \right)^p x - \frac{(2 - p) \left(c - \frac{c}{a x} \right)^p \operatorname{Hypergeometric2F1}\left[1, p, 1 + p, 1 - \frac{1}{a x}\right]}{a p}$$

Result (type 8, 24 leaves):

$$\int e^{2 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^p dx$$

Problem 474: Unable to integrate problem.

$$\int e^{4 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^p dx$$

Optimal (type 5, 93 leaves, 7 steps):

$$-\frac{c(5-p)\left(c-\frac{c}{ax}\right)^{-1+p}}{a(1-p)} + c\left(c-\frac{c}{ax}\right)^{-1+p} x + \frac{(4-p)\left(c-\frac{c}{ax}\right)^p \operatorname{Hypergeometric2F1}\left[1, p, 1+p, 1-\frac{1}{ax}\right]}{ap}$$

Result (type 8, 24 leaves):

$$\int e^{4 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^p dx$$

Problem 484: Unable to integrate problem.

$$\int e^{-\operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^p dx$$

Optimal (type 6, 60 leaves, 3 steps):

$$\frac{\left(c-\frac{c}{ax}\right)^p x (1-ax)^{-p} \operatorname{AppellF1}\left[1-p, -\frac{1}{2}-p, \frac{1}{2}, 2-p, ax, -ax\right]}{1-p}$$

Result (type 8, 24 leaves):

$$\int e^{-\operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^p dx$$

Problem 493: Unable to integrate problem.

$$\int e^{-2 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^p dx$$

Optimal (type 5, 114 leaves, 8 steps):

$$-\frac{\left(c-\frac{c}{ax}\right)^{2+p} x}{c^2} - \frac{\left(c-\frac{c}{ax}\right)^{2+p} \operatorname{Hypergeometric2F1}\left[1, 2+p, 3+p, \frac{a-x}{2a}\right]}{2ac^2(2+p)} + \frac{\left(c-\frac{c}{ax}\right)^{2+p} \operatorname{Hypergeometric2F1}\left[1, 2+p, 3+p, 1-\frac{1}{ax}\right]}{ac^2}$$

Result (type 8, 24 leaves):

$$\int e^{-2 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^p dx$$

Problem 499: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{-2 \operatorname{ArcTanh}[a x]}}{\left(c - \frac{c}{a x} \right)^2} dx$$

Optimal (type 3, 18 leaves, 5 steps):

$$-\frac{x}{c^2} + \frac{\operatorname{ArcTanh}[a x]}{a c^2}$$

Result (type 3, 40 leaves):

$$-\frac{x}{c^2} - \frac{\operatorname{Log}[1 - a x]}{2 a c^2} + \frac{\operatorname{Log}[1 + a x]}{2 a c^2}$$

Problem 510: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{9/2} dx$$

Optimal (type 3, 225 leaves, 8 steps):

$$\begin{aligned} & -\frac{a^3 \left(c - \frac{c}{a x} \right)^{9/2} x^4 (54 - 227 a x) \sqrt{1 + a x}}{105 (1 - a x)^{9/2}} - \frac{10 a^2 \left(c - \frac{c}{a x} \right)^{9/2} x^3 \sqrt{1 + a x}}{21 (1 - a x)^{5/2}} + \\ & \frac{2 a \left(c - \frac{c}{a x} \right)^{9/2} x^2 \sqrt{1 + a x}}{5 (1 - a x)^{3/2}} - \frac{2 \left(c - \frac{c}{a x} \right)^{9/2} x \sqrt{1 + a x}}{7 \sqrt{1 - a x}} - \frac{7 a^{7/2} \left(c - \frac{c}{a x} \right)^{9/2} x^{9/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - a x)^{9/2}} \end{aligned}$$

Result (type 3, 151 leaves):

$$-\frac{c^4 \sqrt{c - \frac{c}{a x}} \sqrt{1 - a^2 x^2} (-30 + 162 a x - 356 a^2 x^2 + 292 a^3 x^3 + 105 a^4 x^4)}{105 a^4 x^3 (-1 + a x)} - \frac{7 i c^{9/2} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}]}{2 a}$$

Problem 511: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{7/2} dx$$

Optimal (type 3, 181 leaves, 7 steps):

$$\frac{2 a \left(c - \frac{c}{a x} \right)^{7/2} x^2 \sqrt{1+a x}}{3 (1-a x)^{3/2}} - \frac{2 \left(c - \frac{c}{a x} \right)^{7/2} x \sqrt{1+a x}}{5 \sqrt{1-a x}} - \frac{a^2 \left(c - \frac{c}{a x} \right)^{7/2} x^3 \sqrt{1+a x} (18+31 a x)}{15 (1-a x)^{7/2}} + \frac{5 a^{5/2} \left(c - \frac{c}{a x} \right)^{7/2} x^{7/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1-a x)^{7/2}}$$

Result (type 3, 143 leaves):

$$-\frac{c^3 \sqrt{c - \frac{c}{a x}} \sqrt{1-a^2 x^2} (6-28 a x+56 a^2 x^2+15 a^3 x^3)}{15 a^3 x^2 (-1+a x)} - \frac{5 i c^{7/2} \text{Log}[-i \sqrt{c} (1+2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1-a^2 x^2}}{-1+a x}]}{2 a}$$

Problem 512: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{5/2} dx$$

Optimal (type 3, 171 leaves, 7 steps):

$$-\frac{3 a^2 \left(c - \frac{c}{a x} \right)^{5/2} x^3 \sqrt{1+a x}}{(1-a x)^{5/2}} - \frac{2 \left(c - \frac{c}{a x} \right)^{5/2} x (1+a x)^{3/2}}{3 (1-a x)^{5/2}} + \frac{4 a \left(c - \frac{c}{a x} \right)^{5/2} x^2 (1+a x)^{3/2}}{(1-a x)^{5/2}} - \frac{3 a^{3/2} \left(c - \frac{c}{a x} \right)^{5/2} x^{5/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1-a x)^{5/2}}$$

Result (type 3, 133 leaves):

$$c^2 \left(-\frac{2 \sqrt{c - \frac{c}{a x}} \sqrt{1-a^2 x^2} (-2+10 a x+3 a^2 x^2)}{x (-1+a x)} - 9 i a \sqrt{c} \text{Log}[-i \sqrt{c} (1+2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1-a^2 x^2}}{-1+a x}] \right)$$

$$6 a^2$$

Problem 513: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{3/2} dx$$

Optimal (type 3, 128 leaves, 7 steps):

$$\frac{a \left(c - \frac{c}{a x} \right)^{3/2} x^2 \sqrt{1+a x}}{(1-a x)^{3/2}} - \frac{2 \left(c - \frac{c}{a x} \right)^{3/2} x (1-a^2 x^2)^{3/2}}{(1-a x)^3} + \frac{\sqrt{a} \left(c - \frac{c}{a x} \right)^{3/2} x^{3/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1-a x)^{3/2}}$$

Result (type 3, 119 leaves):

$$-\frac{c \sqrt{c - \frac{c}{a x}} (2+a x) \sqrt{1-a^2 x^2}}{a (-1+a x)} - \frac{i c^{3/2} \text{Log}[-i \sqrt{c} (1+2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1-a^2 x^2}}{-1+a x}]}{2 a}$$

Problem 514: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} dx$$

Optimal (type 3, 84 leaves, 6 steps):

$$-\frac{c \sqrt{1 - a^2 x^2}}{a \sqrt{c - \frac{c}{a x}}} + \frac{\sqrt{c - \frac{c}{a x}} \sqrt{x} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1 - a x}}$$

Result (type 3, 111 leaves):

$$-\frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} + \frac{i \sqrt{c} \text{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}]}{2 a}$$

Problem 515: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{\sqrt{c - \frac{c}{a x}}} dx$$

Optimal (type 3, 157 leaves, 8 steps):

$$-\frac{\sqrt{1 - a x} \sqrt{1 + a x}}{a \sqrt{c - \frac{c}{a x}}} - \frac{3 \sqrt{1 - a x} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{3/2} \sqrt{c - \frac{c}{a x}} \sqrt{x}} + \frac{2 \sqrt{2} \sqrt{1 - a x} \text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + a x}}\right]}{a^{3/2} \sqrt{c - \frac{c}{a x}} \sqrt{x}}$$

Result (type 3, 203 leaves):

$$\frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{c - a c x} + \frac{3 i \text{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}]}{2 a \sqrt{c}} - \frac{i \sqrt{2} \text{Log}\left[\frac{4 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} - i \sqrt{2} \sqrt{c} (-1 - 2 a x + 3 a^2 x^2)}{4 (-1 + a x)^2}\right]}{a \sqrt{c}}$$

Problem 516: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{\left(c - \frac{c}{a x}\right)^{3/2}} dx$$

Optimal (type 3, 198 leaves, 9 steps):

$$\frac{\sqrt{1-ax}\sqrt{1+ax}}{a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{2(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{5(1-ax)^{3/2}\text{ArcSinh}[\sqrt{a}\sqrt{x}]}{a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}} - \frac{7(1-ax)^{3/2}\text{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right]}{\sqrt{2}a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

Result (type 3, 211 leaves):

$$\frac{1}{4a} \left(-\frac{4a\sqrt{c-\frac{c}{ax}}x(-2+ax)\sqrt{1-a^2x^2}}{c^2(-1+ax)^2} + \frac{10i\text{Log}\left[-i\sqrt{c}(1+2ax) + \frac{2a\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}}{-1+ax}\right]}{c^{3/2}} - \frac{7i\sqrt{2}\text{Log}\left[\frac{4ac\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}-i\sqrt{2}c^{3/2}(-1-2ax+3a^2x^2)}{7(-1+ax)^2}\right]}{c^{3/2}} \right)$$

Problem 517: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{\left(c - \frac{c}{a x}\right)^{5/2}} dx$$

Optimal (type 3, 249 leaves, 10 steps):

$$\frac{\sqrt{1-ax}\sqrt{1+ax}}{2a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{5/2}x} - \frac{23(1-ax)^{5/2}\sqrt{1+ax}}{8a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} - \frac{7(1-ax)^{5/2}\text{ArcSinh}[\sqrt{a}\sqrt{x}]}{a^{7/2}\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} + \frac{79(1-ax)^{5/2}\text{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right]}{8\sqrt{2}a^{7/2}\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

Result (type 3, 222 leaves):

$$\frac{1}{32 a} \left(\frac{4 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} (23 - 35 a x + 8 a^2 x^2)}{c^3 (-1 + a x)^3} + \frac{112 i \operatorname{Log} \left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right]}{c^{5/2}} - \frac{79 i \sqrt{2} \operatorname{Log} \left[\frac{32 a c^2 \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} - 8 i \sqrt{2} c^{5/2} (-1 - 2 a x + 3 a^2 x^2)}{79 (-1 + a x)^2} \right]}{c^{5/2}} \right)$$

Problem 527: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{9/2} dx$$

Optimal (type 3, 223 leaves, 8 steps):

$$-\frac{3 a^3 \left(c - \frac{c}{a x} \right)^{9/2} x^4 \sqrt{1 + a x}}{(1 - a x)^{9/2}} + \frac{3 a^2 \left(c - \frac{c}{a x} \right)^{9/2} x^3 (6 - 17 a x) (1 + a x)^{3/2}}{35 (1 - a x)^{9/2}} + \frac{6 a \left(c - \frac{c}{a x} \right)^{9/2} x^2 (1 + a x)^{3/2}}{35 (1 - a x)^{5/2}} - \frac{2 \left(c - \frac{c}{a x} \right)^{9/2} x (1 + a x)^{3/2}}{7 (1 - a x)^{3/2}} + \frac{3 a^{7/2} \left(c - \frac{c}{a x} \right)^{9/2} x^{9/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - a x)^{9/2}}$$

Result (type 3, 151 leaves):

$$\frac{c^4 \sqrt{c - \frac{c}{a x}} \sqrt{1 - a^2 x^2} (10 - 26 a x - 12 a^2 x^2 + 164 a^3 x^3 + 35 a^4 x^4)}{35 a^4 x^3 (-1 + a x)} + \frac{3 i c^{9/2} \operatorname{Log} \left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right]}{2 a}$$

Problem 528: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{7/2} dx$$

Optimal (type 3, 217 leaves, 8 steps):

$$-\frac{a^3 \left(c - \frac{c}{ax}\right)^{7/2} x^4 \sqrt{1+ax}}{(1-ax)^{7/2}} + \frac{2a^2 \left(c - \frac{c}{ax}\right)^{7/2} x^3 (1+ax)^{3/2}}{3(1-ax)^{7/2}} - \frac{2 \left(c - \frac{c}{ax}\right)^{7/2} x (1+ax)^{5/2}}{5(1-ax)^{7/2}} + \frac{4a \left(c - \frac{c}{ax}\right)^{7/2} x^2 (1+ax)^{5/2}}{3(1-ax)^{7/2}} - \frac{a^{5/2} \left(c - \frac{c}{ax}\right)^{7/2} x^{7/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1-ax)^{7/2}}$$

Result (type 3, 143 leaves):

$$\frac{c^3 \sqrt{c - \frac{c}{ax}} \sqrt{1-a^2 x^2} (-6 + 8ax + 44a^2 x^2 + 15a^3 x^3)}{15a^3 x^2 (-1+ax)} + \frac{i c^{7/2} \text{Log}[-i \sqrt{c} (1+2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2 x^2}}{-1+ax}]}{2a}$$

Problem 529: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \text{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^{5/2} dx$$

Optimal (type 3, 176 leaves, 8 steps):

$$-\frac{a^2 \left(c - \frac{c}{ax}\right)^{5/2} x^3 \sqrt{1+ax}}{(1-ax)^{5/2}} + \frac{2a \left(c - \frac{c}{ax}\right)^{5/2} x^2 (1+ax)^{3/2}}{3(1-ax)^{5/2}} - \frac{2 \left(c - \frac{c}{ax}\right)^{5/2} x (1-a^2 x^2)^{5/2}}{3(1-ax)^5} - \frac{a^{3/2} \left(c - \frac{c}{ax}\right)^{5/2} x^{5/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1-ax)^{5/2}}$$

Result (type 3, 133 leaves):

$$\frac{c^2 \left(\frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1-a^2 x^2} (2+2ax+3a^2 x^2)}{x(-1+ax)} - 3i a \sqrt{c} \text{Log}[-i \sqrt{c} (1+2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2 x^2}}{-1+ax}] \right)}{6a^2}$$

Problem 530: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \text{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^{3/2} dx$$

Optimal (type 3, 127 leaves, 7 steps):

$$\frac{3a \left(c - \frac{c}{ax}\right)^{3/2} x^2 \sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{2 \left(c - \frac{c}{ax}\right)^{3/2} x (1+ax)^{3/2}}{(1-ax)^{3/2}} + \frac{3\sqrt{a} \left(c - \frac{c}{ax}\right)^{3/2} x^{3/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1-ax)^{3/2}}$$

Result (type 3, 118 leaves):

$$\frac{2c \sqrt{c - \frac{c}{ax}} (-2+ax) \sqrt{1-a^2x^2}}{-1+ax} - 3i c^{3/2} \operatorname{Log} \left[-i \sqrt{c} (1+2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2x^2}}{-1+ax} \right]}{2a}$$

Problem 531: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}} dx$$

Optimal (type 3, 155 leaves, 8 steps):

$$-\frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1+ax}}{\sqrt{1-ax}} - \frac{5 \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1-ax}} + \frac{4 \sqrt{2} \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1+ax}}\right]}{\sqrt{a} \sqrt{1-ax}}$$

Result (type 3, 204 leaves):

$$\frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2x^2}}{-1+ax} - \frac{5i \sqrt{c} \operatorname{Log} \left[-i \sqrt{c} (1+2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2x^2}}{-1+ax} \right]}{2a} + \frac{2i \sqrt{2} \sqrt{c} \operatorname{Log} \left[\frac{-4a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2x^2} + i \sqrt{2} \sqrt{c} (-1-2ax+3a^2x^2)}{8c(-1+ax)^2} \right]}{a}$$

Problem 532: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[ax]}}{\sqrt{c - \frac{c}{ax}}} dx$$

Optimal (type 3, 195 leaves, 9 steps):

$$\frac{2 \sqrt{1-ax} \sqrt{1+ax}}{a \sqrt{c - \frac{c}{ax}}} + \frac{(1+ax)^{3/2}}{a \sqrt{c - \frac{c}{ax}} \sqrt{1-ax}} + \frac{7 \sqrt{1-ax} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{3/2} \sqrt{c - \frac{c}{ax}} \sqrt{x}} - \frac{5 \sqrt{2} \sqrt{1-ax} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1+ax}}\right]}{a^{3/2} \sqrt{c - \frac{c}{ax}} \sqrt{x}}$$

Result (type 3, 210 leaves):

$$\frac{1}{2a} \left(\frac{2a \sqrt{c - \frac{c}{ax}} x (-3+ax) \sqrt{1-a^2x^2}}{c(-1+ax)^2} - \frac{7i \operatorname{Log} \left[-i \sqrt{c} (1+2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2x^2}}{-1+ax} \right]}{\sqrt{c}} + \frac{5i \sqrt{2} \operatorname{Log} \left[\frac{-4a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2x^2} + i \sqrt{2} \sqrt{c} (-1-2ax+3a^2x^2)}{10(-1+ax)^2} \right]}{\sqrt{c}} \right)$$

Problem 533: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[a x]}}{\left(c - \frac{c}{a x}\right)^{3/2}} dx$$

Optimal (type 3, 249 leaves, 10 steps):

$$-\frac{21(1-ax)^{3/2}\sqrt{1+ax}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} + \frac{(1+ax)^{3/2}}{2a\left(c-\frac{c}{ax}\right)^{3/2}\sqrt{1-ax}} - \frac{9\sqrt{1-ax}(1+ax)^{3/2}}{8a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{9(1-ax)^{3/2}\operatorname{ArcSinh}[\sqrt{a}\sqrt{x}]}{a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}} + \frac{51(1-ax)^{3/2}\operatorname{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right]}{4\sqrt{2}a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

Result (type 3, 220 leaves):

$$\frac{1}{16a} \left(\frac{4a\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}(15-23ax+4a^2x^2)}{c^2(-1+ax)^3} - \frac{72i\operatorname{Log}\left[-i\sqrt{c}(1+2ax) + \frac{2a\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}}{-1+ax}\right]}{c^{3/2}} + \frac{51i\sqrt{2}\operatorname{Log}\left[\frac{-16ac\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}+4i\sqrt{2}c^{3/2}(-1-2ax+3a^2x^2)}{51(-1+ax)^2}\right]}{c^{3/2}} \right)$$

Problem 534: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[a x]}}{\left(c - \frac{c}{a x}\right)^{5/2}} dx$$

Optimal (type 3, 293 leaves, 11 steps):

$$\frac{103(1-ax)^{5/2}\sqrt{1+ax}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{(1+ax)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{5/2}\sqrt{1-ax}} - \frac{13\sqrt{1-ax}(1+ax)^{3/2}}{24a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{43(1-ax)^{3/2}(1+ax)^{3/2}}{32a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{11(1-ax)^{5/2}\operatorname{ArcSinh}[\sqrt{a}\sqrt{x}]}{a^{7/2}\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} - \frac{249(1-ax)^{5/2}\operatorname{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right]}{16\sqrt{2}a^{7/2}\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

Result (type 3, 232 leaves):

$$\frac{1}{64 a} \left(\frac{4 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} (-219 + 554 a x - 415 a^2 x^2 + 48 a^3 x^3)}{3 c^3 (-1 + a x)^4} - \frac{352 i \operatorname{Log} \left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right]}{c^{5/2}} + \frac{249 i \sqrt{2} \operatorname{Log} \left[\frac{-64 a c^2 \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} + 16 i \sqrt{2} c^{5/2} (-1 - 2 a x + 3 a^2 x^2)}{249 (-1 + a x)^2} \right]}{c^{5/2}} \right)$$

Problem 535: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{9/2} dx$$

Optimal (type 3, 225 leaves, 8 steps):

$$-\frac{94 a^2 \left(c - \frac{c}{a x} \right)^{9/2} x^3 \sqrt{1 + a x}}{21 (1 - a x)^{5/2}} + \frac{6 a \left(c - \frac{c}{a x} \right)^{9/2} x^2 \sqrt{1 + a x}}{5 (1 - a x)^{3/2}} - \frac{2 \left(c - \frac{c}{a x} \right)^{9/2} x \sqrt{1 + a x}}{7 \sqrt{1 - a x}} + \frac{a^3 \left(c - \frac{c}{a x} \right)^{9/2} x^4 \sqrt{1 + a x} (2718 + 521 a x)}{105 (1 - a x)^{9/2}} + \frac{11 a^{7/2} \left(c - \frac{c}{a x} \right)^{9/2} x^{9/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - a x)^{9/2}}$$

Result (type 3, 151 leaves):

$$\frac{c^4 \sqrt{c - \frac{c}{a x}} \sqrt{1 - a^2 x^2} (30 - 246 a x + 1028 a^2 x^2 - 4156 a^3 x^3 + 105 a^4 x^4)}{105 a^4 x^3 (-1 + a x)} + \frac{11 i c^{9/2} \operatorname{Log} \left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right]}{2 a}$$

Problem 536: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{7/2} dx$$

Optimal (type 3, 179 leaves, 7 steps):

$$\frac{2 a \left(c - \frac{c}{a x} \right)^{7/2} x^2 \sqrt{1 + a x}}{(1 - a x)^{3/2}} - \frac{2 \left(c - \frac{c}{a x} \right)^{7/2} x \sqrt{1 + a x}}{5 \sqrt{1 - a x}} - \frac{a^2 \left(c - \frac{c}{a x} \right)^{7/2} x^3 \sqrt{1 + a x} (66 + 7 a x)}{5 (1 - a x)^{7/2}} - \frac{9 a^{5/2} \left(c - \frac{c}{a x} \right)^{7/2} x^{7/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - a x)^{7/2}}$$

Result (type 3, 143 leaves):

$$\frac{c^3 \sqrt{c - \frac{c}{ax}} \sqrt{1 - a^2 x^2} (-2 + 16 ax - 92 a^2 x^2 + 5 a^3 x^3)}{5 a^3 x^2 (-1 + ax)} + \frac{9 i c^{7/2} \operatorname{Log}[-i \sqrt{c} (1 + 2 ax) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax}]}{2 a}$$

Problem 537: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^{5/2} dx$$

Optimal (type 3, 137 leaves, 6 steps):

$$-\frac{2 \left(c - \frac{c}{ax}\right)^{5/2} x \sqrt{1 + ax}}{3 \sqrt{1 - ax}} + \frac{a \left(c - \frac{c}{ax}\right)^{5/2} x^2 (18 - ax) \sqrt{1 + ax}}{3 (1 - ax)^{5/2}} + \frac{7 a^{3/2} \left(c - \frac{c}{ax}\right)^{5/2} x^{5/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - ax)^{5/2}}$$

Result (type 3, 135 leaves):

$$\frac{c^2 \sqrt{c - \frac{c}{ax}} \sqrt{1 - a^2 x^2} (2 - 22 ax + 3 a^2 x^2)}{3 a^2 x (-1 + ax)} + \frac{7 i c^{5/2} \operatorname{Log}[-i \sqrt{c} (1 + 2 ax) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax}]}{2 a}$$

Problem 538: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^{3/2} dx$$

Optimal (type 3, 126 leaves, 6 steps):

$$-\frac{2 \left(c - \frac{c}{ax}\right)^{3/2} x \sqrt{1 + ax}}{(1 - ax)^{3/2}} + \frac{a \left(c - \frac{c}{ax}\right)^{3/2} x^2 \sqrt{1 + ax}}{(1 - ax)^{3/2}} - \frac{5 \sqrt{a} \left(c - \frac{c}{ax}\right)^{3/2} x^{3/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - ax)^{3/2}}$$

Result (type 3, 118 leaves):

$$\frac{2 c \sqrt{c - \frac{c}{ax}} (-2 + ax) \sqrt{1 - a^2 x^2}}{-1 + ax} + \frac{5 i c^{3/2} \operatorname{Log}[-i \sqrt{c} (1 + 2 ax) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax}]}{2 a}$$

Problem 539: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}} dx$$

Optimal (type 3, 90 leaves, 6 steps):

$$-\frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{1 - ax} + \frac{3 \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1 - ax}}$$

Result (type 3, 110 leaves):

$$\frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} + \frac{3 i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 ax) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax}]}{2 a}$$

Problem 540: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-\operatorname{ArcTanh}[ax]}}{\sqrt{c - \frac{c}{ax}}} dx$$

Optimal (type 3, 88 leaves, 6 steps):

$$\frac{\sqrt{1 - ax} \sqrt{1 + ax}}{a \sqrt{c - \frac{c}{ax}}} - \frac{\sqrt{1 - ax} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{3/2} \sqrt{c - \frac{c}{ax}} \sqrt{x}}$$

Result (type 3, 113 leaves):

$$\frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{c (-1 + ax)} + \frac{i \operatorname{Log}[-i \sqrt{c} (1 + 2 ax) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax}]}{2 a \sqrt{c}}$$

Problem 541: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-\operatorname{ArcTanh}[ax]}}{\left(c - \frac{c}{ax}\right)^{3/2}} dx$$

Optimal (type 3, 159 leaves, 9 steps):

$$-\frac{(1 - ax)^{3/2} \sqrt{1 + ax}}{a^2 \left(c - \frac{c}{ax}\right)^{3/2} x} - \frac{(1 - ax)^{3/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{5/2} \left(c - \frac{c}{ax}\right)^{3/2} x^{3/2}} + \frac{\sqrt{2} (1 - ax)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + ax}}\right]}{a^{5/2} \left(c - \frac{c}{ax}\right)^{3/2} x^{3/2}}$$

Result (type 3, 205 leaves):

$$\frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{c^2 (-1 + ax)} - \frac{i \operatorname{Log} \left[-i \sqrt{c} (1 + 2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} \right]}{2a c^{3/2}} + \frac{i \operatorname{Log} \left[\frac{-4ac \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} + i \sqrt{2} c^{3/2} (-1 - 2ax + 3a^2 x^2)}{2(-1 + ax)^2} \right]}{\sqrt{2} a c^{3/2}}$$

Problem 542: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-\operatorname{ArcTanh}[ax]}}{\left(c - \frac{c}{ax}\right)^{5/2}} dx$$

Optimal (type 3, 208 leaves, 9 steps):

$$\frac{(1 - ax)^{3/2} \sqrt{1 + ax}}{2a^2 \left(c - \frac{c}{ax}\right)^{5/2} x} + \frac{3(1 - ax)^{5/2} \sqrt{1 + ax}}{2a^3 \left(c - \frac{c}{ax}\right)^{5/2} x^2} + \frac{3(1 - ax)^{5/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{7/2} \left(c - \frac{c}{ax}\right)^{5/2} x^{5/2}} - \frac{9(1 - ax)^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + ax}}\right]}{2\sqrt{2} a^{7/2} \left(c - \frac{c}{ax}\right)^{5/2} x^{5/2}}$$

Result (type 3, 214 leaves):

$$\frac{1}{8a} \left(\frac{4a \sqrt{c - \frac{c}{ax}} x (-3 + 2ax) \sqrt{1 - a^2 x^2}}{c^3 (-1 + ax)^2} - \frac{12i \operatorname{Log} \left[-i \sqrt{c} (1 + 2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} \right]}{c^{5/2}} + \frac{9i \sqrt{2} \operatorname{Log} \left[\frac{-8ac^2 \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} + 2i \sqrt{2} c^{5/2} (-1 - 2ax + 3a^2 x^2)}{9(-1 + ax)^2} \right]}{c^{5/2}} \right)$$

Problem 543: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-\operatorname{ArcTanh}[ax]}}{\left(c - \frac{c}{ax}\right)^{7/2}} dx$$

Optimal (type 3, 252 leaves, 10 steps):

$$\frac{(1 - ax)^{3/2} \sqrt{1 + ax}}{4a^2 \left(c - \frac{c}{ax}\right)^{7/2} x} - \frac{15(1 - ax)^{5/2} \sqrt{1 + ax}}{16a^3 \left(c - \frac{c}{ax}\right)^{7/2} x^2} - \frac{35(1 - ax)^{7/2} \sqrt{1 + ax}}{16a^4 \left(c - \frac{c}{ax}\right)^{7/2} x^3} - \frac{5(1 - ax)^{7/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{9/2} \left(c - \frac{c}{ax}\right)^{7/2} x^{7/2}} + \frac{115(1 - ax)^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + ax}}\right]}{16\sqrt{2} a^{9/2} \left(c - \frac{c}{ax}\right)^{7/2} x^{7/2}}$$

Result (type 3, 222 leaves):

$$\frac{1}{64 a} \left(\frac{4 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} (35 - 55 a x + 16 a^2 x^2)}{c^4 (-1 + a x)^3} - \frac{160 i \operatorname{Log} \left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right]}{c^{7/2}} + \frac{115 i \sqrt{2} \operatorname{Log} \left[\frac{-64 a c^3 \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} + 16 i \sqrt{2} c^{7/2} (-1 - 2 a x + 3 a^2 x^2)}{115 (-1 + a x)^2} \right]}{c^{7/2}} \right)$$

Problem 554: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{9/2} dx$$

Optimal (type 3, 267 leaves, 9 steps):

$$\frac{5 a^4 \left(c - \frac{c}{a x} \right)^{9/2} x^5 (587 - 109 a x)}{7 (1 - a x)^{9/2} \sqrt{1 + a x}} + \frac{70 a^3 \left(c - \frac{c}{a x} \right)^{9/2} x^4}{(1 - a x)^{5/2} \sqrt{1 + a x}} - \frac{50 a^2 \left(c - \frac{c}{a x} \right)^{9/2} x^3}{7 (1 - a x)^{3/2} \sqrt{1 + a x}} + \frac{10 a \left(c - \frac{c}{a x} \right)^{9/2} x^2}{7 \sqrt{1 - a x} \sqrt{1 + a x}} - \frac{2 \left(c - \frac{c}{a x} \right)^{9/2} x \sqrt{1 - a x}}{7 \sqrt{1 + a x}} - \frac{15 a^{7/2} \left(c - \frac{c}{a x} \right)^{9/2} x^{9/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - a x)^{9/2}}$$

Result (type 3, 152 leaves):

$$\frac{c^4 \sqrt{c - \frac{c}{a x}} (-2 + 20 a x - 110 a^2 x^2 + 720 a^3 x^3 + 1755 a^4 x^4 + 7 a^5 x^5)}{7 a^4 x^3 \sqrt{1 - a^2 x^2}} - \frac{15 i c^{9/2} \operatorname{Log} \left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right]}{2 a}$$

Problem 555: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{7/2} dx$$

Optimal (type 3, 225 leaves, 8 steps):

$$\begin{aligned}
& - \frac{a^3 \left(c - \frac{c}{ax}\right)^{7/2} x^4 (2525 - 427 ax)}{15 (1 - ax)^{7/2} \sqrt{1 + ax}} - \frac{398 a^2 \left(c - \frac{c}{ax}\right)^{7/2} x^3}{15 (1 - ax)^{3/2} \sqrt{1 + ax}} + \\
& \frac{38 a \left(c - \frac{c}{ax}\right)^{7/2} x^2}{15 \sqrt{1 - ax} \sqrt{1 + ax}} - \frac{2 \left(c - \frac{c}{ax}\right)^{7/2} x \sqrt{1 - ax}}{5 \sqrt{1 + ax}} + \frac{13 a^{5/2} \left(c - \frac{c}{ax}\right)^{7/2} x^{7/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - ax)^{7/2}}
\end{aligned}$$

Result (type 3, 144 leaves):

$$\frac{c^3 \sqrt{c - \frac{c}{ax}} (6 - 62 ax + 548 a^2 x^2 + 1591 a^3 x^3 + 15 a^4 x^4)}{15 a^3 x^2 \sqrt{1 - a^2 x^2}} - \frac{13 i c^{7/2} \text{Log}\left[-i \sqrt{c} (1 + 2 ax) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax}\right]}{2 a}$$

Problem 556: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \text{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^{5/2} dx$$

Optimal (type 3, 181 leaves, 7 steps):

$$\frac{a^2 \left(c - \frac{c}{ax}\right)^{5/2} x^3 (191 - 25 ax)}{3 (1 - ax)^{5/2} \sqrt{1 + ax}} + \frac{26 a \left(c - \frac{c}{ax}\right)^{5/2} x^2}{3 \sqrt{1 - ax} \sqrt{1 + ax}} - \frac{2 \left(c - \frac{c}{ax}\right)^{5/2} x \sqrt{1 - ax}}{3 \sqrt{1 + ax}} - \frac{11 a^{3/2} \left(c - \frac{c}{ax}\right)^{5/2} x^{5/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - ax)^{5/2}}$$

Result (type 3, 134 leaves):

$$\frac{c^2 \left(\frac{2 \sqrt{c - \frac{c}{ax}} (-2 + 32 ax + 133 a^2 x^2 + 3 a^3 x^3)}{x \sqrt{1 - a^2 x^2}} - 33 i a \sqrt{c} \text{Log}\left[-i \sqrt{c} (1 + 2 ax) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax}\right] \right)}{6 a^2}$$

Problem 557: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \text{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^{3/2} dx$$

Optimal (type 3, 133 leaves, 6 steps):

$$- \frac{2 \left(c - \frac{c}{ax}\right)^{3/2} x \sqrt{1 - ax}}{\sqrt{1 + ax}} - \frac{a \left(c - \frac{c}{ax}\right)^{3/2} x^2 (23 - ax)}{(1 - ax)^{3/2} \sqrt{1 + ax}} + \frac{9 \sqrt{a} \left(c - \frac{c}{ax}\right)^{3/2} x^{3/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - ax)^{3/2}}$$

Result (type 3, 119 leaves):

$$\frac{2c \sqrt{c - \frac{c}{ax}} (2 + 19ax + a^2x^2) - 9i c^{3/2} \operatorname{Log}\left[-i\sqrt{c} (1 + 2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2x^2}}{-1 + ax}\right]}{\sqrt{1 - a^2x^2}}}{2a}$$

Problem 558: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}} dx$$

Optimal (type 3, 123 leaves, 6 steps):

$$\frac{8 \sqrt{c - \frac{c}{ax}} x}{\sqrt{1 - ax} \sqrt{1 + ax}} + \frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1 + ax}}{\sqrt{1 - ax}} - \frac{7 \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1 - ax}}$$

Result (type 3, 108 leaves):

$$\frac{\sqrt{c - \frac{c}{ax}} x (9 + ax)}{\sqrt{1 - a^2x^2}} - \frac{7i \sqrt{c} \operatorname{Log}\left[-i\sqrt{c} (1 + 2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2x^2}}{-1 + ax}\right]}{2a}$$

Problem 559: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-3 \operatorname{ArcTanh}[ax]}}{\sqrt{c - \frac{c}{ax}}} dx$$

Optimal (type 3, 127 leaves, 7 steps):

$$-\frac{5 \sqrt{1 - ax}}{a \sqrt{c - \frac{c}{ax}} \sqrt{1 + ax}} - \frac{x (1 - ax)}{\sqrt{c - \frac{c}{ax}} \sqrt{1 - a^2x^2}} + \frac{5 \sqrt{1 - ax} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{3/2} \sqrt{c - \frac{c}{ax}} \sqrt{x}}$$

Result (type 3, 140 leaves):

$$\frac{\sqrt{\frac{c(-1+ax)}{ax}} \sqrt{1 - a^2x^2} \left(-\frac{1}{c} - \frac{3}{c(-1+ax)} - \frac{2}{c(1+ax)}\right)}{a} - \frac{5i \operatorname{Log}\left[-\frac{i(c+2acx)}{\sqrt{c}} + \frac{2ax \sqrt{\frac{c(-1+ax)}{ax}} \sqrt{1 - a^2x^2}}{-1 + ax}\right]}{2a \sqrt{c}}$$

Problem 560: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-3 \operatorname{ArcTanh}[a x]}}{\left(c - \frac{c}{a x}\right)^{3/2}} dx$$

Optimal (type 3, 131 leaves, 7 steps):

$$-\frac{2(1-ax)^{3/2}}{a\left(c - \frac{c}{ax}\right)^{3/2}\sqrt{1+ax}} + \frac{3(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c - \frac{c}{ax}\right)^{3/2}x} - \frac{3(1-ax)^{3/2}\operatorname{ArcSinh}[\sqrt{a}\sqrt{x}]}{a^{5/2}\left(c - \frac{c}{ax}\right)^{3/2}x^{3/2}}$$

Result (type 3, 111 leaves):

$$\frac{\sqrt{c - \frac{c}{ax}} x (3 + ax)}{c^2 \sqrt{1 - a^2 x^2}} - \frac{3 i \operatorname{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + a x}]}{2 a c^{3/2}}$$

Problem 561: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-3 \operatorname{ArcTanh}[a x]}}{\left(c - \frac{c}{a x}\right)^{5/2}} dx$$

Optimal (type 3, 199 leaves, 9 steps):

$$\frac{(1-ax)^{5/2}}{a^2\left(c - \frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}} - \frac{2(1-ax)^{5/2}\sqrt{1+ax}}{a^3\left(c - \frac{c}{ax}\right)^{5/2}x^2} + \frac{(1-ax)^{5/2}\operatorname{ArcSinh}[\sqrt{a}\sqrt{x}]}{a^{7/2}\left(c - \frac{c}{ax}\right)^{5/2}x^{5/2}} + \frac{(1-ax)^{5/2}\operatorname{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right]}{\sqrt{2}a^{7/2}\left(c - \frac{c}{ax}\right)^{5/2}x^{5/2}}$$

Result (type 3, 205 leaves):

$$\frac{1}{4c^3} \left(\frac{4\sqrt{c - \frac{c}{ax}} x (2 + ax)}{\sqrt{1 - a^2 x^2}} - \frac{2 i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + a x}]}{a} - \frac{i \sqrt{2} \sqrt{c} \operatorname{Log}\left[\frac{4 a c^2 \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} - i \sqrt{2} c^{5/2} (-1 - 2 a x + 3 a^2 x^2)}{(-1 + a x)^2}\right]}{a} \right)$$

Problem 562: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-3 \operatorname{ArcTanh}[a x]}}{\left(c - \frac{c}{a x}\right)^{7/2}} dx$$

Optimal (type 3, 251 leaves, 10 steps):

$$\frac{(1-ax)^{5/2}}{2a^2\left(c-\frac{c}{ax}\right)^{7/2}x\sqrt{1+ax}} - \frac{(1-ax)^{7/2}}{4a^3\left(c-\frac{c}{ax}\right)^{7/2}x^2\sqrt{1+ax}} + \frac{7(1-ax)^{7/2}\sqrt{1+ax}}{4a^4\left(c-\frac{c}{ax}\right)^{7/2}x^3} + \frac{(1-ax)^{7/2}\text{ArcSinh}[\sqrt{a}\sqrt{x}]}{a^{9/2}\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}} - \frac{11(1-ax)^{7/2}\text{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right]}{4\sqrt{2}a^{9/2}\left(c-\frac{c}{ax}\right)^{7/2}x^{7/2}}$$

Result (type 3, 228 leaves):

$$\frac{1}{16a} \left(\frac{4a\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}(-7+ax+4a^2x^2)}{c^4(-1+ax)^2(1+ax)} + \frac{8i\text{Log}\left[-i\sqrt{c}(1+2ax) + \frac{2a\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}}{-1+ax}\right]}{c^{7/2}} - \frac{11i\sqrt{2}\text{Log}\left[\frac{16ac^3\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}-4i\sqrt{2}c^{7/2}(-1-2ax+3a^2x^2)}{11(-1+ax)^2}\right]}{c^{7/2}} \right)$$

Problem 565: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcTanh}[ax]} \sqrt{c-\frac{c}{ax}} x^2 dx$$

Optimal (type 3, 179 leaves, 8 steps):

$$-\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{8a^2\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^2\sqrt{1+ax}}{12a\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}x^3\sqrt{1+ax}}{3\sqrt{1-ax}} + \frac{\sqrt{c-\frac{c}{ax}}\sqrt{x}\text{ArcSinh}[\sqrt{a}\sqrt{x}]}{8a^{5/2}\sqrt{1-ax}}$$

Result (type 3, 128 leaves):

$$-\frac{2a\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}(-3+2ax+8a^2x^2)}{-1+ax} + \frac{3i\sqrt{c}\text{Log}\left[-i\sqrt{c}(1+2ax) + \frac{2a\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}}{-1+ax}\right]}{48a^3}$$

Problem 566: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} x \, dx$$

Optimal (type 3, 135 leaves, 7 steps):

$$\frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{4 a \sqrt{1 - a x}} + \frac{\sqrt{c - \frac{c}{a x}} x^2 \sqrt{1 + a x}}{2 \sqrt{1 - a x}} - \frac{\sqrt{c - \frac{c}{a x}} \sqrt{x} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{4 a^{3/2} \sqrt{1 - a x}}$$

Result (type 3, 120 leaves):

$$\frac{2 a \sqrt{c - \frac{c}{a x}} x (1 + 2 a x) \sqrt{1 - a^2 x^2}}{-1 + a x} + \frac{i \sqrt{c} \text{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}]}{8 a^2}$$

Problem 567: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} \, dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$\frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{\sqrt{1 - a x}} + \frac{\sqrt{c - \frac{c}{a x}} \sqrt{x} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1 - a x}}$$

Result (type 3, 111 leaves):

$$\frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} + \frac{i \sqrt{c} \text{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}]}{2 a}$$

Problem 568: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\text{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}}}{x} \, dx$$

Optimal (type 3, 86 leaves, 6 steps):

$$-\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1+ax}}{\sqrt{1-ax}} + \frac{2\sqrt{a}\sqrt{c-\frac{c}{ax}}\sqrt{x}\operatorname{ArcSinh}[\sqrt{a}\sqrt{x}]}{\sqrt{1-ax}}$$

Result (type 3, 105 leaves):

$$\frac{2\sqrt{c-\frac{c}{ax}}\sqrt{1-a^2x^2}}{-1+ax} + i\sqrt{c}\operatorname{Log}[-i\sqrt{c}(1+2ax)] + \frac{2a\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}}{-1+ax}$$

Problem 582: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3\operatorname{ArcTanh}[ax]}\sqrt{c-\frac{c}{ax}}x^3dx$$

Optimal (type 3, 292 leaves, 11 steps):

$$\begin{aligned} &-\frac{107\sqrt{c-\frac{c}{ax}}x\sqrt{1+ax}}{64a^3\sqrt{1-ax}} - \frac{21\sqrt{c-\frac{c}{ax}}x(1+ax)^{3/2}}{32a^3\sqrt{1-ax}} - \frac{11\sqrt{c-\frac{c}{ax}}x^2(1+ax)^{3/2}}{24a^2\sqrt{1-ax}} - \\ &\frac{\sqrt{c-\frac{c}{ax}}x^3(1+ax)^{3/2}}{4a\sqrt{1-ax}} - \frac{363\sqrt{c-\frac{c}{ax}}\sqrt{x}\operatorname{ArcSinh}[\sqrt{a}\sqrt{x}]}{64a^{7/2}\sqrt{1-ax}} + \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\sqrt{x}\operatorname{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right]}{a^{7/2}\sqrt{1-ax}} \end{aligned}$$

Result (type 3, 231 leaves):

$$\begin{aligned} &\frac{1}{384a^4} \left(\frac{2a\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}(447+214ax+136a^2x^2+48a^3x^3)}{-1+ax} - 1089i\sqrt{c}\operatorname{Log}[-i\sqrt{c}(1+2ax)] + \frac{2a\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}}{-1+ax} \right) + \\ &\left. \frac{768i\sqrt{2}\sqrt{c}\operatorname{Log}\left[\frac{i a^4 \left(4 i a \sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2} + \sqrt{2}\sqrt{c}(-1-2ax+3a^2x^2) \right)}{8c(-1+ax)^2}\right]}{\right) \end{aligned}$$

Problem 583: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \operatorname{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} x^2 dx$$

Optimal (type 3, 248 leaves, 10 steps):

$$\frac{13 \sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{8 a^2 \sqrt{1 - a x}} - \frac{3 \sqrt{c - \frac{c}{a x}} x (1 + a x)^{3/2}}{4 a^2 \sqrt{1 - a x}} - \frac{\sqrt{c - \frac{c}{a x}} x^2 (1 + a x)^{3/2}}{3 a \sqrt{1 - a x}} - \frac{45 \sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{8 a^{5/2} \sqrt{1 - a x}} + \frac{4 \sqrt{2} \sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + a x}}\right]}{a^{5/2} \sqrt{1 - a x}}$$

Result (type 3, 223 leaves):

$$\frac{1}{48 a^3} \left(\frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} (57 + 26 a x + 8 a^2 x^2)}{-1 + a x} - 135 i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x)] + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right) + 96 i \sqrt{2} \sqrt{c} \operatorname{Log}\left[\frac{i a^3 \left(4 i a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} + \sqrt{2} \sqrt{c} (-1 - 2 a x + 3 a^2 x^2) \right)}{8 c (-1 + a x)^2} \right]$$

Problem 584: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \operatorname{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} x dx$$

Optimal (type 3, 204 leaves, 9 steps):

$$\frac{7 \sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{4 a \sqrt{1 - a x}} - \frac{\sqrt{c - \frac{c}{a x}} x (1 + a x)^{3/2}}{2 a \sqrt{1 - a x}} - \frac{23 \sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{4 a^{3/2} \sqrt{1 - a x}} + \frac{4 \sqrt{2} \sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + a x}}\right]}{a^{3/2} \sqrt{1 - a x}}$$

Result (type 3, 211 leaves):

$$\frac{1}{8a^2} \left(\frac{2a \sqrt{c - \frac{c}{ax}} x (9 + 2ax) \sqrt{1 - a^2 x^2}}{-1 + ax} - 23i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2ax)] + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} \right) + 16i \sqrt{2} \sqrt{c} \operatorname{Log} \left[\frac{-4a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} + i \sqrt{2} \sqrt{c} (-1 - 2ax + 3a^2 x^2)}{8c (-1 + ax)^2} \right]$$

Problem 585: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}} dx$$

Optimal (type 3, 155 leaves, 8 steps):

$$-\frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1 + ax}}{\sqrt{1 - ax}} - \frac{5 \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1 - ax}} + \frac{4 \sqrt{2} \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + ax}}\right]}{\sqrt{a} \sqrt{1 - ax}}$$

Result (type 3, 204 leaves):

$$\frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} - \frac{5i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2ax)] + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax}}{2a} + \frac{2i \sqrt{2} \sqrt{c} \operatorname{Log}\left[\frac{-4a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} + i \sqrt{2} \sqrt{c} (-1 - 2ax + 3a^2 x^2)}{8c (-1 + ax)^2}\right]}{a}$$

Problem 586: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}}}{x} dx$$

Optimal (type 3, 154 leaves, 8 steps):

$$-\frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1 + ax}}{\sqrt{1 - ax}} - \frac{2 \sqrt{a} \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{1 - ax}} + \frac{4 \sqrt{2} \sqrt{a} \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + ax}}\right]}{\sqrt{1 - ax}}$$

Result (type 3, 196 leaves):

$$\frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1 - a^2 x^2}}{-1 + ax} - i \sqrt{c} \operatorname{Log} \left[-i \sqrt{c} (1 + 2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} \right] +$$

$$2i \sqrt{2} \sqrt{c} \operatorname{Log} \left[\frac{-4a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} + i \sqrt{2} \sqrt{c} (-1 - 2ax + 3a^2 x^2)}{8c (-1 + ax)^2} \right]$$

Problem 587: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}}}{x^2} dx$$

Optimal (type 3, 147 leaves, 6 steps):

$$-\frac{4a \sqrt{c - \frac{c}{ax}} \sqrt{1+ax}}{\sqrt{1-ax}} - \frac{2 \sqrt{c - \frac{c}{ax}} (1+ax)^{3/2}}{3x \sqrt{1-ax}} + \frac{4 \sqrt{2} a^{3/2} \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1+ax}} \right]}{\sqrt{1-ax}}$$

Result (type 3, 145 leaves):

$$\frac{2 \sqrt{c - \frac{c}{ax}} (1 + 7ax) \sqrt{1 - a^2 x^2}}{3x (-1 + ax)} + 2i \sqrt{2} a \sqrt{c} \operatorname{Log} \left[\frac{-4a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} + i \sqrt{2} \sqrt{c} (-1 - 2ax + 3a^2 x^2)}{8ac (-1 + ax)^2} \right]$$

Problem 588: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}}}{x^3} dx$$

Optimal (type 3, 191 leaves, 7 steps):

$$-\frac{4a^2 \sqrt{c - \frac{c}{ax}} \sqrt{1+ax}}{\sqrt{1-ax}} - \frac{2a \sqrt{c - \frac{c}{ax}} (1+ax)^{3/2}}{3x \sqrt{1-ax}} - \frac{2 \sqrt{c - \frac{c}{ax}} (1+ax)^{5/2}}{5x^2 \sqrt{1-ax}} + \frac{4 \sqrt{2} a^{5/2} \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1+ax}} \right]}{\sqrt{1-ax}}$$

Result (type 3, 155 leaves):

$$\frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1 - a^2 x^2} (3 + 11 ax + 38 a^2 x^2)}{15 x^2 (-1 + ax)} + 2 i \sqrt{2} a^2 \sqrt{c} \operatorname{Log} \left[\frac{-4 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} + i \sqrt{2} \sqrt{c} (-1 - 2 ax + 3 a^2 x^2)}{8 a^2 c (-1 + ax)^2} \right]$$

Problem 589: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}}}{x^4} dx$$

Optimal (type 3, 237 leaves, 9 steps):

$$\frac{104 a^3 \sqrt{c - \frac{c}{ax}} \sqrt{1 + ax}}{21 \sqrt{1 - ax}} - \frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1 + ax}}{7 x^3 \sqrt{1 - ax}} - \frac{6 a \sqrt{c - \frac{c}{ax}} \sqrt{1 + ax}}{7 x^2 \sqrt{1 - ax}} - \frac{32 a^2 \sqrt{c - \frac{c}{ax}} \sqrt{1 + ax}}{21 x \sqrt{1 - ax}} + \frac{4 \sqrt{2} a^{7/2} \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + ax}} \right]}{\sqrt{1 - ax}}$$

Result (type 3, 163 leaves):

$$\frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1 - a^2 x^2} (3 + 9 ax + 16 a^2 x^2 + 52 a^3 x^3)}{21 x^3 (-1 + ax)} + 2 i \sqrt{2} a^3 \sqrt{c} \operatorname{Log} \left[\frac{-4 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} + i \sqrt{2} \sqrt{c} (-1 - 2 ax + 3 a^2 x^2)}{8 a^3 c (-1 + ax)^2} \right]$$

Problem 590: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}}}{x^5} dx$$

Optimal (type 3, 281 leaves, 10 steps):

$$\frac{1576 a^4 \sqrt{c - \frac{c}{ax}} \sqrt{1 + ax}}{315 \sqrt{1 - ax}} - \frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1 + ax}}{9 x^4 \sqrt{1 - ax}} - \frac{38 a \sqrt{c - \frac{c}{ax}} \sqrt{1 + ax}}{63 x^3 \sqrt{1 - ax}} - \frac{92 a^2 \sqrt{c - \frac{c}{ax}} \sqrt{1 + ax}}{105 x^2 \sqrt{1 - ax}} - \frac{472 a^3 \sqrt{c - \frac{c}{ax}} \sqrt{1 + ax}}{315 x \sqrt{1 - ax}} + \frac{4 \sqrt{2} a^{9/2} \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + ax}} \right]}{\sqrt{1 - ax}}$$

Result (type 3, 171 leaves):

$$\frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1 - a^2 x^2} (35 + 95 ax + 138 a^2 x^2 + 236 a^3 x^3 + 788 a^4 x^4)}{315 x^4 (-1 + ax)} + 2 i \sqrt{2} a^4 \sqrt{c} \operatorname{Log} \left[\frac{-4 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} + i \sqrt{2} \sqrt{c} (-1 - 2 ax + 3 a^2 x^2)}{8 a^4 c (-1 + ax)^2} \right]$$

Problem 592: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}} x^2 dx$$

Optimal (type 3, 182 leaves, 8 steps):

$$-\frac{11 \sqrt{c - \frac{c}{ax}} x \sqrt{1 + ax}}{8 a^2 \sqrt{1 - ax}} + \frac{11 \sqrt{c - \frac{c}{ax}} x^2 \sqrt{1 + ax}}{12 a \sqrt{1 - ax}} - \frac{\sqrt{c - \frac{c}{ax}} x^3 \sqrt{1 - a^2 x^2}}{3 (1 - ax)} + \frac{11 \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{8 a^{5/2} \sqrt{1 - ax}}$$

Result (type 3, 128 leaves):

$$\frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} (33 - 22 ax + 8 a^2 x^2)}{-1 + ax} + 33 i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 ax) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax}]$$

$$48 a^3$$

Problem 593: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}} x dx$$

Optimal (type 3, 138 leaves, 7 steps):

$$\frac{7 \sqrt{c - \frac{c}{ax}} x \sqrt{1 + ax}}{4 a \sqrt{1 - ax}} - \frac{\sqrt{c - \frac{c}{ax}} x^2 \sqrt{1 - a^2 x^2}}{2 (1 - ax)} - \frac{7 \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{4 a^{3/2} \sqrt{1 - ax}}$$

Result (type 3, 120 leaves):

$$\frac{2 a \sqrt{c - \frac{c}{ax}} x (-7 + 2 ax) \sqrt{1 - a^2 x^2}}{-1 + ax} - 7 i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 ax) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax}]$$

$$8 a^2$$

Problem 594: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\text{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} dx$$

Optimal (type 3, 90 leaves, 6 steps):

$$-\frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{1 - a x} + \frac{3 \sqrt{c - \frac{c}{a x}} \sqrt{x} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1 - a x}}$$

Result (type 3, 110 leaves):

$$\frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} + \frac{3 i \sqrt{c} \text{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}]}{2 a}$$

Problem 595: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-\text{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}}}{x} dx$$

Optimal (type 3, 89 leaves, 6 steps):

$$-\frac{2 \sqrt{c - \frac{c}{a x}} \sqrt{1 - a^2 x^2}}{1 - a x} - \frac{2 \sqrt{a} \sqrt{c - \frac{c}{a x}} \sqrt{x} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{1 - a x}}$$

Result (type 3, 105 leaves):

$$\frac{2 \sqrt{c - \frac{c}{a x}} \sqrt{1 - a^2 x^2}}{-1 + a x} - i \sqrt{c} \text{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}]$$

Problem 608: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \text{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} x^3 dx$$

Optimal (type 3, 262 leaves, 9 steps):

$$\frac{8 \sqrt{c - \frac{c}{ax}} x^4}{\sqrt{1-ax} \sqrt{1+ax}} - \frac{1115 \sqrt{c - \frac{c}{ax}} x \sqrt{1+ax}}{64 a^3 \sqrt{1-ax}} + \frac{1115 \sqrt{c - \frac{c}{ax}} x^2 \sqrt{1+ax}}{96 a^2 \sqrt{1-ax}} -$$

$$\frac{223 \sqrt{c - \frac{c}{ax}} x^3 \sqrt{1+ax}}{24 a \sqrt{1-ax}} + \frac{\sqrt{c - \frac{c}{ax}} x^4 \sqrt{1+ax}}{4 \sqrt{1-ax}} + \frac{1115 \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{64 a^{7/2} \sqrt{1-ax}}$$

Result (type 3, 137 leaves):

$$\frac{2 a \sqrt{c - \frac{c}{ax}} x (-3345 - 1115 a x + 446 a^2 x^2 - 200 a^3 x^3 + 48 a^4 x^4)}{\sqrt{1-a^2 x^2}} + 3345 i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x)] + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2 x^2}}{-1+ax}]$$

$$384 a^4$$

Problem 609: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}} x^2 dx$$

Optimal (type 3, 218 leaves, 8 steps):

$$\frac{8 \sqrt{c - \frac{c}{ax}} x^3}{\sqrt{1-ax} \sqrt{1+ax}} + \frac{119 \sqrt{c - \frac{c}{ax}} x \sqrt{1+ax}}{8 a^2 \sqrt{1-ax}} - \frac{119 \sqrt{c - \frac{c}{ax}} x^2 \sqrt{1+ax}}{12 a \sqrt{1-ax}} + \frac{\sqrt{c - \frac{c}{ax}} x^3 \sqrt{1+ax}}{3 \sqrt{1-ax}} - \frac{119 \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{8 a^{5/2} \sqrt{1-ax}}$$

Result (type 3, 129 leaves):

$$\frac{2 a \sqrt{c - \frac{c}{ax}} x (357 + 119 a x - 38 a^2 x^2 + 8 a^3 x^3)}{\sqrt{1-a^2 x^2}} - 357 i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x)] + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2 x^2}}{-1+ax}]$$

$$48 a^3$$

Problem 610: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}} x dx$$

Optimal (type 3, 174 leaves, 7 steps):

$$\frac{8 \sqrt{c - \frac{c}{ax}} x^2}{\sqrt{1-ax} \sqrt{1+ax}} - \frac{47 \sqrt{c - \frac{c}{ax}} x \sqrt{1+ax}}{4a \sqrt{1-ax}} + \frac{\sqrt{c - \frac{c}{ax}} x^2 \sqrt{1+ax}}{2 \sqrt{1-ax}} + \frac{47 \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{4a^{3/2} \sqrt{1-ax}}$$

Result (type 3, 121 leaves):

$$\frac{2a \sqrt{c - \frac{c}{ax}} x (-47 - 13ax + 2a^2 x^2)}{\sqrt{1-a^2 x^2}} + \frac{47 i \sqrt{c} \operatorname{Log}\left[-i \sqrt{c} (1 + 2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2 x^2}}{-1+ax}\right]}{8a^2}$$

Problem 611: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}} dx$$

Optimal (type 3, 123 leaves, 6 steps):

$$\frac{8 \sqrt{c - \frac{c}{ax}} x}{\sqrt{1-ax} \sqrt{1+ax}} + \frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1+ax}}{\sqrt{1-ax}} - \frac{7 \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1-ax}}$$

Result (type 3, 108 leaves):

$$\frac{\sqrt{c - \frac{c}{ax}} x (9 + ax)}{\sqrt{1-a^2 x^2}} - \frac{7 i \sqrt{c} \operatorname{Log}\left[-i \sqrt{c} (1 + 2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2 x^2}}{-1+ax}\right]}{2a}$$

Problem 612: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}}}{x} dx$$

Optimal (type 3, 124 leaves, 6 steps):

$$-\frac{2 \sqrt{c - \frac{c}{ax}}}{\sqrt{1-ax} \sqrt{1+ax}} - \frac{10a \sqrt{c - \frac{c}{ax}} x}{\sqrt{1-ax} \sqrt{1+ax}} + \frac{2 \sqrt{a} \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{1-ax}}$$

Result (type 3, 104 leaves):

$$-\frac{2\sqrt{c - \frac{c}{ax}}(1 + 5ax)}{\sqrt{1 - a^2x^2}} + i\sqrt{c} \operatorname{Log}\left[-i\sqrt{c}(1 + 2ax)\right] + \frac{2a\sqrt{c - \frac{c}{ax}}x\sqrt{1 - a^2x^2}}{-1 + ax}$$

Problem 617: Unable to integrate problem.

$$\int e^{n \operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$\frac{\left(c - \frac{c}{ax}\right)^p x (1 - ax)^{-p} \operatorname{AppellF1}\left[1 - p, \frac{1}{2}(n - 2p), -\frac{n}{2}, 2 - p, ax, -ax\right]}{1 - p}$$

Result (type 8, 24 leaves):

$$\int e^{n \operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Problem 618: Unable to integrate problem.

$$\int e^{-2p \operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Optimal (type 6, 54 leaves, 3 steps):

$$\frac{\left(c - \frac{c}{ax}\right)^p x (1 - ax)^{-p} \operatorname{AppellF1}\left[1 - p, -2p, p, 2 - p, ax, -ax\right]}{1 - p}$$

Result (type 8, 25 leaves):

$$\int e^{-2p \operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Problem 619: Unable to integrate problem.

$$\int e^{2p \operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Optimal (type 5, 50 leaves, 3 steps):

$$\frac{\left(c - \frac{c}{ax}\right)^p x (1 - ax)^{-p} \operatorname{Hypergeometric2F1}\left[1 - p, -p, 2 - p, -ax\right]}{1 - p}$$

Result (type 8, 25 leaves):

$$\int e^{2 p \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^p dx$$

Problem 624: Attempted integration timed out after 120 seconds.

$$\int e^{n \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{3/2} dx$$

Optimal (type 6, 54 leaves, 3 steps):

$$\frac{2 \left(c - \frac{c}{a x} \right)^{3/2} x \operatorname{AppellF1} \left[-\frac{1}{2}, \frac{1}{2} (-3 + n), -\frac{n}{2}, \frac{1}{2}, a x, -a x \right]}{(1 - a x)^{3/2}}$$

Result (type 1, 1 leaves):

???

Problem 625: Attempted integration timed out after 120 seconds.

$$\int e^{n \operatorname{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} dx$$

Optimal (type 6, 54 leaves, 3 steps):

$$\frac{2 \sqrt{c - \frac{c}{a x}} x \operatorname{AppellF1} \left[\frac{1}{2}, \frac{1}{2} (-1 + n), -\frac{n}{2}, \frac{3}{2}, a x, -a x \right]}{\sqrt{1 - a x}}$$

Result (type 1, 1 leaves):

???

Problem 626: Attempted integration timed out after 120 seconds.

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]} dx}{\sqrt{c - \frac{c}{a x}}}$$

Optimal (type 6, 56 leaves, 3 steps):

$$\frac{2 x \sqrt{1-a x} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+n}{2}, -\frac{n}{2}, \frac{5}{2}, a x, -a x\right]}{3 \sqrt{c-\frac{c}{a x}}}$$

Result (type 1, 1 leaves):

???

Problem 627: Attempted integration timed out after 120 seconds.

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]}}{\left(c-\frac{c}{a x}\right)^{3/2}} dx$$

Optimal (type 6, 56 leaves, 3 steps):

$$\frac{2 x (1-a x)^{3/2} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3+n}{2}, -\frac{n}{2}, \frac{7}{2}, a x, -a x\right]}{5 \left(c-\frac{c}{a x}\right)^{3/2}}$$

Result (type 1, 1 leaves):

???

Problem 789: Unable to integrate problem.

$$\int e^{-2 p \operatorname{ArcTanh}[a x]} \left(c-\frac{c}{a^2 x^2}\right)^p dx$$

Optimal (type 5, 53 leaves, 3 steps):

$$\frac{\left(c-\frac{c}{a^2 x^2}\right)^p x \left(1-a^2 x^2\right)^{-p} \operatorname{Hypergeometric2F1}\left[1-2 p, -2 p, 2-2 p, a x\right]}{1-2 p}$$

Result (type 8, 25 leaves):

$$\int e^{-2 p \operatorname{ArcTanh}[a x]} \left(c-\frac{c}{a^2 x^2}\right)^p dx$$

Problem 790: Unable to integrate problem.

$$\int e^{2 p \operatorname{ArcTanh}[a x]} \left(c-\frac{c}{a^2 x^2}\right)^p dx$$

Optimal (type 5, 54 leaves, 3 steps):

$$\frac{\left(c - \frac{c}{a^2 x^2}\right)^p x \left(1 - a^2 x^2\right)^{-p} \text{Hypergeometric2F1}\left[1 - 2p, -2p, 2 - 2p, -ax\right]}{1 - 2p}$$

Result (type 8, 25 leaves):

$$\int e^{2p \text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2}\right)^p dx$$

Problem 800: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2}\right)^p dx$$

Optimal (type 6, 72 leaves, 3 steps):

$$\frac{\left(c - \frac{c}{a^2 x^2}\right)^p x \left(1 - a^2 x^2\right)^{-p} \text{AppellF1}\left[1 - 2p, \frac{1}{2}(n - 2p), -\frac{n}{2} - p, 2 - 2p, ax, -ax\right]}{1 - 2p}$$

Result (type 8, 24 leaves):

$$\int e^{n \text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2}\right)^p dx$$

Problem 801: Result unnecessarily involves higher level functions.

$$\int e^{4 \text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2}\right)^p dx$$

Optimal (type 5, 339 leaves, 13 steps):

$$\frac{2a \left(c - \frac{c}{a^2 x^2}\right)^p x^2}{(1-p)(1-ax)(1+ax)} + \frac{\left(c - \frac{c}{a^2 x^2}\right)^p x (1-ax)^{-p} (1+ax)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2}(1-2p), 2-p, \frac{1}{2}(3-2p), a^2 x^2\right]}{1-2p} +$$

$$\frac{6a^2 \left(c - \frac{c}{a^2 x^2}\right)^p x^3 (1-ax)^{-p} (1+ax)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2}(3-2p), 2-p, \frac{1}{2}(5-2p), a^2 x^2\right]}{3-2p} +$$

$$\frac{a^4 \left(c - \frac{c}{a^2 x^2}\right)^p x^5 (1-ax)^{-p} (1+ax)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2}(5-2p), 2-p, \frac{1}{2}(7-2p), a^2 x^2\right]}{5-2p} +$$

$$\frac{2a^3 \left(c - \frac{c}{a^2 x^2}\right)^p x^4 (1-ax)^{-p} (1+ax)^{-p} \text{Hypergeometric2F1}\left[2-p, 2-p, 3-p, a^2 x^2\right]}{2-p}$$

Result (type 6, 319 leaves):

$$\left(c - \frac{c}{a^2 x^2} \right)^p x \left(\frac{1}{1-2p} \left(4 (-1+ax)^p \left(\frac{1-ax}{1+ax} \right)^{-p} (1+ax)^{-1+p} (-1+a^2 x^2)^{-p} \text{Hypergeometric2F1} \left[1-2p, 2-p, 2-2p, \frac{2ax}{1+ax} \right] + \right. \right. \\ \left. \left. (1-a^2 x^2)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2}-p, -p, \frac{3}{2}-p, a^2 x^2 \right] \right) - \right. \\ \left. \left(8 (-1+p) (1-ax)^{-p} (-1+ax)^{-1+p} (1-a^2 x^2)^p (-1+a^2 x^2)^{-p} \text{AppellF1} [1-2p, 1-p, -p, 2-2p, ax, -ax] \right) / \right. \\ \left. \left((-1+2p) (2(-1+p) \text{AppellF1} [1-2p, 1-p, -p, 2-2p, ax, -ax] + \right. \right. \\ \left. \left. ax ((-1+p) \text{AppellF1} [2-2p, 2-p, -p, 3-2p, ax, -ax] - p \text{HypergeometricPFQ} [\{1-p, 1-p\}, \{2-p\}, a^2 x^2]) \right) \right)$$

Problem 802: Unable to integrate problem.

$$\int e^{3 \text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Optimal (type 5, 217 leaves, 7 steps):

$$\frac{\left(c - \frac{c}{a^2 x^2} \right)^p x}{(1-2p) \sqrt{1-a^2 x^2}} - \frac{a \left(c - \frac{c}{a^2 x^2} \right)^p x^2}{\sqrt{1-a^2 x^2}} + \frac{3 a^2 \left(c - \frac{c}{a^2 x^2} \right)^p x^3 (1-a^2 x^2)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2} (3-2p), \frac{3}{2}-p, \frac{1}{2} (5-2p), a^2 x^2 \right]}{3-2p} + \\ \frac{a (5-2p) \left(c - \frac{c}{a^2 x^2} \right)^p x^2 (1-a^2 x^2)^{-p} \text{Hypergeometric2F1} \left[1-p, \frac{3}{2}-p, 2-p, a^2 x^2 \right]}{2(1-p)}$$

Result (type 8, 24 leaves):

$$\int e^{3 \text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Problem 803: Result unnecessarily involves higher level functions.

$$\int e^{2 \text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Optimal (type 5, 217 leaves, 10 steps):

$$\frac{\left(c - \frac{c}{a^2 x^2} \right)^p x (1-ax)^{-p} (1+ax)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2} (1-2p), 1-p, \frac{1}{2} (3-2p), a^2 x^2 \right]}{1-2p} + \\ \frac{a^2 \left(c - \frac{c}{a^2 x^2} \right)^p x^3 (1-ax)^{-p} (1+ax)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2} (3-2p), 1-p, \frac{1}{2} (5-2p), a^2 x^2 \right]}{3-2p} + \\ \frac{a \left(c - \frac{c}{a^2 x^2} \right)^p x^2 (1-ax)^{-p} (1+ax)^{-p} \text{Hypergeometric2F1} [1-p, 1-p, 2-p, a^2 x^2]}{1-p}$$

Result (type 6, 235 leaves):

$$\frac{1}{-1+2p} \left(c - \frac{c}{a^2 x^2} \right)^p x (1-a^2 x^2)^{-p} \left(\text{Hypergeometric2F1} \left[\frac{1}{2} - p, -p, \frac{3}{2} - p, a^2 x^2 \right] + \right. \\ \left. \left(4(-1+p)(1-ax)^{-p}(-1+ax)^{-1+p}(1-a^2 x^2)^{2p}(-1+a^2 x^2)^{-p} \text{AppellF1} [1-2p, 1-p, -p, 2-2p, ax, -ax] \right) / \right. \\ \left. \left(2(-1+p) \text{AppellF1} [1-2p, 1-p, -p, 2-2p, ax, -ax] + \right. \right. \\ \left. \left. ax \left((-1+p) \text{AppellF1} [2-2p, 2-p, -p, 3-2p, ax, -ax] - p \text{HypergeometricPFQ} [\{1-p, 1-p\}, \{2-p\}, a^2 x^2] \right) \right) \right)$$

Problem 804: Unable to integrate problem.

$$\int e^{\text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Optimal (type 5, 137 leaves, 5 steps):

$$\frac{\left(c - \frac{c}{a^2 x^2} \right)^p x (1-a^2 x^2)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2} (1-2p), \frac{1}{2} - p, \frac{1}{2} (3-2p), a^2 x^2 \right]}{1-2p} + \\ \frac{a \left(c - \frac{c}{a^2 x^2} \right)^p x^2 (1-a^2 x^2)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2} - p, 1-p, 2-p, a^2 x^2 \right]}{2(1-p)}$$

Result (type 8, 22 leaves):

$$\int e^{\text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Problem 805: Unable to integrate problem.

$$\int e^{-\text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Optimal (type 5, 137 leaves, 5 steps):

$$\frac{\left(c - \frac{c}{a^2 x^2} \right)^p x (1-a^2 x^2)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2} (1-2p), \frac{1}{2} - p, \frac{1}{2} (3-2p), a^2 x^2 \right]}{1-2p} - \\ \frac{a \left(c - \frac{c}{a^2 x^2} \right)^p x^2 (1-a^2 x^2)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2} - p, 1-p, 2-p, a^2 x^2 \right]}{2(1-p)}$$

Result (type 8, 24 leaves):

$$\int e^{-\text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Problem 806: Result unnecessarily involves higher level functions.

$$\int e^{-2 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Optimal (type 5, 218 leaves, 10 steps):

$$\frac{\left(c - \frac{c}{a^2 x^2} \right)^p x (1 - a x)^{-p} (1 + a x)^{-p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(1 - 2p), 1 - p, \frac{1}{2}(3 - 2p), a^2 x^2\right]}{1 - 2p} +$$

$$\frac{a^2 \left(c - \frac{c}{a^2 x^2} \right)^p x^3 (1 - a x)^{-p} (1 + a x)^{-p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(3 - 2p), 1 - p, \frac{1}{2}(5 - 2p), a^2 x^2\right]}{3 - 2p} -$$

$$\frac{a \left(c - \frac{c}{a^2 x^2} \right)^p x^2 (1 - a x)^{-p} (1 + a x)^{-p} \operatorname{Hypergeometric2F1}[1 - p, 1 - p, 2 - p, a^2 x^2]}{1 - p}$$

Result (type 6, 226 leaves):

$$\left(c - \frac{c}{a^2 x^2} \right)^p x \left(\frac{(1 - a^2 x^2)^{-p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2} - p, -p, \frac{3}{2} - p, a^2 x^2\right]}{-1 + 2p} + \right.$$

$$\left. \frac{\left(4(-1 + p)(-1 + a x)^p (1 + a x)^{-1+p} (-1 + a^2 x^2)^{-p} \operatorname{AppellF1}[1 - 2p, -p, 1 - p, 2 - 2p, a x, -a x] \right)}{\left((1 - 2p)(2(-1 + p) \operatorname{AppellF1}[1 - 2p, -p, 1 - p, 2 - 2p, a x, -a x] + \right.} \right.$$

$$\left. \left. a x (-1 + p) \operatorname{AppellF1}[2 - 2p, -p, 2 - p, 3 - 2p, a x, -a x] + p \operatorname{HypergeometricPFQ}[\{1 - p, 1 - p\}, \{2 - p\}, a^2 x^2] \right) \right)$$

Problem 807: Unable to integrate problem.

$$\int e^{-3 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Optimal (type 5, 216 leaves, 7 steps):

$$\frac{\left(c - \frac{c}{a^2 x^2} \right)^p x}{(1 - 2p) \sqrt{1 - a^2 x^2}} + \frac{a \left(c - \frac{c}{a^2 x^2} \right)^p x^2}{\sqrt{1 - a^2 x^2}} + \frac{3 a^2 \left(c - \frac{c}{a^2 x^2} \right)^p x^3 (1 - a^2 x^2)^{-p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}(3 - 2p), \frac{3}{2} - p, \frac{1}{2}(5 - 2p), a^2 x^2\right]}{3 - 2p} -$$

$$\frac{a(5 - 2p) \left(c - \frac{c}{a^2 x^2} \right)^p x^2 (1 - a^2 x^2)^{-p} \operatorname{Hypergeometric2F1}[1 - p, \frac{3}{2} - p, 2 - p, a^2 x^2]}{2(1 - p)}$$

Result (type 8, 24 leaves):

$$\int e^{-3 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Problem 808: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[x]} x \sqrt{1+x} \operatorname{Sin}[x] dx$$

Optimal (type 4, 240 leaves, 16 steps):

$$\begin{aligned} & 3 \sqrt{1-x} \operatorname{Cos}[x] - (1-x)^{3/2} \operatorname{Cos}[x] - 3 \sqrt{\frac{\pi}{2}} \operatorname{Cos}[1] \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] - \\ & \frac{3}{2} \sqrt{\frac{\pi}{2}} \operatorname{Cos}[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] + 2 \sqrt{2\pi} \operatorname{Cos}[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] + \frac{3}{2} \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \operatorname{Sin}[1] - \\ & 2 \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \operatorname{Sin}[1] - 3 \sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \operatorname{Sin}[1] - \frac{3}{2} \sqrt{1-x} \operatorname{Sin}[x] \end{aligned}$$

Result (type 4, 185 leaves):

$$\begin{aligned} & \frac{1}{8 \sqrt{1-x^2}} i \sqrt{1+x} \left((-11-i) \sqrt{\frac{\pi}{2}} \sqrt{-1+x} \operatorname{Erfi}\left[\frac{(1+i) \sqrt{-1+x}}{\sqrt{2}}\right] (\operatorname{Cos}[1] + i \operatorname{Sin}[1]) + ((-4-3i) + (2+3i)x + 2x^2) (2i \operatorname{Cos}[x] - 2 \operatorname{Sin}[x]) + \right. \\ & \left. \left(2((-3-4i) + (3+2i)x + 2ix^2) (\operatorname{Cos}[1] + i \operatorname{Sin}[1]) - (1+11i) \sqrt{\frac{\pi}{2}} \sqrt{-1+x} \operatorname{Erf}\left[\frac{(1+i) \sqrt{-1+x}}{\sqrt{2}}\right] (\operatorname{Cos}[x] + i \operatorname{Sin}[x]) \right) \right) \\ & (\operatorname{Cos}[1+x] - i \operatorname{Sin}[1+x]) \end{aligned}$$

Problem 809: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[x]} \sqrt{1+x} \operatorname{Sin}[x] dx$$

Optimal (type 4, 141 leaves, 11 steps):

$$\sqrt{1-x} \cos[x] - \sqrt{\frac{\pi}{2}} \cos[1] \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] + 2\sqrt{2\pi} \cos[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] -$$

$$2\sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \sin[1] - \sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \sin[1]$$

Result (type 4, 129 leaves):

$$\frac{1}{4} \left((1+4i) (-1)^{3/4} e^{-i} \sqrt{\pi} \operatorname{Erfi}\left[(-1)^{1/4} \sqrt{1-x}\right] + \frac{e^{-ix} \sqrt{1-x^2} \left(2(1+e^{2ix}) \sqrt{-1+x} + (1-4i) (-1)^{3/4} e^{i(1+x)} \sqrt{\pi} \operatorname{Erfi}\left[(-1)^{1/4} \sqrt{-1+x}\right]\right)}{\sqrt{-1+x} \sqrt{1+x}} \right)$$

Problem 810: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[x]} \sqrt{1-x} x \sin[x] dx$$

Optimal (type 4, 163 leaves, 13 steps):

$$\sqrt{1+x} \cos[x] - (1+x)^{3/2} \cos[x] - \sqrt{\frac{\pi}{2}} \cos[1] \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] - \frac{3}{2} \sqrt{\frac{\pi}{2}} \cos[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] +$$

$$\frac{3}{2} \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] \sin[1] - \sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] \sin[1] + \frac{3}{2} \sqrt{1+x} \sin[x]$$

Result (type 4, 168 leaves):

$$\frac{1}{\sqrt{1-x^2}} \left(\frac{1}{16} + \frac{i}{16} \right) e^{-i(1+x)} \sqrt{1-x} \left((-3-2i) e^{ix} \sqrt{2\pi} \sqrt{-1-x} \operatorname{Erf}\left[\frac{(1+i) \sqrt{-1-x}}{\sqrt{2}}\right] + \right.$$

$$\left. e^i \left((2+2i) (3+e^{2ix} (-3+2ix) + 2ix) (1+x) + (3-2i) e^{i(1+x)} \sqrt{2\pi} \sqrt{-1-x} \operatorname{Erfi}\left[\frac{(1+i) \sqrt{-1-x}}{\sqrt{2}}\right] \right) \right)$$

Problem 811: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[x]} \sqrt{1-x} \sin[x] dx$$

Optimal (type 4, 72 leaves, 7 steps):

$$-\sqrt{1+x} \operatorname{Cos}[x] + \sqrt{\frac{\pi}{2}} \operatorname{Cos}[1] \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] + \sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] \operatorname{Sin}[1]$$

Result (type 4, 138 leaves):

$$-\frac{1}{4\sqrt{-1-x}\sqrt{1-x}} e^{-i(1+x)} \sqrt{1-x^2} \left(2e^i (1+e^{2ix}) \sqrt{-1-x} + (-1)^{3/4} e^{i(2+x)} \sqrt{\pi} \operatorname{Erfi}\left[(-1)^{1/4} \sqrt{-1-x}\right] + (-1)^{1/4} e^{ix} \sqrt{\pi} \operatorname{Erfi}\left[(-1)^{3/4} \sqrt{-1-x}\right] \right)$$

Problem 812: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[x]} x (1+x)^{3/2} \operatorname{Sin}[x] dx$$

Optimal (type 4, 335 leaves, 22 steps):

$$\begin{aligned} & \frac{17}{4} \sqrt{1-x} \operatorname{Cos}[x] - 5(1-x)^{3/2} \operatorname{Cos}[x] + (1-x)^{5/2} \operatorname{Cos}[x] + \frac{15}{4} \sqrt{\frac{\pi}{2}} \operatorname{Cos}[1] \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] - \\ & 4\sqrt{2\pi} \operatorname{Cos}[1] \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] - \frac{15}{2} \sqrt{\frac{\pi}{2}} \operatorname{Cos}[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] + \\ & 4\sqrt{2\pi} \operatorname{Cos}[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] + \frac{15}{2} \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \operatorname{Sin}[1] - 4\sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \operatorname{Sin}[1] + \\ & \frac{15}{4} \sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \operatorname{Sin}[1] - 4\sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \operatorname{Sin}[1] - \frac{15}{2} \sqrt{1-x} \operatorname{Sin}[x] + \frac{5}{2} (1-x)^{3/2} \operatorname{Sin}[x] \end{aligned}$$

Result (type 4, 201 leaves):

$$\frac{1}{\sqrt{1-x^2}} \left(\frac{1}{32} + \frac{i}{32} \right) \sqrt{1+x}$$

$$\left((-2-17i) \sqrt{2\pi} \sqrt{-1+x} \operatorname{Erfi} \left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}} \right] (\cos[1] + i \sin[1]) - (2-2i) \left((-1-20i) - (11-10i)x + (8+10i)x^2 + 4x^3 \right) \right.$$

$$\left. (\cos[x] + i \sin[x]) - (1+i) \left(2 \left((-1+20i) - (11+10i)x + (8-10i)x^2 + 4x^3 \right) (-i \cos[1] + \sin[1]) + \right.$$

$$\left. (15+19i) \sqrt{\frac{\pi}{2}} \sqrt{-1+x} \operatorname{Erf} \left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}} \right] (\cos[x] + i \sin[x]) \right) (\cos[1+x] - i \sin[1+x]) \Bigg)$$

Problem 813: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[x]} (1+x)^{3/2} \sin[x] \, dx$$

Optimal (type 4, 236 leaves, 16 steps):

$$4\sqrt{1-x} \cos[x] - (1-x)^{3/2} \cos[x] - 2\sqrt{2\pi} \cos[1] \operatorname{FresnelC} \left[\sqrt{\frac{2}{\pi}} \sqrt{1-x} \right] -$$

$$\frac{3}{2} \sqrt{\frac{\pi}{2}} \cos[1] \operatorname{FresnelS} \left[\sqrt{\frac{2}{\pi}} \sqrt{1-x} \right] + 4\sqrt{2\pi} \cos[1] \operatorname{FresnelS} \left[\sqrt{\frac{2}{\pi}} \sqrt{1-x} \right] + \frac{3}{2} \sqrt{\frac{\pi}{2}} \operatorname{FresnelC} \left[\sqrt{\frac{2}{\pi}} \sqrt{1-x} \right] \sin[1] -$$

$$4\sqrt{2\pi} \operatorname{FresnelC} \left[\sqrt{\frac{2}{\pi}} \sqrt{1-x} \right] \sin[1] - 2\sqrt{2\pi} \operatorname{FresnelS} \left[\sqrt{\frac{2}{\pi}} \sqrt{1-x} \right] \sin[1] - \frac{3}{2} \sqrt{1-x} \sin[x]$$

Result (type 4, 178 leaves):

$$\frac{1}{8\sqrt{-1+x}\sqrt{1+x}} \sqrt{1-x^2} \left((5+21i) \sqrt{\frac{\pi}{2}} \operatorname{Erfi} \left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}} \right] (\cos[1] + i \sin[1]) + 2\sqrt{-1+x} \left((6+3i) + 2x \right) (\cos[x] + i \sin[x]) - \right.$$

$$\left. i \left(2 \left((3+6i) + 2ix \right) \sqrt{-1+x} (\cos[1] + i \sin[1]) + (21+5i) \sqrt{\frac{\pi}{2}} \operatorname{Erf} \left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}} \right] (-i \cos[x] + \sin[x]) \right) (\cos[1+x] - i \sin[1+x]) \right)$$

Problem 814: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[x]} (1-x)^{3/2} x \sin[x] \, dx$$

Optimal (type 4, 193 leaves, 19 steps):

$$-\frac{7}{4}\sqrt{1+x}\cos[x] - 3(1+x)^{3/2}\cos[x] + (1+x)^{5/2}\cos[x] + \frac{7}{4}\sqrt{\frac{\pi}{2}}\cos[1]\operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right] - \frac{9}{2}\sqrt{\frac{\pi}{2}}\cos[1]\operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right] +$$

$$\frac{9}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right]\sin[1] + \frac{7}{4}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right]\sin[1] + \frac{9}{2}\sqrt{1+x}\sin[x] - \frac{5}{2}(1+x)^{3/2}\sin[x]$$

Result (type 4, 215 leaves):

$$\frac{1}{16\sqrt{1-x^2}}\sqrt{1-x}\left(e^{-i}\left((18-7i)\sqrt{\pi}\sqrt{-i(1+x)} + 2e^{i(1+x)}\left((-15-8i) - (19-2i)x + 10ix^2 + 4x^3\right) - (18-7i)\sqrt{\pi}\sqrt{-i(1+x)}\operatorname{Erf}\left[\sqrt{-i(1+x)}\right]\right) +\right.$$

$$\left.e^{-ix}\left((-30+16i) - (38+4i)x - 20ix^2 + 8x^3 + (18+7i)e^{i(1+x)}\sqrt{\pi}\sqrt{i(1+x)} - (18+7i)e^{i(1+x)}\sqrt{\pi}\sqrt{i(1+x)}\operatorname{Erf}\left[\sqrt{i(1+x)}\right]\right)\right)$$

Problem 815: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[x]}(1-x)^{3/2}\sin[x]dx$$

Optimal (type 4, 157 leaves, 13 steps):

$$-2\sqrt{1+x}\cos[x] + (1+x)^{3/2}\cos[x] + \sqrt{2\pi}\cos[1]\operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right] + \frac{3}{2}\sqrt{\frac{\pi}{2}}\cos[1]\operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right] -$$

$$\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right]\sin[1] + \sqrt{2\pi}\operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right]\sin[1] - \frac{3}{2}\sqrt{1+x}\sin[x]$$

Result (type 4, 176 leaves):

$$\frac{1}{\sqrt{-1-x}\sqrt{1-x}}\left(\frac{1}{16} + \frac{i}{16}\right)e^{-ix}\sqrt{1-x^2}\left((2+2i)\sqrt{-1-x}\left((-3+2i) + e^{2ix}\left((3+2i) - 2ix\right) - 2ix\right) -\right.$$

$$\left.(3+4i)e^{ix}\sqrt{2\pi}\operatorname{Erf}\left[\frac{(1+i)\sqrt{-1-x}}{\sqrt{2}}\right](\cos[1] - i\sin[1]) + (4+3i)e^{ix}\sqrt{2\pi}\operatorname{Erfi}\left[\frac{(1+i)\sqrt{-1-x}}{\sqrt{2}}\right](-i\cos[1] + \sin[1])\right)$$

Problem 816: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\text{ArcTanh}[x]} x \text{Sin}[x]}{\sqrt{1+x}} dx$$

Optimal (type 4, 140 leaves, 11 steps):

$$\sqrt{1-x} \text{Cos}[x] - \sqrt{\frac{\pi}{2}} \text{Cos}[1] \text{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] + \sqrt{2\pi} \text{Cos}[1] \text{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] -$$

$$\sqrt{2\pi} \text{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \text{Sin}[1] - \sqrt{\frac{\pi}{2}} \text{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \text{Sin}[1]$$

Result (type 4, 165 leaves):

$$\frac{1}{\sqrt{1-x^2}} \left(\frac{1}{8} + \frac{i}{8} \right) \sqrt{1+x} \left((-2-i) \sqrt{2\pi} \sqrt{-1+x} \text{Erfi}\left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}}\right] (\text{Cos}[1] + i \text{Sin}[1]) - (2-2i) (-1+x) (\text{Cos}[x] + i \text{Sin}[x]) - \right.$$

$$\left. (1-i) \left(2(-1+x) (\text{Cos}[1] + i \text{Sin}[1]) - (3+i) \sqrt{\frac{\pi}{2}} \sqrt{-1+x} \text{Erf}\left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}}\right] (\text{Cos}[x] + i \text{Sin}[x]) \right) (\text{Cos}[1+x] - i \text{Sin}[1+x]) \right)$$

Problem 817: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\text{ArcTanh}[x]} \text{Sin}[x]}{\sqrt{1+x}} dx$$

Optimal (type 4, 62 leaves, 6 steps):

$$\sqrt{2\pi} \text{Cos}[1] \text{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] - \sqrt{2\pi} \text{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \text{Sin}[1]$$

Result (type 4, 98 leaves):

$$\frac{1}{\sqrt{1-x^2}} \left(\frac{1}{2} + \frac{i}{2} \right) \sqrt{\frac{\pi}{2}} \sqrt{-1+x} \sqrt{1+x} \left(\text{Erf}\left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}}\right] (\text{Cos}[1] - i \text{Sin}[1]) - \text{Erfi}\left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}}\right] (\text{Cos}[1] + i \text{Sin}[1]) \right)$$

Problem 872: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a+bx]}}{1-a^2-2abx-b^2x^2} dx$$

Optimal (type 2, 27 leaves, 2 steps):

$$\frac{\sqrt{1+a+bx}}{b\sqrt{1-a-bx}}$$

Result (type 3, 12 leaves):

$$\frac{e^{\text{ArcTanh}[a+bx]}}{b}$$

Problem 875: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[a+bx]} x^m dx$$

Optimal (type 6, 109 leaves, 4 steps):

$$\frac{x^{1+m} (1-a-bx)^{-n/2} (1+a+bx)^{n/2} \left(1 - \frac{bx}{1-a}\right)^{n/2} \left(1 + \frac{bx}{1+a}\right)^{-n/2} \text{AppellF1}\left[1+m, \frac{n}{2}, -\frac{n}{2}, 2+m, \frac{bx}{1-a}, -\frac{bx}{1+a}\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{n \text{ArcTanh}[a+bx]} x^m dx$$

Problem 880: Unable to integrate problem.

$$\int \frac{e^{n \text{ArcTanh}[a+bx]}}{x} dx$$

Optimal (type 5, 135 leaves, 5 steps):

$$\frac{2 (1-a-bx)^{-n/2} (1+a+bx)^{n/2} \text{Hypergeometric2F1}\left[1, -\frac{n}{2}, 1-\frac{n}{2}, \frac{(1+a)(1-a-bx)}{(1-a)(1+a+bx)}\right]}{n} - \frac{2^{1+\frac{n}{2}} (1-a-bx)^{-n/2} \text{Hypergeometric2F1}\left[-\frac{n}{2}, -\frac{n}{2}, 1-\frac{n}{2}, \frac{1}{2} (1-a-bx)\right]}{n}$$

Result (type 8, 16 leaves):

$$\int \frac{e^{n \text{ArcTanh}[a+bx]}}{x} dx$$

Problem 881: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{ArcTanh}[a+bx]}}{x^2} dx$$

Optimal (type 5, 92 leaves, 2 steps):

$$\frac{4b(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)} \operatorname{Hypergeometric2F1}\left[2, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{(1+a)(1-ax)}{(1-a)(1+ax)}\right]}{(1-a)^2(2-n)}$$

Result (type 8, 16 leaves):

$$\int \frac{e^{n \operatorname{ArcTanh}[a+bx]}}{x^2} dx$$

Problem 882: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{ArcTanh}[a+bx]}}{x^3} dx$$

Optimal (type 5, 152 leaves, 3 steps):

$$\frac{(1-ax-bx)^{1-\frac{n}{2}}(1+ax)^{\frac{2+n}{2}}}{2(1-a^2)x^2} - \frac{2b^2(2a+n)(1-ax)^{1-\frac{n}{2}}(1+ax)^{\frac{1}{2}(-2+n)} \operatorname{Hypergeometric2F1}\left[2, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{(1+a)(1-ax)}{(1-a)(1+ax)}\right]}{(1-a)^3(1+a)(2-n)}$$

Result (type 8, 16 leaves):

$$\int \frac{e^{n \operatorname{ArcTanh}[a+bx]}}{x^3} dx$$

Problem 924: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e^{\operatorname{ArcTanh}[ax]}}{\sqrt{1-a^2x^2}} dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$\frac{\operatorname{Log}[1-ax]}{a}$$

Result (type 4, 52 leaves):

$$\frac{2 \operatorname{Im} \sqrt{-a^2} \operatorname{EllipticF}[\operatorname{Im} \operatorname{ArcSinh}[\sqrt{-a^2} x], 1] - a \operatorname{Log}[-1 + a^2 x^2]}{2 a^2}$$

Problem 961: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]}}{\sqrt{c - a^2 c x^2}} dx$$

Optimal (type 3, 41 leaves, 3 steps):

$$-\frac{\sqrt{1 - a^2 x^2} \operatorname{Log}[1 - a x]}{a \sqrt{c - a^2 c x^2}}$$

Result (type 4, 87 leaves):

$$\frac{a \sqrt{1 - a^2 x^2} \left(2 \operatorname{Im} a \operatorname{EllipticF}[\operatorname{Im} \operatorname{ArcSinh}[\sqrt{-a^2} x], 1] + \sqrt{-a^2} \operatorname{Log}[-1 + a^2 x^2] \right)}{2 (-a^2)^{3/2} \sqrt{c - a^2 c x^2}}$$

Problem 970: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]} x}{(c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 91 leaves, 5 steps):

$$\frac{\sqrt{1 - a^2 x^2}}{2 a^2 c (1 - a x) \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2} \operatorname{ArcTanh}[a x]}{2 a^2 c \sqrt{c - a^2 c x^2}}$$

Result (type 4, 93 leaves):

$$-\frac{\operatorname{Im} \sqrt{1 - a^2 x^2} \left(\operatorname{Im} \sqrt{-a^2} + a (-1 + a x) \operatorname{EllipticF}[\operatorname{Im} \operatorname{ArcSinh}[\sqrt{-a^2} x], 1] \right)}{2 (-a^2)^{3/2} c (-1 + a x) \sqrt{c - a^2 c x^2}}$$

Problem 971: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]}}{(c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 91 leaves, 5 steps):

$$\frac{\sqrt{1-a^2 x^2}}{2 a c (1-a x) \sqrt{c-a^2 c x^2}} + \frac{\sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]}{2 a c \sqrt{c-a^2 c x^2}}$$

Result (type 4, 91 leaves):

$$\frac{a \sqrt{1-a^2 x^2} \left(\sqrt{-a^2} + i a (-1+a x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] \right)}{2 (-a^2)^{3/2} c (-1+a x) \sqrt{c-a^2 c x^2}}$$

Problem 972: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]}}{x (c-a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 165 leaves, 4 steps):

$$\frac{\sqrt{1-a^2 x^2}}{2 c (1-a x) \sqrt{c-a^2 c x^2}} + \frac{\sqrt{1-a^2 x^2} \operatorname{Log}[x]}{c \sqrt{c-a^2 c x^2}} - \frac{3 \sqrt{1-a^2 x^2} \operatorname{Log}[1-a x]}{4 c \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2} \operatorname{Log}[1+a x]}{4 c \sqrt{c-a^2 c x^2}}$$

Result (type 4, 121 leaves):

$$\left(\sqrt{c-a^2 c x^2} \left(-i a (-1+a x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] + \sqrt{-a^2} (-1+(-1+a x) \operatorname{Log}[x^2] + (1-a x) \operatorname{Log}[1-a^2 x^2]) \right) \right) / \left(2 \sqrt{-a^2} c^2 (-1+a x) \sqrt{1-a^2 x^2} \right)$$

Problem 973: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]}}{x^2 (c-a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 206 leaves, 4 steps):

$$-\frac{\sqrt{1-a^2 x^2}}{c x \sqrt{c-a^2 c x^2}} + \frac{a \sqrt{1-a^2 x^2}}{2 c (1-a x) \sqrt{c-a^2 c x^2}} + \frac{a \sqrt{1-a^2 x^2} \operatorname{Log}[x]}{c \sqrt{c-a^2 c x^2}} - \frac{5 a \sqrt{1-a^2 x^2} \operatorname{Log}[1-a x]}{4 c \sqrt{c-a^2 c x^2}} + \frac{a \sqrt{1-a^2 x^2} \operatorname{Log}[1+a x]}{4 c \sqrt{c-a^2 c x^2}}$$

Result (type 4, 135 leaves):

$$\left(\sqrt{c-a^2 c x^2} \left(-3 i a^2 x (-1+a x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] + \sqrt{-a^2} (2-3 a x+a x (-1+a x) \operatorname{Log}[x^2] + a x (1-a x) \operatorname{Log}[1-a^2 x^2]) \right) \right) / \left(2 \sqrt{-a^2} c^2 x (-1+a x) \sqrt{1-a^2 x^2} \right)$$

Problem 974: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{x^3 (c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 255 leaves, 4 steps):

$$-\frac{\sqrt{1-a^2 x^2}}{2 c x^2 \sqrt{c-a^2 c x^2}} - \frac{a \sqrt{1-a^2 x^2}}{c x \sqrt{c-a^2 c x^2}} + \frac{a^2 \sqrt{1-a^2 x^2}}{2 c (1-a x) \sqrt{c-a^2 c x^2}} + \frac{2 a^2 \sqrt{1-a^2 x^2} \text{Log}[x]}{c \sqrt{c-a^2 c x^2}} - \frac{7 a^2 \sqrt{1-a^2 x^2} \text{Log}[1-a x]}{4 c \sqrt{c-a^2 c x^2}} - \frac{a^2 \sqrt{1-a^2 x^2} \text{Log}[1+a x]}{4 c \sqrt{c-a^2 c x^2}}$$

Result (type 4, 153 leaves):

$$\left(\sqrt{c-a^2 c x^2} \left(-3 i a^3 x^2 (-1+a x) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-a^2} x \right], 1 \right] + \sqrt{-a^2} \left(1+a x - 3 a^2 x^2 + 2 a^2 x^2 (-1+a x) \text{Log}[x^2] - 2 a^2 x^2 (-1+a x) \text{Log}[1-a^2 x^2] \right) \right) \right) / \left(2 \sqrt{-a^2} c^2 x^2 (-1+a x) \sqrt{1-a^2 x^2} \right)$$

Problem 975: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{x^4 (c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 297 leaves, 4 steps):

$$-\frac{\sqrt{1-a^2 x^2}}{3 c x^3 \sqrt{c-a^2 c x^2}} - \frac{a \sqrt{1-a^2 x^2}}{2 c x^2 \sqrt{c-a^2 c x^2}} - \frac{2 a^2 \sqrt{1-a^2 x^2}}{c x \sqrt{c-a^2 c x^2}} + \frac{a^3 \sqrt{1-a^2 x^2}}{2 c (1-a x) \sqrt{c-a^2 c x^2}} + \frac{2 a^3 \sqrt{1-a^2 x^2} \text{Log}[x]}{c \sqrt{c-a^2 c x^2}} - \frac{9 a^3 \sqrt{1-a^2 x^2} \text{Log}[1-a x]}{4 c \sqrt{c-a^2 c x^2}} + \frac{a^3 \sqrt{1-a^2 x^2} \text{Log}[1+a x]}{4 c \sqrt{c-a^2 c x^2}}$$

Result (type 4, 161 leaves):

$$\left(\sqrt{c-a^2 c x^2} \left(-15 i a^4 x^3 (-1+a x) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-a^2} x \right], 1 \right] + \sqrt{-a^2} \left(2+a x + 9 a^2 x^2 - 15 a^3 x^3 + 6 a^3 x^3 (-1+a x) \text{Log}[x^2] - 6 a^3 x^3 (-1+a x) \text{Log}[1-a^2 x^2] \right) \right) \right) / \left(6 \sqrt{-a^2} c^2 x^3 (-1+a x) \sqrt{1-a^2 x^2} \right)$$

Problem 979: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a x]} x^3}{(c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 184 leaves, 5 steps):

$$\frac{\sqrt{1-a^2 x^2}}{8 a^4 c^2 (1-a x)^2 \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2}}{2 a^4 c^2 (1-a x) \sqrt{c-a^2 c x^2}} + \frac{\sqrt{1-a^2 x^2}}{8 a^4 c^2 (1+a x) \sqrt{c-a^2 c x^2}} + \frac{3 \sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]}{8 a^4 c^2 \sqrt{c-a^2 c x^2}}$$

Result (type 4, 122 leaves):

$$\frac{\sqrt{1-a^2 x^2} \left(\sqrt{-a^2} (-2-a x+5 a^2 x^2) - 3 i a (-1+a x)^2 (1+a x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] \right)}{8 a^4 \sqrt{-a^2} c^2 (-1+a x)^2 (1+a x) \sqrt{c-a^2 c x^2}}$$

Problem 980: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]} x^2}{(c-a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 184 leaves, 5 steps):

$$\frac{\sqrt{1-a^2 x^2}}{8 a^3 c^2 (1-a x)^2 \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2}}{4 a^3 c^2 (1-a x) \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2}}{8 a^3 c^2 (1+a x) \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]}{8 a^3 c^2 \sqrt{c-a^2 c x^2}}$$

Result (type 4, 119 leaves):

$$\frac{a \sqrt{1-a^2 x^2} \left(\sqrt{-a^2} (-2+3 a x+a^2 x^2) + i a (-1+a x)^2 (1+a x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] \right)}{8 (-a^2)^{5/2} c^2 (-1+a x)^2 (1+a x) \sqrt{c-a^2 c x^2}}$$

Problem 981: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]} x}{(c-a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 137 leaves, 5 steps):

$$\frac{\sqrt{1-a^2 x^2}}{8 a^2 c^2 (1-a x)^2 \sqrt{c-a^2 c x^2}} + \frac{\sqrt{1-a^2 x^2}}{8 a^2 c^2 (1+a x) \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]}{8 a^2 c^2 \sqrt{c-a^2 c x^2}}$$

Result (type 4, 118 leaves):

$$\frac{\sqrt{1-a^2 x^2} \left(\sqrt{-a^2} (2-a x+a^2 x^2) + i a (-1+a x)^2 (1+a x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] \right)}{8 (-a^2)^{3/2} c^2 (-1+a x)^2 (1+a x) \sqrt{c-a^2 c x^2}}$$

Problem 982: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{(c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 184 leaves, 5 steps):

$$\frac{\sqrt{1 - a^2 x^2}}{8 a c^2 (1 - a x)^2 \sqrt{c - a^2 c x^2}} + \frac{\sqrt{1 - a^2 x^2}}{4 a c^2 (1 - a x) \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2}}{8 a c^2 (1 + a x) \sqrt{c - a^2 c x^2}} + \frac{3 \sqrt{1 - a^2 x^2} \text{ArcTanh}[a x]}{8 a c^2 \sqrt{c - a^2 c x^2}}$$

Result (type 4, 120 leaves):

$$\frac{a \sqrt{1 - a^2 x^2} \left(\sqrt{-a^2} (2 + 3 a x - 3 a^2 x^2) - 3 i a (-1 + a x)^2 (1 + a x) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] \right)}{8 (-a^2)^{3/2} c^2 (-1 + a x)^2 (1 + a x) \sqrt{c - a^2 c x^2}}$$

Problem 983: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{x (c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 252 leaves, 4 steps):

$$\frac{\sqrt{1 - a^2 x^2}}{8 c^2 (1 - a x)^2 \sqrt{c - a^2 c x^2}} + \frac{\sqrt{1 - a^2 x^2}}{2 c^2 (1 - a x) \sqrt{c - a^2 c x^2}} +$$

$$\frac{\sqrt{1 - a^2 x^2}}{8 c^2 (1 + a x) \sqrt{c - a^2 c x^2}} + \frac{\sqrt{1 - a^2 x^2} \text{Log}[x]}{c^2 \sqrt{c - a^2 c x^2}} - \frac{11 \sqrt{1 - a^2 x^2} \text{Log}[1 - a x]}{16 c^2 \sqrt{c - a^2 c x^2}} - \frac{5 \sqrt{1 - a^2 x^2} \text{Log}[1 + a x]}{16 c^2 \sqrt{c - a^2 c x^2}}$$

Result (type 4, 162 leaves):

$$\left(\sqrt{c - a^2 c x^2} \left(-3 i a (-1 + a x)^2 (1 + a x) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] + \right. \right.$$

$$\left. \left. \sqrt{-a^2} \left(6 - a x - 3 a^2 x^2 + 4 (-1 + a x)^2 (1 + a x) \text{Log}[x^2] - 4 (-1 + a x)^2 (1 + a x) \text{Log}[1 - a^2 x^2] \right) \right) \right) / \left(8 \sqrt{-a^2} c^3 (-1 + a x)^2 (1 + a x) \sqrt{1 - a^2 x^2} \right)$$

Problem 984: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{x^2 (c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 295 leaves, 4 steps):

$$\begin{aligned}
& -\frac{\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}} + \frac{a\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}} + \frac{3a\sqrt{1-a^2x^2}}{4c^2(1-ax)\sqrt{c-a^2cx^2}} - \\
& \frac{a\sqrt{1-a^2x^2}}{8c^2(1+ax)\sqrt{c-a^2cx^2}} + \frac{a\sqrt{1-a^2x^2}\operatorname{Log}[x]}{c^2\sqrt{c-a^2cx^2}} - \frac{23a\sqrt{1-a^2x^2}\operatorname{Log}[1-ax]}{16c^2\sqrt{c-a^2cx^2}} + \frac{7a\sqrt{1-a^2x^2}\operatorname{Log}[1+ax]}{16c^2\sqrt{c-a^2cx^2}}
\end{aligned}$$

Result (type 4, 180 leaves):

$$\begin{aligned}
& \left(\sqrt{c-a^2cx^2} \left(-15i a^2x(-1+ax)^2(1+ax) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2}x\right], 1\right] + \right. \right. \\
& \quad \left. \left. \sqrt{-a^2} \left(-8+14ax+11a^2x^2-15a^3x^3+4ax(-1+ax)^2(1+ax) \operatorname{Log}[x^2] - 4ax(-1+ax)^2(1+ax) \operatorname{Log}[1-a^2x^2] \right) \right) \right) / \\
& \left(8\sqrt{-a^2}c^3x(-1+ax)^2(1+ax)\sqrt{1-a^2x^2} \right)
\end{aligned}$$

Problem 985: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\operatorname{ArcTanh}[ax]}}{x^3(c-a^2cx^2)^{5/2}} dx$$

Optimal (type 3, 345 leaves, 4 steps):

$$\begin{aligned}
& -\frac{\sqrt{1-a^2x^2}}{2c^2x^2\sqrt{c-a^2cx^2}} - \frac{a\sqrt{1-a^2x^2}}{c^2x\sqrt{c-a^2cx^2}} + \frac{a^2\sqrt{1-a^2x^2}}{8c^2(1-ax)^2\sqrt{c-a^2cx^2}} + \frac{a^2\sqrt{1-a^2x^2}}{c^2(1-ax)\sqrt{c-a^2cx^2}} + \\
& \frac{a^2\sqrt{1-a^2x^2}}{8c^2(1+ax)\sqrt{c-a^2cx^2}} + \frac{3a^2\sqrt{1-a^2x^2}\operatorname{Log}[x]}{c^2\sqrt{c-a^2cx^2}} - \frac{39a^2\sqrt{1-a^2x^2}\operatorname{Log}[1-ax]}{16c^2\sqrt{c-a^2cx^2}} - \frac{9a^2\sqrt{1-a^2x^2}\operatorname{Log}[1+ax]}{16c^2\sqrt{c-a^2cx^2}}
\end{aligned}$$

Result (type 4, 198 leaves):

$$\begin{aligned}
& \left(\sqrt{c-a^2cx^2} \left(-15i a^3x^2(-1+ax)^2(1+ax) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2}x\right], 1\right] + \right. \right. \\
& \quad \left. \left. \sqrt{-a^2} \left(-4-4ax+22a^2x^2+3a^3x^3-15a^4x^4+12a^2x^2(-1+ax)^2(1+ax) \operatorname{Log}[x^2] - 12a^2x^2(-1+ax)^2(1+ax) \operatorname{Log}[1-a^2x^2] \right) \right) \right) / \\
& \left(8\sqrt{-a^2}c^3x^2(-1+ax)^2(1+ax)\sqrt{1-a^2x^2} \right)
\end{aligned}$$

Problem 986: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\operatorname{ArcTanh}[ax]}}{(c-a^2cx^2)^{7/2}} dx$$

Optimal (type 3, 277 leaves, 5 steps):

$$\frac{\sqrt{1-a^2 x^2}}{24 a c^3 (1-a x)^3 \sqrt{c-a^2 c x^2}} + \frac{3 \sqrt{1-a^2 x^2}}{32 a c^3 (1-a x)^2 \sqrt{c-a^2 c x^2}} + \frac{3 \sqrt{1-a^2 x^2}}{16 a c^3 (1-a x) \sqrt{c-a^2 c x^2}} -$$

$$\frac{\sqrt{1-a^2 x^2}}{32 a c^3 (1+a x)^2 \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2}}{8 a c^3 (1+a x) \sqrt{c-a^2 c x^2}} + \frac{5 \sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]}{16 a c^3 \sqrt{c-a^2 c x^2}}$$

Result (type 4, 138 leaves):

$$-\left(\left(a \sqrt{1-a^2 x^2} \left(\sqrt{-a^2} (-8-25 a x+25 a^2 x^2+15 a^3 x^3-15 a^4 x^4) -15 i a (-1+a x)^3 (1+a x)^2 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] \right) \right) / \right.$$

$$\left. \left(48 (-a^2)^{3/2} c^3 (-1+a x)^3 (1+a x)^2 \sqrt{c-a^2 c x^2} \right) \right)$$

Problem 989: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]} x^m}{c-a^2 c x^2} dx$$

Optimal (type 5, 80 leaves, 4 steps):

$$\frac{x^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{c(1+m)} + \frac{a x^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{c(2+m)}$$

Result (type 6, 391 leaves):

$$\frac{1}{2c(1+m)} \left((2+m)x^{1+m} \left(\left(2\sqrt{-1-ax} \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, -ax, ax\right] \right) / \left((-1+ax)^{3/2} \left(2(2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, -ax, ax\right] + \right. \right. \right. \right. \\ \left. \left. \left. \left. ax \left(3 \operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{5}{2}, 3+m, -ax, ax\right] + \operatorname{AppellF1}\left[2+m, \frac{1}{2}, \frac{3}{2}, 3+m, -ax, ax\right] \right) \right) \right) \right) + \frac{1}{\sqrt{1+ax}} \sqrt{1-ax} \right. \\ \left. \left(\left(\sqrt{-1-ax} \sqrt{1-a^2x^2} \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -ax, ax\right] \right) / \left((-1+ax)^{3/2} \left(2(2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -ax, ax\right] + \right. \right. \right. \right. \\ \left. \left. \left. \left. ax \left(\operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{3}{2}, 3+m, -ax, ax\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2x^2\right] \right) \right) \right) \right) + \right. \\ \left. \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, ax, -ax\right] / \left(2(2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, ax, -ax\right] - \right. \right. \\ \left. \left. \left. \left. ax \left(\operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{3}{2}, 3+m, ax, -ax\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2x^2\right] \right) \right) \right) \right) \right)$$

Problem 990: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e^{\operatorname{ArcTanh}[ax]} x^m}{(c - a^2 c x^2)^2} dx$$

Optimal (type 5, 80 leaves, 4 steps):

$$\frac{x^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{c^2 (1+m)} + \frac{a x^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{c^2 (2+m)}$$

Result (type 6, 711 leaves):

$$\begin{aligned}
& \left((2+m) x^{1+m} \sqrt{-1-ax} \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, -ax, ax\right] \right) / \\
& \left(2c^2(1+m)(-1+ax)^{3/2} \left(2(2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, -ax, ax\right] + ax \right. \right. \\
& \left. \left. \left(3 \operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{5}{2}, 3+m, -ax, ax\right] + \operatorname{AppellF1}\left[2+m, \frac{1}{2}, \frac{3}{2}, 3+m, -ax, ax\right] \right) \right) \right) + \\
& \left((2+m) x^{1+m} \sqrt{1-ax} \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, ax, -ax\right] \right) / \left(4c^2(1+m)(1+ax)^{3/2} \left(2(2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, ax, -ax\right] - \right. \right. \\
& \left. \left. ax \left(3 \operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{5}{2}, 3+m, ax, -ax\right] + \operatorname{AppellF1}\left[2+m, \frac{1}{2}, \frac{3}{2}, 3+m, ax, -ax\right] \right) \right) \right) - \\
& \left((2+m) x^{1+m} \sqrt{-1-ax} \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{5}{2}, 2+m, -ax, ax\right] \right) / \left(2c^2(1+m)(-1+ax)^{5/2} \left(2(2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{5}{2}, 2+m, -ax, ax\right] + \right. \right. \\
& \left. \left. ax \left(5 \operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{7}{2}, 3+m, -ax, ax\right] + \operatorname{AppellF1}\left[2+m, \frac{1}{2}, \frac{5}{2}, 3+m, -ax, ax\right] \right) \right) \right) + \\
& \left(3(2+m) x^{1+m} \sqrt{-1-ax} \sqrt{1-ax} \sqrt{1-a^2x^2} \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -ax, ax\right] \right) / \\
& \left(8c^2(1+m)(-1+ax)^{3/2} \sqrt{1+ax} \left(2(2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -ax, ax\right] + \right. \right. \\
& \left. \left. ax \left(\operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{3}{2}, 3+m, -ax, ax\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2x^2\right] \right) \right) \right) + \\
& \left(3(2+m) x^{1+m} \sqrt{1-ax} \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, ax, -ax\right] \right) / \left(8c^2(1+m) \sqrt{1+ax} \left(2(2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, ax, -ax\right] - \right. \right. \\
& \left. \left. ax \left(\operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{3}{2}, 3+m, ax, -ax\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2x^2\right] \right) \right) \right)
\end{aligned}$$

Problem 991: Unable to integrate problem.

$$\int \frac{e^{\operatorname{ArcTanh}[ax]} x^m}{(c - a^2 c x^2)^3} dx$$

Optimal (type 5, 80 leaves, 4 steps):

$$\frac{x^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{7}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{c^3 (1+m)} + \frac{a x^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{7}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{c^3 (2+m)}$$

Result (type 8, 25 leaves):

$$\int \frac{e^{\operatorname{ArcTanh}[ax]} x^m}{(c - a^2 c x^2)^3} dx$$

Problem 1001: Unable to integrate problem.

$$\int \frac{e^{\text{ArcTanh}[a x]} x^m}{\sqrt{c - a^2 c x^2}} dx$$

Optimal (type 5, 51 leaves, 3 steps):

$$\frac{x^{1+m} \sqrt{1 - a^2 x^2} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, a x\right]}{(1+m) \sqrt{c - a^2 c x^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{e^{\text{ArcTanh}[a x]} x^m}{\sqrt{c - a^2 c x^2}} dx$$

Problem 1002: Unable to integrate problem.

$$\int \frac{e^{\text{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 5, 134 leaves, 7 steps):

$$\frac{x^{1+m} \sqrt{1 - a^2 x^2} \text{Hypergeometric2F1}\left[2, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{c (1+m) \sqrt{c - a^2 c x^2}} + \frac{a x^{2+m} \sqrt{1 - a^2 x^2} \text{Hypergeometric2F1}\left[2, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{c (2+m) \sqrt{c - a^2 c x^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{e^{\text{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^{3/2}} dx$$

Problem 1003: Unable to integrate problem.

$$\int \frac{e^{\text{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 5, 134 leaves, 7 steps):

$$\frac{x^{1+m} \sqrt{1 - a^2 x^2} \text{Hypergeometric2F1}\left[3, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{c^2 (1+m) \sqrt{c - a^2 c x^2}} + \frac{a x^{2+m} \sqrt{1 - a^2 x^2} \text{Hypergeometric2F1}\left[3, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{c^2 (2+m) \sqrt{c - a^2 c x^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{e^{\text{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^{5/2}} dx$$

Problem 1004: Unable to integrate problem.

$$\int e^{\text{ArcTanh}[a x]} x^m (c - a^2 c x^2)^p dx$$

Optimal (type 5, 136 leaves, 5 steps):

$$\frac{x^{1+m} (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1}{2} - p, \frac{3+m}{2}, a^2 x^2\right]}{1+m} + \frac{a x^{2+m} (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[\frac{2+m}{2}, \frac{1}{2} - p, \frac{4+m}{2}, a^2 x^2\right]}{2+m}$$

Result (type 8, 25 leaves):

$$\int e^{\text{ArcTanh}[a x]} x^m (c - a^2 c x^2)^p dx$$

Problem 1005: Result more than twice size of optimal antiderivative.

$$\int e^{\text{ArcTanh}[a x]} x^3 (1 - a^2 x^2)^p dx$$

Optimal (type 5, 85 leaves, 6 steps):

$$-\frac{(1 - a^2 x^2)^{\frac{1}{2}+p}}{a^4 (1 + 2p)} + \frac{(1 - a^2 x^2)^{\frac{3}{2}+p}}{a^4 (3 + 2p)} + \frac{1}{5} a x^5 \text{Hypergeometric2F1}\left[\frac{5}{2}, \frac{1}{2} - p, \frac{7}{2}, a^2 x^2\right]$$

Result (type 5, 183 leaves):

$$\frac{1}{3 a^4} \left(-3 a x \text{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{1}{2} - p, \frac{3}{2}, a^2 x^2\right] + \frac{1}{3 + 2p} \right. \\ \left. \left(-3 + 3 (1 - a^2 x^2)^{\frac{1}{2}+p} - 3 a^2 x^2 (1 - a^2 x^2)^{\frac{1}{2}+p} - a^3 (3 + 2p) x^3 \text{Hypergeometric2F1}\left[\frac{3}{2}, -\frac{1}{2} - p, \frac{5}{2}, a^2 x^2\right] + \right. \right. \\ \left. \left. 3 (1 - a x)^{-\frac{1}{2}-p} (1 + a x) (2 - 2 a^2 x^2)^{\frac{1}{2}+p} \text{Hypergeometric2F1}\left[\frac{1}{2} - p, \frac{3}{2} + p, \frac{5}{2} + p, \frac{1}{2} (1 + a x)\right] \right) \right)$$

Problem 1009: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{\text{ArcTanh}[a x]} (1 - a^2 x^2)^p}{x} dx$$

Optimal (type 5, 72 leaves, 5 steps):

$$a x \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - p, \frac{3}{2}, a^2 x^2\right] - \frac{(1 - a^2 x^2)^{\frac{1}{2}+p} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]}{1 + 2 p}$$

Result (type 5, 147 leaves):

$$(1 - a^2 x^2)^{\frac{1}{2}+p} \left(\frac{\operatorname{Hypergeometric2F1}\left[-\frac{1}{2} - p, -\frac{1}{2} - p, \frac{1}{2} - p, \frac{1}{a^2 x^2}\right]}{\left(1 - \frac{1}{a^2 x^2}\right)^{\frac{1}{2}+p} + 2 p \left(1 - \frac{1}{a^2 x^2}\right)^{\frac{1}{2}+p}} + \frac{2^{\frac{1}{2}+p} (1 - a x)^{-\frac{1}{2}-p} (1 + a x) \operatorname{Hypergeometric2F1}\left[\frac{1}{2} - p, \frac{3}{2} + p, \frac{5}{2} + p, \frac{1}{2} (1 + a x)\right]}{3 + 2 p} \right)$$

Problem 1010: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]} (1 - a^2 x^2)^p}{x^2} dx$$

Optimal (type 5, 75 leaves, 5 steps):

$$-\frac{\operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2} - p, \frac{1}{2}, a^2 x^2\right]}{x} - \frac{a (1 - a^2 x^2)^{\frac{1}{2}+p} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]}{1 + 2 p}$$

Result (type 5, 170 leaves):

$$-\frac{\operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, -\frac{1}{2} - p, \frac{1}{2}, a^2 x^2\right]}{x} + \frac{a \left(1 - \frac{1}{a^2 x^2}\right)^{-\frac{1}{2}-p} (1 - a^2 x^2)^{\frac{1}{2}+p} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2} - p, -\frac{1}{2} - p, \frac{1}{2} - p, \frac{1}{a^2 x^2}\right]}{1 + 2 p} + \frac{a (1 - a x)^{-\frac{1}{2}-p} (1 + a x) (2 - 2 a^2 x^2)^{\frac{1}{2}+p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2} - p, \frac{3}{2} + p, \frac{5}{2} + p, \frac{1}{2} (1 + a x)\right]}{3 + 2 p}$$

Problem 1011: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]} (1 - a^2 x^2)^p}{x^3} dx$$

Optimal (type 5, 78 leaves, 5 steps):

$$-\frac{a \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2} - p, \frac{1}{2}, a^2 x^2\right]}{x} - \frac{a^2 (1 - a^2 x^2)^{\frac{1}{2}+p} \operatorname{Hypergeometric2F1}\left[2, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]}{1 + 2 p}$$

Result (type 5, 262 leaves):

$$\begin{aligned}
& - \frac{a \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, -\frac{1}{2} - p, \frac{1}{2}, a^2 x^2\right]}{x} + \frac{a^2 (1 - a^2 x^2)^{\frac{1}{2}+p} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2} - p, -\frac{1}{2} - p, \frac{1}{2} - p, \frac{1}{a^2 x^2}\right]}{\left(1 - \frac{1}{a^2 x^2}\right)^{\frac{1}{2}+p} + 2p \left(1 - \frac{1}{a^2 x^2}\right)^{\frac{1}{2}+p}} + \\
& \frac{\left(1 - \frac{1}{a^2 x^2}\right)^{-\frac{1}{2}-p} (1 - a^2 x^2)^{\frac{1}{2}+p} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2} - p, \frac{1}{2} - p, \frac{3}{2} - p, \frac{1}{a^2 x^2}\right]}{(-1 + 2p) x^2} + \\
& \frac{a^2 (1 - a x)^{-\frac{1}{2}-p} (1 + a x) (2 - 2 a^2 x^2)^{\frac{1}{2}+p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2} - p, \frac{3}{2} + p, \frac{5}{2} + p, \frac{1}{2} (1 + a x)\right]}{3 + 2p}
\end{aligned}$$

Problem 1012: Result more than twice size of optimal antiderivative.

$$\int e^{\operatorname{ArcTanh}[a x]} x^3 (c - a^2 c x^2)^p dx$$

Optimal (type 5, 134 leaves, 7 steps):

$$-\frac{\sqrt{1 - a^2 x^2} (c - a^2 c x^2)^p}{a^4 (1 + 2p)} + \frac{(1 - a^2 x^2)^{3/2} (c - a^2 c x^2)^p}{a^4 (3 + 2p)} + \frac{1}{5} a x^5 (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, \frac{1}{2} - p, \frac{7}{2}, a^2 x^2\right]$$

Result (type 5, 295 leaves):

$$\begin{aligned}
& \frac{1}{a^4 (3 + 2p) (5 + 2p) (7 + 2p) (9 + 2p)} 4^{1+p} e^{3 \operatorname{ArcTanh}[a x]} \left(\frac{e^{\operatorname{ArcTanh}[a x]}}{1 + e^{2 \operatorname{ArcTanh}[a x]}} \right)^{2p} (1 + e^{2 \operatorname{ArcTanh}[a x]})^{2p} \\
& (1 - a^2 x^2)^{-p} (c (1 - a^2 x^2))^p \left(- (315 + 286 p + 84 p^2 + 8 p^3) \operatorname{Hypergeometric2F1}\left[\frac{3}{2} + p, 5 + 2p, \frac{5}{2} + p, -e^{2 \operatorname{ArcTanh}[a x]}\right] + \right. \\
& e^{2 \operatorname{ArcTanh}[a x]} (3 + 2p) \left(3 (63 + 32 p + 4 p^2) \operatorname{Hypergeometric2F1}\left[\frac{5}{2} + p, 5 + 2p, \frac{7}{2} + p, -e^{2 \operatorname{ArcTanh}[a x]}\right] + \right. \\
& e^{2 \operatorname{ArcTanh}[a x]} (5 + 2p) \left(-3 (9 + 2p) \operatorname{Hypergeometric2F1}\left[\frac{7}{2} + p, 5 + 2p, \frac{9}{2} + p, -e^{2 \operatorname{ArcTanh}[a x]}\right] + \right. \\
& \left. \left. \left. e^{2 \operatorname{ArcTanh}[a x]} (7 + 2p) \operatorname{Hypergeometric2F1}\left[\frac{9}{2} + p, 5 + 2p, \frac{11}{2} + p, -e^{2 \operatorname{ArcTanh}[a x]}\right]\right)\right)\right)
\end{aligned}$$

Problem 1016: Unable to integrate problem.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x} dx$$

Optimal (type 5, 110 leaves, 6 steps):

$$a x (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - p, \frac{3}{2}, a^2 x^2\right] - \frac{\sqrt{1 - a^2 x^2} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]}{1 + 2 p}$$

Result (type 8, 25 leaves):

$$\int \frac{e^{\operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x} dx$$

Problem 1017: Unable to integrate problem.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^2} dx$$

Optimal (type 5, 113 leaves, 6 steps):

$$-\frac{(1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2} - p, \frac{1}{2}, a^2 x^2\right]}{x} - \frac{a \sqrt{1 - a^2 x^2} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]}{1 + 2 p}$$

Result (type 8, 25 leaves):

$$\int \frac{e^{\operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^2} dx$$

Problem 1018: Unable to integrate problem.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^3} dx$$

Optimal (type 5, 116 leaves, 6 steps):

$$-\frac{a (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2} - p, \frac{1}{2}, a^2 x^2\right]}{x} - \frac{a^2 \sqrt{1 - a^2 x^2} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[2, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]}{1 + 2 p}$$

Result (type 8, 25 leaves):

$$\int \frac{e^{\operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^3} dx$$

Problem 1035: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{2 \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^2}{x^3} dx$$

Optimal (type 1, 17 leaves, 2 steps):

$$-\frac{c^2 (1 + a x)^4}{2 x^2}$$

Result (type 1, 42 leaves):

$$-\frac{c^2}{2 x^2} - \frac{2 a c^2}{x} - 2 a^3 c^2 x - \frac{1}{2} a^4 c^2 x^2$$

Problem 1053: Result unnecessarily involves higher level functions.

$$\int \frac{e^{2 \operatorname{ArcTanh}[a x]}}{c - a^2 c x^2} dx$$

Optimal (type 1, 15 leaves, 2 steps):

$$\frac{1}{a c (1 - a x)}$$

Result (type 3, 18 leaves):

$$\frac{e^{2 \operatorname{ArcTanh}[a x]}}{2 a c}$$

Problem 1130: Result unnecessarily involves higher level functions.

$$\int \frac{e^{2 \operatorname{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^3} dx$$

Optimal (type 5, 203 leaves, 8 steps):

$$-\frac{(2-m)(4-m)x^{1+m}}{24c^3(1+ax)} + \frac{x^{1+m}}{6c^3(1-ax)^3(1+ax)} + \frac{(4-m)x^{1+m}}{12c^3(1-ax)^2(1+ax)} + \frac{(7-2m)(2-m)x^{1+m}}{24c^3(1-ax)(1+ax)} +$$

$$\frac{(2-m)x^{1+m} \operatorname{Hypergeometric2F1}[1, 1+m, 2+m, -ax]}{16c^3(1+m)} + \frac{(2-m)(3-8m+2m^2)x^{1+m} \operatorname{Hypergeometric2F1}[1, 1+m, 2+m, ax]}{48c^3(1+m)}$$

Result (type 6, 109 leaves):

$$\left((2+m)x^{1+m} \operatorname{AppellF1}[1+m, 4, 2, 2+m, ax, -ax] \right) / \left(c^3(1+m)(-1+ax)^4(1+ax)^2 \right)$$

$$\left((2+m) \operatorname{AppellF1}[1+m, 4, 2, 2+m, ax, -ax] - 2ax \left(\operatorname{AppellF1}[2+m, 4, 3, 3+m, ax, -ax] - 2 \operatorname{AppellF1}[2+m, 5, 2, 3+m, ax, -ax] \right) \right)$$

Problem 1133: Result unnecessarily involves higher level functions.

$$\int e^{2 \operatorname{ArcTanh}[a x]} x^m \sqrt{c - a^2 c x^2} dx$$

Optimal (type 5, 172 leaves, 7 steps):

$$-\frac{x^{1+m} \sqrt{c - a^2 c x^2}}{2 + m} + \frac{c (3 + 2 m) x^{1+m} \sqrt{1 - a^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{(1 + m) (2 + m) \sqrt{c - a^2 c x^2}} + \frac{2 a c x^{2+m} \sqrt{1 - a^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{(2 + m) \sqrt{c - a^2 c x^2}}$$

Result (type 6, 193 leaves):

$$\frac{1}{1 + m} x^{1+m} \left(-\frac{\sqrt{c - a^2 c x^2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{\sqrt{1 - a^2 x^2}} - \left(4 (2 + m) \sqrt{-c (1 + a x)} \operatorname{AppellF1}\left[1 + m, \frac{1}{2}, -\frac{1}{2}, 2 + m, a x, -a x\right] \right) / \left(\sqrt{-1 + a x} \left(2 (2 + m) \operatorname{AppellF1}\left[1 + m, \frac{1}{2}, -\frac{1}{2}, 2 + m, a x, -a x\right] + a x \left(\operatorname{AppellF1}\left[2 + m, \frac{3}{2}, -\frac{1}{2}, 3 + m, a x, -a x\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1 + \frac{m}{2}\right\}, \left\{2 + \frac{m}{2}\right\}, a^2 x^2\right]\right) \right) \right) \right)$$

Problem 1134: Result unnecessarily involves higher level functions.

$$\int \frac{e^{2 \operatorname{ArcTanh}[a x]} x^m}{\sqrt{c - a^2 c x^2}} dx$$

Optimal (type 5, 169 leaves, 7 steps):

$$\frac{2 x^{1+m} (1 + a x)}{\sqrt{c - a^2 c x^2}} - \frac{(1 + 2 m) x^{1+m} \sqrt{1 - a^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{(1 + m) \sqrt{c - a^2 c x^2}} - \frac{2 a (1 + m) x^{2+m} \sqrt{1 - a^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{(2 + m) \sqrt{c - a^2 c x^2}}$$

Result (type 6, 133 leaves):

$$\left(2 (2 + m) x^{1+m} \sqrt{-c (1 + a x)} \operatorname{AppellF1}\left[1 + m, \frac{3}{2}, -\frac{1}{2}, 2 + m, a x, -a x\right] \right) / \left(c (1 + m) (-1 + a x)^{3/2} \left(2 (2 + m) \operatorname{AppellF1}\left[1 + m, \frac{3}{2}, -\frac{1}{2}, 2 + m, a x, -a x\right] + a x \left(\operatorname{AppellF1}\left[2 + m, \frac{3}{2}, \frac{1}{2}, 3 + m, a x, -a x\right] + 3 \operatorname{AppellF1}\left[2 + m, \frac{5}{2}, -\frac{1}{2}, 3 + m, a x, -a x\right] \right) \right) \right)$$

Problem 1135: Result unnecessarily involves higher level functions and more than twice size of optimal

antiderivative.

$$\int \frac{e^{2 \operatorname{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 5, 183 leaves, 7 steps):

$$\frac{2 x^{1+m} (1 + a x)}{3 (c - a^2 c x^2)^{3/2}} + \frac{(1 - 2 m) x^{1+m} \sqrt{1 - a^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{3 c (1+m) \sqrt{c - a^2 c x^2}} +$$

$$\frac{2 a (1 - m) x^{2+m} \sqrt{1 - a^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{3 c (2+m) \sqrt{c - a^2 c x^2}}$$

Result (type 6, 582 leaves):

$$\left((2+m) x^{1+m} \sqrt{-c(1+ax)} \operatorname{AppellF1}\left[1+m, \frac{3}{2}, -\frac{1}{2}, 2+m, ax, -ax\right] \right) /$$

$$\left(2c^2(1+m)(-1+ax)^{3/2} \left(2(2+m) \operatorname{AppellF1}\left[1+m, \frac{3}{2}, -\frac{1}{2}, 2+m, ax, -ax\right] + \right. \right.$$

$$\left. \left. ax \left(\operatorname{AppellF1}\left[2+m, \frac{3}{2}, \frac{1}{2}, 3+m, ax, -ax\right] + 3 \operatorname{AppellF1}\left[2+m, \frac{5}{2}, -\frac{1}{2}, 3+m, ax, -ax\right] \right) \right) \right) -$$

$$\left((2+m) x^{1+m} \sqrt{-c(1+ax)} \operatorname{AppellF1}\left[1+m, \frac{5}{2}, -\frac{1}{2}, 2+m, ax, -ax\right] \right) /$$

$$\left(c^2(1+m)(-1+ax)^{5/2} \left(2(2+m) \operatorname{AppellF1}\left[1+m, \frac{5}{2}, -\frac{1}{2}, 2+m, ax, -ax\right] + \right. \right.$$

$$\left. \left. ax \left(\operatorname{AppellF1}\left[2+m, \frac{5}{2}, \frac{1}{2}, 3+m, ax, -ax\right] + 5 \operatorname{AppellF1}\left[2+m, \frac{7}{2}, -\frac{1}{2}, 3+m, ax, -ax\right] \right) \right) \right) +$$

$$\left((2+m) x^{1+m} \sqrt{c-ax} \operatorname{AppellF1}\left[1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, -ax, ax\right] \right) / \left(4c^2(1+m) \sqrt{1+ax} \left(2(2+m) \operatorname{AppellF1}\left[1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, -ax, ax\right] - \right. \right.$$

$$\left. \left. ax \left(\operatorname{AppellF1}\left[2+m, \frac{3}{2}, -\frac{1}{2}, 3+m, -ax, ax\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2 x^2\right] \right) \right) \right) +$$

$$\left((2+m) x^{1+m} \sqrt{1-ax} \sqrt{-c(1+ax)} \sqrt{1-a^2 x^2} \operatorname{AppellF1}\left[1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, ax, -ax\right] \right) /$$

$$\left(4c^2(1+m)(-1+ax)^{3/2} \sqrt{1+ax} \left(2(2+m) \operatorname{AppellF1}\left[1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, ax, -ax\right] + \right. \right.$$

$$\left. \left. ax \left(\operatorname{AppellF1}\left[2+m, \frac{3}{2}, -\frac{1}{2}, 3+m, ax, -ax\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2 x^2\right] \right) \right) \right)$$

Problem 1136: Result more than twice size of optimal antiderivative.

$$\int e^{2 \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p dx$$

Optimal (type 5, 55 leaves, 3 steps):

$$\frac{2^{1+p} (1 + a x)^{-p} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[-1 - p, p, 1 + p, \frac{1}{2} (1 - a x)\right]}{a^p}$$

Result (type 5, 133 leaves):

$$\frac{1}{a (1 + p)} \left(-(-1 + a x)^2 \right)^{-p} (-2 + 2 a x)^p (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p$$

$$\left(-a (1 + p) x \left(\frac{1}{2} - \frac{a x}{2} \right)^p \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, a^2 x^2\right] + (1 + a x) (1 - a^2 x^2)^p \operatorname{Hypergeometric2F1}\left[1 - p, 1 + p, 2 + p, \frac{1}{2} (1 + a x)\right] \right)$$

Problem 1171: Result unnecessarily involves higher level functions.

$$\int \frac{e^{3 \operatorname{ArcTanh}[a x]}}{(c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 185 leaves, 5 steps):

$$\frac{\sqrt{1 - a^2 x^2}}{6 a c^2 (1 - a x)^3 \sqrt{c - a^2 c x^2}} + \frac{\sqrt{1 - a^2 x^2}}{8 a c^2 (1 - a x)^2 \sqrt{c - a^2 c x^2}} + \frac{\sqrt{1 - a^2 x^2}}{8 a c^2 (1 - a x) \sqrt{c - a^2 c x^2}} + \frac{\sqrt{1 - a^2 x^2} \operatorname{ArcTanh}[a x]}{8 a c^2 \sqrt{c - a^2 c x^2}}$$

Result (type 4, 108 leaves):

$$\frac{a \sqrt{1 - a^2 x^2} \left(\sqrt{-a^2} (-10 + 9 a x - 3 a^2 x^2) - 3 i a (-1 + a x)^3 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] \right)}{24 (-a^2)^{3/2} c^2 (-1 + a x)^3 \sqrt{c - a^2 c x^2}}$$

Problem 1172: Result unnecessarily involves higher level functions.

$$\int \frac{e^{3 \operatorname{ArcTanh}[a x]}}{(c - a^2 c x^2)^{7/2}} dx$$

Optimal (type 3, 278 leaves, 5 steps):

$$\frac{\sqrt{1-a^2x^2}}{16ac^3(1-ax)^4\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-a^2x^2}}{12ac^3(1-ax)^3\sqrt{c-a^2cx^2}} + \frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}} +$$

$$\frac{\sqrt{1-a^2x^2}}{8ac^3(1-ax)\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}}{32ac^3(1+ax)\sqrt{c-a^2cx^2}} + \frac{5\sqrt{1-a^2x^2}\operatorname{ArcTanh}[ax]}{32ac^3\sqrt{c-a^2cx^2}}$$

Result (type 4, 136 leaves):

$$-\left(\left(a\sqrt{1-a^2x^2}\left(\sqrt{-a^2}\left(32-15ax-35a^2x^2+45a^3x^3-15a^4x^4\right)-15ia(-1+ax)^4(1+ax)\operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{-a^2}x\right],1\right]\right)\right)/\right.$$

$$\left.\left(96(-a^2)^{3/2}c^3(-1+ax)^4(1+ax)\sqrt{c-a^2cx^2}\right)\right)$$

Problem 1173: Unable to integrate problem.

$$\int e^{3\operatorname{ArcTanh}[ax]} x^m \sqrt{c-a^2cx^2} dx$$

Optimal (type 5, 136 leaves, 5 steps):

$$-\frac{3x^{1+m}\sqrt{c-a^2cx^2}}{(1+m)\sqrt{1-a^2x^2}} - \frac{ax^{2+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-a^2x^2}} + \frac{4x^{1+m}\sqrt{c-a^2cx^2}\operatorname{Hypergeometric2F1}[1,1+m,2+m,ax]}{(1+m)\sqrt{1-a^2x^2}}$$

Result (type 8, 29 leaves):

$$\int e^{3\operatorname{ArcTanh}[ax]} x^m \sqrt{c-a^2cx^2} dx$$

Problem 1174: Unable to integrate problem.

$$\int e^{3\operatorname{ArcTanh}[ax]} x^m (c-a^2cx^2)^p dx$$

Optimal (type 5, 251 leaves, 7 steps):

$$-\frac{3x^{1+m}(c-a^2cx^2)^p}{(m+2p)\sqrt{1-a^2x^2}} - \frac{ax^{2+m}(c-a^2cx^2)^p}{(1+m+2p)\sqrt{1-a^2x^2}} + \frac{(3+4m+2p)x^{1+m}(1-a^2x^2)^{-p}(c-a^2cx^2)^p\operatorname{Hypergeometric2F1}\left[\frac{1+m}{2},\frac{3}{2}-p,\frac{3+m}{2},a^2x^2\right]}{(1+m)(m+2p)} +$$

$$\frac{a(5+4m+6p)x^{2+m}(1-a^2x^2)^{-p}(c-a^2cx^2)^p\operatorname{Hypergeometric2F1}\left[\frac{2+m}{2},\frac{3}{2}-p,\frac{4+m}{2},a^2x^2\right]}{(2+m)(1+m+2p)}$$

Result (type 8, 27 leaves):

$$\int e^{3 \operatorname{ArcTanh}[a x]} x^m (c - a^2 c x^2)^p dx$$

Problem 1179: Unable to integrate problem.

$$\int \frac{e^{3 \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x} dx$$

Optimal (type 5, 193 leaves, 8 steps):

$$\frac{4 (c - a^2 c x^2)^p}{(1 - 2 p) \sqrt{1 - a^2 x^2}} - \frac{a x (c - a^2 c x^2)^p}{2 p \sqrt{1 - a^2 x^2}} + \frac{a (1 + 6 p) x (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{2} - p, \frac{3}{2}, a^2 x^2\right]}{2 p} - \frac{\sqrt{1 - a^2 x^2} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]}{1 + 2 p}$$

Result (type 8, 27 leaves):

$$\int \frac{e^{3 \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x} dx$$

Problem 1180: Unable to integrate problem.

$$\int \frac{e^{3 \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^2} dx$$

Optimal (type 5, 187 leaves, 9 steps):

$$\frac{4 a (c - a^2 c x^2)^p}{(1 - 2 p) \sqrt{1 - a^2 x^2}} - \frac{(c - a^2 c x^2)^p}{x \sqrt{1 - a^2 x^2}} + a^2 (5 - 2 p) x (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{2} - p, \frac{3}{2}, a^2 x^2\right] - \frac{3 a \sqrt{1 - a^2 x^2} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]}{1 + 2 p}$$

Result (type 8, 27 leaves):

$$\int \frac{e^{3 \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^2} dx$$

Problem 1181: Unable to integrate problem.

$$\int \frac{e^{3 \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^3} dx$$

Optimal (type 5, 194 leaves, 8 steps):

$$-\frac{(c - a^2 c x^2)^p}{2 x^2 \sqrt{1 - a^2 x^2}} - \frac{3 a (c - a^2 c x^2)^p}{x \sqrt{1 - a^2 x^2}} + a^3 (7 - 6 p) x (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{2} - p, \frac{3}{2}, a^2 x^2\right] + \frac{a^2 (9 - 2 p) (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[1, -\frac{1}{2} + p, \frac{1}{2} + p, 1 - a^2 x^2\right]}{2 (1 - 2 p) \sqrt{1 - a^2 x^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{e^{3 \text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^3} dx$$

Problem 1185: Result more than twice size of optimal antiderivative.

$$\int e^{4 \text{ArcTanh}[a x]} (c - a^2 c x^2)^2 dx$$

Optimal (type 1, 17 leaves, 2 steps):

$$\frac{c^2 (1 + a x)^5}{5 a}$$

Result (type 1, 49 leaves):

$$c^2 x + 2 a c^2 x^2 + 2 a^2 c^2 x^3 + a^3 c^2 x^4 + \frac{1}{5} a^4 c^2 x^5$$

Problem 1187: Result unnecessarily involves higher level functions.

$$\int \frac{e^{4 \text{ArcTanh}[a x]}}{c - a^2 c x^2} dx$$

Optimal (type 1, 13 leaves, 2 steps):

$$\frac{x}{c (1 - a x)^2}$$

Result (type 3, 18 leaves):

$$\frac{e^{4 \text{ArcTanh}[a x]}}{4 a c}$$

Problem 1191: Result more than twice size of optimal antiderivative.

$$\int e^{4 \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p dx$$

Optimal (type 5, 63 leaves, 3 steps):

$$\frac{2^{2+p} c (1 + a x)^{1-p} (c - a^2 c x^2)^{-1+p} \operatorname{Hypergeometric2F1}\left[-2-p, -1+p, p, \frac{1}{2}(1 - a x)\right]}{a(1-p)}$$

Result (type 5, 159 leaves):

$$\frac{1}{a(1+p)} \left(-(-1 + a x)^2 \right)^{-p} (-2 + 2 a x)^p (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \left(a(1+p) x \left(\frac{1}{2} - \frac{a x}{2} \right)^p \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, a^2 x^2\right] - \right. \\ \left. (1 + a x) (1 - a^2 x^2)^p \left(2 \operatorname{Hypergeometric2F1}\left[1-p, 1+p, 2+p, \frac{1}{2}(1 + a x)\right] - \operatorname{Hypergeometric2F1}\left[2-p, 1+p, 2+p, \frac{1}{2}(1 + a x)\right] \right) \right)$$

Problem 1211: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e^{-\operatorname{ArcTanh}[a x]}}{\sqrt{c - a^2 c x^2}} dx$$

Optimal (type 3, 39 leaves, 3 steps):

$$\frac{\sqrt{1 - a^2 x^2} \operatorname{Log}[1 + a x]}{a \sqrt{c - a^2 c x^2}}$$

Result (type 4, 87 leaves):

$$\frac{a \sqrt{1 - a^2 x^2} \left(-2 i a \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2} x \right], 1 \right] + \sqrt{-a^2} \operatorname{Log}\left[-1 + a^2 x^2 \right] \right)}{2 (-a^2)^{3/2} \sqrt{c - a^2 c x^2}}$$

Problem 1212: Result unnecessarily involves higher level functions.

$$\int \frac{e^{-\operatorname{ArcTanh}[a x]}}{(c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 90 leaves, 5 steps):

$$- \frac{\sqrt{1-a^2 x^2}}{2 a c (1+a x) \sqrt{c-a^2 c x^2}} + \frac{\sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]}{2 a c \sqrt{c-a^2 c x^2}}$$

Result (type 4, 89 leaves):

$$\frac{a \sqrt{1-a^2 x^2} \left(\sqrt{-a^2} + i a (1+a x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2} x \right], 1 \right] \right)}{2 (-a^2)^{3/2} (c+a c x) \sqrt{c-a^2 c x^2}}$$

Problem 1213: Result unnecessarily involves higher level functions.

$$\int \frac{e^{-\operatorname{ArcTanh}[a x]}}{(c-a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 183 leaves, 5 steps):

$$\frac{\sqrt{1-a^2 x^2}}{8 a c^2 (1-a x) \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2}}{8 a c^2 (1+a x)^2 \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2}}{4 a c^2 (1+a x) \sqrt{c-a^2 c x^2}} + \frac{3 \sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]}{8 a c^2 \sqrt{c-a^2 c x^2}}$$

Result (type 4, 118 leaves):

$$- \frac{a \sqrt{1-a^2 x^2} \left(\sqrt{-a^2} (2-3 a x-3 a^2 x^2) - 3 i a (-1+a x) (1+a x)^2 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2} x \right], 1 \right] \right)}{8 (-a^2)^{3/2} (-1+a x) (c+a c x)^2 \sqrt{c-a^2 c x^2}}$$

Problem 1214: Result unnecessarily involves higher level functions.

$$\int \frac{e^{-\operatorname{ArcTanh}[a x]}}{(c-a^2 c x^2)^{7/2}} dx$$

Optimal (type 3, 276 leaves, 5 steps):

$$\frac{\sqrt{1-a^2 x^2}}{32 a c^3 (1-a x)^2 \sqrt{c-a^2 c x^2}} + \frac{\sqrt{1-a^2 x^2}}{8 a c^3 (1-a x) \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2}}{24 a c^3 (1+a x)^3 \sqrt{c-a^2 c x^2}} -$$

$$\frac{3 \sqrt{1-a^2 x^2}}{32 a c^3 (1+a x)^2 \sqrt{c-a^2 c x^2}} - \frac{3 \sqrt{1-a^2 x^2}}{16 a c^3 (1+a x) \sqrt{c-a^2 c x^2}} + \frac{5 \sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]}{16 a c^3 \sqrt{c-a^2 c x^2}}$$

Result (type 4, 136 leaves):

$$- \left(\left(a \sqrt{1 - a^2 x^2} \left(\sqrt{-a^2} (-8 + 25 a x + 25 a^2 x^2 - 15 a^3 x^3 - 15 a^4 x^4) - 15 i a (-1 + a x)^2 (1 + a x)^3 \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{-a^2} x \right], 1 \right] \right) \right) \right) / \left(48 (-a^2)^{3/2} (-1 + a x)^2 (c + a c x)^3 \sqrt{c - a^2 c x^2} \right)$$

Problem 1215: Unable to integrate problem.

$$\int e^{-\operatorname{ArcTanh}[a x]} x^m (c - a^2 c x^2)^p dx$$

Optimal (type 5, 137 leaves, 5 steps):

$$\frac{x^{1+m} (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{1}{2} - p, \frac{3+m}{2}, a^2 x^2 \right]}{1+m} - \frac{a x^{2+m} (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1} \left[\frac{2+m}{2}, \frac{1}{2} - p, \frac{4+m}{2}, a^2 x^2 \right]}{2+m}$$

Result (type 8, 27 leaves):

$$\int e^{-\operatorname{ArcTanh}[a x]} x^m (c - a^2 c x^2)^p dx$$

Problem 1216: Result more than twice size of optimal antiderivative.

$$\int e^{-\operatorname{ArcTanh}[a x]} x^3 (1 - a^2 x^2)^p dx$$

Optimal (type 5, 85 leaves, 6 steps):

$$- \frac{(1 - a^2 x^2)^{\frac{1}{2}+p}}{a^4 (1 + 2p)} + \frac{(1 - a^2 x^2)^{\frac{3}{2}+p}}{a^4 (3 + 2p)} - \frac{1}{5} a x^5 \operatorname{Hypergeometric2F1} \left[\frac{5}{2}, \frac{1}{2} - p, \frac{7}{2}, a^2 x^2 \right]$$

Result (type 5, 183 leaves):

$$\frac{1}{3 a^4} \left(3 a x \operatorname{Hypergeometric2F1} \left[\frac{1}{2}, -\frac{1}{2} - p, \frac{3}{2}, a^2 x^2 \right] + \frac{1}{3 + 2p} \right. \\ \left. \left(-3 + 3 (1 - a^2 x^2)^{\frac{1}{2}+p} - 3 a^2 x^2 (1 - a^2 x^2)^{\frac{1}{2}+p} + a^3 (3 + 2p) x^3 \operatorname{Hypergeometric2F1} \left[\frac{3}{2}, -\frac{1}{2} - p, \frac{5}{2}, a^2 x^2 \right] + \right. \right. \\ \left. \left. 3 (1 - a x) (1 + a x)^{-\frac{1}{2}-p} (2 - 2 a^2 x^2)^{\frac{1}{2}+p} \operatorname{Hypergeometric2F1} \left[\frac{1}{2} - p, \frac{3}{2} + p, \frac{5}{2} + p, \frac{1}{2} - \frac{a x}{2} \right] \right) \right)$$

Problem 1220: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{-\text{ArcTanh}[a x]} (1 - a^2 x^2)^p}{x} dx$$

Optimal (type 5, 73 leaves, 5 steps):

$$-a x \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - p, \frac{3}{2}, a^2 x^2\right] - \frac{(1 - a^2 x^2)^{\frac{1}{2}+p} \text{Hypergeometric2F1}\left[1, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]}{1 + 2 p}$$

Result (type 5, 148 leaves):

$$(1 - a^2 x^2)^{\frac{1}{2}+p} \left(\frac{\text{Hypergeometric2F1}\left[-\frac{1}{2} - p, -\frac{1}{2} - p, \frac{1}{2} - p, \frac{1}{a^2 x^2}\right]}{\left(1 - \frac{1}{a^2 x^2}\right)^{\frac{1}{2}+p} + 2 p \left(1 - \frac{1}{a^2 x^2}\right)^{\frac{1}{2}+p}} + \frac{2^{\frac{1}{2}+p} (1 - a x) (1 + a x)^{-\frac{1}{2}-p} \text{Hypergeometric2F1}\left[\frac{1}{2} - p, \frac{3}{2} + p, \frac{5}{2} + p, \frac{1}{2} - \frac{a x}{2}\right]}{3 + 2 p} \right)$$

Problem 1221: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{-\text{ArcTanh}[a x]} (1 - a^2 x^2)^p}{x^2} dx$$

Optimal (type 5, 74 leaves, 5 steps):

$$-\frac{\text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2} - p, \frac{1}{2}, a^2 x^2\right]}{x} + \frac{a (1 - a^2 x^2)^{\frac{1}{2}+p} \text{Hypergeometric2F1}\left[1, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]}{1 + 2 p}$$

Result (type 5, 171 leaves):

$$-\frac{\text{Hypergeometric2F1}\left[-\frac{1}{2}, -\frac{1}{2} - p, \frac{1}{2}, a^2 x^2\right]}{x} - \frac{a \left(1 - \frac{1}{a^2 x^2}\right)^{-\frac{1}{2}-p} (1 - a^2 x^2)^{\frac{1}{2}+p} \text{Hypergeometric2F1}\left[-\frac{1}{2} - p, -\frac{1}{2} - p, \frac{1}{2} - p, \frac{1}{a^2 x^2}\right]}{1 + 2 p} + \frac{a (-1 + a x) (1 + a x)^{-\frac{1}{2}-p} (2 - 2 a^2 x^2)^{\frac{1}{2}+p} \text{Hypergeometric2F1}\left[\frac{1}{2} - p, \frac{3}{2} + p, \frac{5}{2} + p, \frac{1}{2} - \frac{a x}{2}\right]}{3 + 2 p}$$

Problem 1222: Result more than twice size of optimal antiderivative.

$$\int e^{-\text{ArcTanh}[a x]} x^3 (c - a^2 c x^2)^p dx$$

Optimal (type 5, 134 leaves, 7 steps):

$$-\frac{\sqrt{1-a^2x^2}(c-a^2cx^2)^p}{a^4(1+2p)} + \frac{(1-a^2x^2)^{3/2}(c-a^2cx^2)^p}{a^4(3+2p)} - \frac{1}{5}ax^5(1-a^2x^2)^{-p}(c-a^2cx^2)^p \text{Hypergeometric2F1}\left[\frac{5}{2}, \frac{1}{2}-p, \frac{7}{2}, a^2x^2\right]$$

Result (type 5, 290 leaves):

$$\frac{1}{a^4(1+2p)(3+2p)(5+2p)(7+2p)} 4^{1+p} \left(\frac{e^{\text{ArcTanh}[ax]}}{1+e^{2\text{ArcTanh}[ax]}} \right)^{1+2p} (1+e^{2\text{ArcTanh}[ax]})^{1+2p} \\ (1-a^2x^2)^{-p}(c-a^2cx^2)^p \left(- (105+142p+60p^2+8p^3) \text{Hypergeometric2F1}\left[\frac{1}{2}+p, 5+2p, \frac{3}{2}+p, -e^{2\text{ArcTanh}[ax]}\right] + \right. \\ \left. e^{2\text{ArcTanh}[ax]}(1+2p) \left(3(35+24p+4p^2) \text{Hypergeometric2F1}\left[\frac{3}{2}+p, 5+2p, \frac{5}{2}+p, -e^{2\text{ArcTanh}[ax]}\right] + \right. \right. \\ \left. \left. e^{2\text{ArcTanh}[ax]}(3+2p) \left(-3(7+2p) \text{Hypergeometric2F1}\left[\frac{5}{2}+p, 5+2p, \frac{7}{2}+p, -e^{2\text{ArcTanh}[ax]}\right] + \right. \right. \right. \\ \left. \left. \left. e^{2\text{ArcTanh}[ax]}(5+2p) \text{Hypergeometric2F1}\left[\frac{7}{2}+p, 5+2p, \frac{9}{2}+p, -e^{2\text{ArcTanh}[ax]}\right] \right) \right) \right) \right)$$

Problem 1226: Unable to integrate problem.

$$\int \frac{e^{-\text{ArcTanh}[ax]}(c-a^2cx^2)^p}{x} dx$$

Optimal (type 5, 111 leaves, 6 steps):

$$-ax(1-a^2x^2)^{-p}(c-a^2cx^2)^p \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}-p, \frac{3}{2}, a^2x^2\right] - \frac{\sqrt{1-a^2x^2}(c-a^2cx^2)^p \text{Hypergeometric2F1}\left[1, \frac{1}{2}+p, \frac{3}{2}+p, 1-a^2x^2\right]}{1+2p}$$

Result (type 8, 27 leaves):

$$\int \frac{e^{-\text{ArcTanh}[ax]}(c-a^2cx^2)^p}{x} dx$$

Problem 1227: Unable to integrate problem.

$$\int \frac{e^{-\text{ArcTanh}[ax]}(c-a^2cx^2)^p}{x^2} dx$$

Optimal (type 5, 112 leaves, 6 steps):

$$-\frac{(1-a^2x^2)^{-p}(c-a^2cx^2)^p \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2}-p, \frac{1}{2}, a^2x^2\right]}{x} + \frac{a\sqrt{1-a^2x^2}(c-a^2cx^2)^p \text{Hypergeometric2F1}\left[1, \frac{1}{2}+p, \frac{3}{2}+p, 1-a^2x^2\right]}{1+2p}$$

Result (type 8, 27 leaves):

$$\int \frac{e^{-\text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^2} dx$$

Problem 1232: Result unnecessarily involves higher level functions.

$$\int \frac{e^{-2 \text{ArcTanh}[a x]}}{c - a^2 c x^2} dx$$

Optimal (type 1, 15 leaves, 2 steps):

$$-\frac{1}{a c (1 + a x)}$$

Result (type 3, 18 leaves):

$$-\frac{e^{-2 \text{ArcTanh}[a x]}}{2 a c}$$

Problem 1252: Result unnecessarily involves higher level functions.

$$\int e^{-2 \text{ArcTanh}[a x]} x^m \sqrt{c - a^2 c x^2} dx$$

Optimal (type 5, 172 leaves, 7 steps):

$$-\frac{x^{1+m} \sqrt{c - a^2 c x^2}}{2 + m} + \frac{c (3 + 2 m) x^{1+m} \sqrt{1 - a^2 x^2} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{(1+m) (2+m) \sqrt{c - a^2 c x^2}} - \frac{2 a c x^{2+m} \sqrt{1 - a^2 x^2} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{(2+m) \sqrt{c - a^2 c x^2}}$$

Result (type 6, 192 leaves):

$$\frac{1}{1+m} x^{1+m} \left(-\frac{\sqrt{c - a^2 c x^2} \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{\sqrt{1 - a^2 x^2}} - \left(4 (2+m) \sqrt{c - a c x} \text{AppellF1}\left[1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, -a x, a x\right] \right) / \left(\sqrt{1+a x} \left(-2 (2+m) \text{AppellF1}\left[1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, -a x, a x\right] + a x \left(\text{AppellF1}\left[2+m, \frac{3}{2}, -\frac{1}{2}, 3+m, -a x, a x\right] + \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2 x^2\right]\right) \right) \right) \right)$$

Problem 1253: Result more than twice size of optimal antiderivative.

$$\int e^{-2 \text{ArcTanh}[a x]} (c - a^2 c x^2)^p dx$$

Optimal (type 5, 54 leaves, 3 steps):

$$\frac{2^{1+p} (1 - a x)^{-p} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[-1 - p, p, 1 + p, \frac{1}{2} (1 + a x)\right]}{a p}$$

Result (type 5, 125 leaves):

$$\frac{1}{a (1 + p)} 2^p (1 + a x)^{-p} (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p$$

$$\left(-a (1 + p) x \left(\frac{1}{2} + \frac{a x}{2} \right)^p \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, a^2 x^2\right] + (-1 + a x) (1 - a^2 x^2)^p \text{Hypergeometric2F1}\left[1 - p, 1 + p, 2 + p, \frac{1}{2} - \frac{a x}{2}\right] \right)$$

Problem 1277: Result unnecessarily involves higher level functions.

$$\int \frac{e^{-3 \text{ArcTanh}[a x]}}{(c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 182 leaves, 5 steps):

$$-\frac{\sqrt{1 - a^2 x^2}}{6 a c^2 (1 + a x)^3 \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2}}{8 a c^2 (1 + a x)^2 \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2}}{8 a c^2 (1 + a x) \sqrt{c - a^2 c x^2}} + \frac{\sqrt{1 - a^2 x^2} \text{ArcTanh}[a x]}{8 a c^2 \sqrt{c - a^2 c x^2}}$$

Result (type 4, 108 leaves):

$$\frac{a \sqrt{1 - a^2 x^2} \left(\sqrt{-a^2} (10 + 9 a x + 3 a^2 x^2) + 3 i a (1 + a x)^3 \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] \right)}{24 (-a^2)^{3/2} c^2 (1 + a x)^3 \sqrt{c - a^2 c x^2}}$$

Problem 1278: Result unnecessarily involves higher level functions.

$$\int \frac{e^{-3 \text{ArcTanh}[a x]}}{(c - a^2 c x^2)^{7/2}} dx$$

Optimal (type 3, 275 leaves, 5 steps):

$$\frac{\sqrt{1 - a^2 x^2}}{32 a c^3 (1 - a x) \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2}}{16 a c^3 (1 + a x)^4 \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2}}{12 a c^3 (1 + a x)^3 \sqrt{c - a^2 c x^2}} -$$

$$\frac{3 \sqrt{1 - a^2 x^2}}{32 a c^3 (1 + a x)^2 \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2}}{8 a c^3 (1 + a x) \sqrt{c - a^2 c x^2}} + \frac{5 \sqrt{1 - a^2 x^2} \text{ArcTanh}[a x]}{32 a c^3 \sqrt{c - a^2 c x^2}}$$

Result (type 4, 136 leaves):

$$- \left(\left(a \sqrt{1 - a^2 x^2} \left(\sqrt{-a^2} (32 + 15 a x - 35 a^2 x^2 - 45 a^3 x^3 - 15 a^4 x^4) - 15 i a (-1 + a x) (1 + a x)^4 \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{-a^2} x \right], 1 \right] \right) \right) \right) / \left(96 (-a^2)^{3/2} c^3 (-1 + a x) (1 + a x)^4 \sqrt{c - a^2 c x^2} \right)$$

Problem 1279: Unable to integrate problem.

$$\int e^{-3 \text{ArcTanh}[a x]} x^m \sqrt{c - a^2 c x^2} dx$$

Optimal (type 5, 136 leaves, 5 steps):

$$-\frac{3 x^{1+m} \sqrt{c - a^2 c x^2}}{(1+m) \sqrt{1 - a^2 x^2}} + \frac{a x^{2+m} \sqrt{c - a^2 c x^2}}{(2+m) \sqrt{1 - a^2 x^2}} + \frac{4 x^{1+m} \sqrt{c - a^2 c x^2} \text{Hypergeometric2F1}[1, 1+m, 2+m, -a x]}{(1+m) \sqrt{1 - a^2 x^2}}$$

Result (type 8, 29 leaves):

$$\int e^{-3 \text{ArcTanh}[a x]} x^m \sqrt{c - a^2 c x^2} dx$$

Problem 1281: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \text{ArcTanh}[a x]} (1 - a^2 x^2)^{5/2} dx$$

Optimal (type 3, 359 leaves, 18 steps):

$$\begin{aligned} & \frac{231 (1 - a x)^{1/4} (1 + a x)^{3/4}}{512 a} + \frac{231 (1 - a x)^{5/4} (1 + a x)^{3/4}}{1280 a} + \frac{77 (1 - a x)^{9/4} (1 + a x)^{3/4}}{960 a} - \\ & \frac{77 (1 - a x)^{13/4} (1 + a x)^{3/4}}{480 a} - \frac{11 (1 - a x)^{13/4} (1 + a x)^{7/4}}{60 a} - \frac{(1 - a x)^{13/4} (1 + a x)^{11/4}}{6 a} + \frac{231 \text{ArcTan} \left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{512 \sqrt{2} a} - \\ & \frac{231 \text{ArcTan} \left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{512 \sqrt{2} a} + \frac{231 \text{Log} \left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{1024 \sqrt{2} a} - \frac{231 \text{Log} \left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{1024 \sqrt{2} a} \end{aligned}$$

Result (type 7, 422 leaves):

$$\frac{1}{1920 a \left(1 + e^{2 \operatorname{ArcTanh}[a x]}\right)^6}$$

$$\left(960 e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} \left(1 + e^{2 \operatorname{ArcTanh}[a x]}\right)^4 \left(-1 + 3 e^{2 \operatorname{ArcTanh}[a x]}\right) - 360 \left(1 + e^{2 \operatorname{ArcTanh}[a x]}\right)^6 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \& \right) +$$

$$80 \left(1 + e^{2 \operatorname{ArcTanh}[a x]}\right)^2 \left(\frac{39 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \&}{(-1 + a^2 x^2)^2} - \right.$$

$$\left. \left(\operatorname{Cosh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right] + \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right] \right) \left(13 \operatorname{Cosh}\left[3 \operatorname{ArcTanh}[a x]\right] + \frac{7 - 166 a x + 26 \sqrt{1 - a^2 x^2} \operatorname{Sinh}\left[3 \operatorname{ArcTanh}[a x]\right]}{\sqrt{1 - a^2 x^2}} \right) \right)$$

$$\left(\operatorname{Cosh}\left[4 \operatorname{ArcTanh}[a x]\right] + \operatorname{Sinh}\left[4 \operatorname{ArcTanh}[a x]\right] \right) - \left(- \frac{3300 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \&}{(-1 + a^2 x^2)^3} - \right.$$

$$\left. \left(\operatorname{Cosh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right] + \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right] \right) \left(\frac{286}{\sqrt{1 - a^2 x^2}} + \frac{12556 a x}{\sqrt{1 - a^2 x^2}} - 129 \operatorname{Cosh}\left[3 \operatorname{ArcTanh}[a x]\right] + 275 \operatorname{Cosh}\left[5 \operatorname{ArcTanh}[a x]\right] - \right.$$

$$\left. \left. 7374 \operatorname{Sinh}\left[3 \operatorname{ArcTanh}[a x]\right] + 550 \operatorname{Sinh}\left[5 \operatorname{ArcTanh}[a x]\right] \right) \right) \left(\operatorname{Cosh}\left[6 \operatorname{ArcTanh}[a x]\right] + \operatorname{Sinh}\left[6 \operatorname{ArcTanh}[a x]\right] \right)$$

Problem 1282: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} \left(1 - a^2 x^2\right)^{3/2} dx$$

Optimal (type 3, 307 leaves, 16 steps):

$$\frac{35 (1 - a x)^{1/4} (1 + a x)^{3/4}}{64 a} + \frac{7 (1 - a x)^{5/4} (1 + a x)^{3/4}}{32 a} - \frac{7 (1 - a x)^{9/4} (1 + a x)^{3/4}}{24 a} - \frac{(1 - a x)^{9/4} (1 + a x)^{7/4}}{4 a} +$$

$$\frac{35 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a} - \frac{35 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a} + \frac{35 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a} - \frac{35 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a}$$

Result (type 7, 249 leaves):

$$\frac{1}{48 a \left(1 + e^{2 \operatorname{ArcTanh}[a x]}\right)^4} \left(24 e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} \left(1 + e^{2 \operatorname{ArcTanh}[a x]}\right)^2 \left(-1 + 3 e^{2 \operatorname{ArcTanh}[a x]}\right) - 9 \left(1 + e^{2 \operatorname{ArcTanh}[a x]}\right)^4 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \& \right) + \left(\frac{39 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1}\right]}{\left(-1 + a^2 x^2\right)^2} \& \right) - \left(\operatorname{Cosh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right] + \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right] \right) \left(13 \operatorname{Cosh}\left[3 \operatorname{ArcTanh}[a x]\right] + \frac{7 - 166 a x + 26 \sqrt{1 - a^2 x^2} \operatorname{Sinh}\left[3 \operatorname{ArcTanh}[a x]\right]}{\sqrt{1 - a^2 x^2}} \right) \left(\operatorname{Cosh}\left[4 \operatorname{ArcTanh}[a x]\right] + \operatorname{Sinh}\left[4 \operatorname{ArcTanh}[a x]\right] \right)$$

Problem 1283: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} \sqrt{1 - a^2 x^2} \, dx$$

Optimal (type 3, 255 leaves, 14 steps):

$$\frac{3 (1 - a x)^{1/4} (1 + a x)^{3/4}}{4 a} - \frac{(1 - a x)^{5/4} (1 + a x)^{3/4}}{2 a} + \frac{3 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a} - \frac{3 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a} + \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a} - \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a}$$

Result (type 7, 83 leaves):

$$\frac{8 e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} \left(-1 + 3 e^{2 \operatorname{ArcTanh}[a x]}\right)}{\left(1 + e^{2 \operatorname{ArcTanh}[a x]}\right)^2} - 3 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \& \right]}{16 a}$$

Problem 1284: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}}{\sqrt{1 - a^2 x^2}} \, dx$$

Optimal (type 3, 193 leaves, 12 steps):

$$\frac{\sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{a} - \frac{\sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{a} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2} a} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2} a}$$

Result (type 7, 46 leaves):

$$\frac{\operatorname{RootSum}\left[1 + \#1^4 \&, \frac{-\operatorname{ArcTanh}[ax] + 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[ax]} - \#1\right]}{\#1}\right]}{2 a}$$

Problem 1289: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[ax]} (c - a^2 c x^2)^{5/2} dx$$

Optimal (type 3, 679 leaves, 19 steps):

$$\begin{aligned} & \frac{231 c^2 (1-ax)^{1/4} (1+ax)^{3/4} \sqrt{c - a^2 c x^2}}{512 a \sqrt{1 - a^2 x^2}} + \frac{231 c^2 (1-ax)^{5/4} (1+ax)^{3/4} \sqrt{c - a^2 c x^2}}{1280 a \sqrt{1 - a^2 x^2}} + \\ & \frac{77 c^2 (1-ax)^{9/4} (1+ax)^{3/4} \sqrt{c - a^2 c x^2}}{960 a \sqrt{1 - a^2 x^2}} - \frac{77 c^2 (1-ax)^{13/4} (1+ax)^{3/4} \sqrt{c - a^2 c x^2}}{480 a \sqrt{1 - a^2 x^2}} - \frac{11 c^2 (1-ax)^{13/4} (1+ax)^{7/4} \sqrt{c - a^2 c x^2}}{60 a \sqrt{1 - a^2 x^2}} - \\ & \frac{c^2 (1-ax)^{13/4} (1+ax)^{11/4} \sqrt{c - a^2 c x^2}}{6 a \sqrt{1 - a^2 x^2}} + \frac{231 c^2 \sqrt{c - a^2 c x^2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{512 \sqrt{2} a \sqrt{1 - a^2 x^2}} - \frac{231 c^2 \sqrt{c - a^2 c x^2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{512 \sqrt{2} a \sqrt{1 - a^2 x^2}} + \\ & \frac{231 c^2 \sqrt{c - a^2 c x^2} \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{1024 \sqrt{2} a \sqrt{1 - a^2 x^2}} - \frac{231 c^2 \sqrt{c - a^2 c x^2} \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{1024 \sqrt{2} a \sqrt{1 - a^2 x^2}} \end{aligned}$$

Result (type 7, 167 leaves):

$$\begin{aligned} & - \left(\left(c^3 \sqrt{1 - a^2 x^2} \left(-8 e^{\frac{3}{2} \operatorname{ArcTanh}[ax]} (-1155 - 6435 e^{2 \operatorname{ArcTanh}[ax]} - 14670 e^{4 \operatorname{ArcTanh}[ax]} + 48202 e^{6 \operatorname{ArcTanh}[ax]} + 20097 e^{8 \operatorname{ArcTanh}[ax]} + 3465 e^{10 \operatorname{ArcTanh}[ax]}) + \right. \right. \\ & \left. \left. 3465 (1 + e^{2 \operatorname{ArcTanh}[ax]})^6 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[ax] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[ax]} - \#1\right]}{\#1}\right] \&\right] \right) \Bigg/ \left(30720 a (1 + e^{2 \operatorname{ArcTanh}[ax]})^6 \sqrt{c - a^2 c x^2} \right) \end{aligned}$$

Problem 1290: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[ax]} (c - a^2 c x^2)^{3/2} dx$$

Optimal (type 3, 547 leaves, 17 steps):

$$\begin{aligned} & \frac{35 c (1-a x)^{1/4} (1+a x)^{3/4} \sqrt{c-a^2 c x^2}}{64 a \sqrt{1-a^2 x^2}} + \frac{7 c (1-a x)^{5/4} (1+a x)^{3/4} \sqrt{c-a^2 c x^2}}{32 a \sqrt{1-a^2 x^2}} - \frac{7 c (1-a x)^{9/4} (1+a x)^{3/4} \sqrt{c-a^2 c x^2}}{24 a \sqrt{1-a^2 x^2}} \\ & - \frac{c (1-a x)^{9/4} (1+a x)^{7/4} \sqrt{c-a^2 c x^2}}{4 a \sqrt{1-a^2 x^2}} + \frac{35 c \sqrt{c-a^2 c x^2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{64 \sqrt{2} a \sqrt{1-a^2 x^2}} - \frac{35 c \sqrt{c-a^2 c x^2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{64 \sqrt{2} a \sqrt{1-a^2 x^2}} + \\ & - \frac{35 c \sqrt{c-a^2 c x^2} \operatorname{Log}\left[1 + \frac{\sqrt{1-a x}}{\sqrt{1+a x}} - \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{128 \sqrt{2} a \sqrt{1-a^2 x^2}} - \frac{35 c \sqrt{c-a^2 c x^2} \operatorname{Log}\left[1 + \frac{\sqrt{1-a x}}{\sqrt{1+a x}} + \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{128 \sqrt{2} a \sqrt{1-a^2 x^2}} \end{aligned}$$

Result (type 7, 147 leaves):

$$\begin{aligned} & - \left(\left(c^2 \sqrt{1-a^2 x^2} \left(-8 e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} \left(-35 - 125 e^{2 \operatorname{ArcTanh}[a x]} + 399 e^{4 \operatorname{ArcTanh}[a x]} + 105 e^{6 \operatorname{ArcTanh}[a x]} \right) + \right. \right. \\ & \left. \left. 105 \left(1 + e^{2 \operatorname{ArcTanh}[a x]} \right)^4 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1} \&\right] \right) \right) / \left(768 a \left(1 + e^{2 \operatorname{ArcTanh}[a x]} \right)^4 \sqrt{c-a^2 c x^2} \right) \end{aligned}$$

Problem 1291: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} \sqrt{c-a^2 c x^2} dx$$

Optimal (type 3, 429 leaves, 15 steps):

$$\begin{aligned} & \frac{3 (1-a x)^{1/4} (1+a x)^{3/4} \sqrt{c-a^2 c x^2}}{4 a \sqrt{1-a^2 x^2}} - \frac{(1-a x)^{5/4} (1+a x)^{3/4} \sqrt{c-a^2 c x^2}}{2 a \sqrt{1-a^2 x^2}} + \frac{3 \sqrt{c-a^2 c x^2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{4 \sqrt{2} a \sqrt{1-a^2 x^2}} - \\ & - \frac{3 \sqrt{c-a^2 c x^2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{4 \sqrt{2} a \sqrt{1-a^2 x^2}} + \frac{3 \sqrt{c-a^2 c x^2} \operatorname{Log}\left[1 + \frac{\sqrt{1-a x}}{\sqrt{1+a x}} - \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{8 \sqrt{2} a \sqrt{1-a^2 x^2}} - \frac{3 \sqrt{c-a^2 c x^2} \operatorname{Log}\left[1 + \frac{\sqrt{1-a x}}{\sqrt{1+a x}} + \frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{8 \sqrt{2} a \sqrt{1-a^2 x^2}} \end{aligned}$$

Result (type 7, 126 leaves):

$$\begin{aligned} & \left(c \sqrt{1-a^2 x^2} \left(8 e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} \left(-1 + 3 e^{2 \operatorname{ArcTanh}[a x]} \right) - 3 \left(1 + e^{2 \operatorname{ArcTanh}[a x]} \right)^2 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1} \&\right] \right) \right) / \\ & \left(16 a \left(1 + e^{2 \operatorname{ArcTanh}[a x]} \right)^2 \sqrt{c(1-a^2 x^2)} \right) \end{aligned}$$

Problem 1292: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}}{\sqrt{c - a^2 c x^2}} dx$$

Optimal (type 3, 309 leaves, 13 steps):

$$\frac{\sqrt{2} \sqrt{1 - a^2 x^2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{a \sqrt{c - a^2 c x^2}} - \frac{\sqrt{2} \sqrt{1 - a^2 x^2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{a \sqrt{c - a^2 c x^2}} +$$

$$\frac{\sqrt{1 - a^2 x^2} \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2} a \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2} \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2} a \sqrt{c - a^2 c x^2}}$$

Result (type 7, 79 leaves):

$$\frac{\sqrt{c (1 - a^2 x^2)} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{-\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \&]}{2 a c \sqrt{1 - a^2 x^2}}$$

Problem 1307: Unable to integrate problem.

$$\int \frac{e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}}{x (c - a^2 c x^2)^{9/8}} dx$$

Optimal (type 6, 73 leaves, 3 steps):

$$\frac{2 \times 2^{5/8} (1 + a x)^{1/8} (1 - a^2 x^2)^{1/8} \operatorname{AppellF1}\left[\frac{1}{8}, \frac{11}{8}, 1, \frac{9}{8}, \frac{1}{2} (1 + a x), 1 + a x\right]}{c (c - a^2 c x^2)^{1/8}}$$

Result (type 8, 31 leaves):

$$\int \frac{e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}}{x (c - a^2 c x^2)^{9/8}} dx$$

Problem 1309: Result more than twice size of optimal antiderivative.

$$\int e^{n \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^2 dx$$

Optimal (type 5, 70 leaves, 2 steps):

$$\frac{2^{3+\frac{n}{2}} c^2 (1-ax)^{3-\frac{n}{2}} \text{Hypergeometric2F1}\left[-2-\frac{n}{2}, 3-\frac{n}{2}, 4-\frac{n}{2}, \frac{1}{2}(1-ax)\right]}{a(6-n)}$$

Result (type 5, 184 leaves):

$$\frac{1}{120a} c^2 e^{n \text{ArcTanh}[ax]} \left(22n - n^3 + 120ax - 22a^2n^2x + a^4n^4x - 28a^2n^2x^2 + a^2n^3x^2 - 80a^3x^3 + 2a^3n^2x^3 + 6a^4nx^4 + 24a^5x^5 - e^{2 \text{ArcTanh}[ax]} n (32 - 16n - 2n^2 + n^3) \right. \\ \left. \text{Hypergeometric2F1}\left[1, 1 + \frac{n}{2}, 2 + \frac{n}{2}, -e^{2 \text{ArcTanh}[ax]}\right] + (64 - 20n^2 + n^4) \text{Hypergeometric2F1}\left[1, \frac{n}{2}, 1 + \frac{n}{2}, -e^{2 \text{ArcTanh}[ax]}\right] \right)$$

Problem 1310: Result more than twice size of optimal antiderivative.

$$\int e^{n \text{ArcTanh}[ax]} (c - a^2 c x^2)^3 dx$$

Optimal (type 5, 70 leaves, 2 steps):

$$\frac{2^{4+\frac{n}{2}} c^3 (1-ax)^{4-\frac{n}{2}} \text{Hypergeometric2F1}\left[-3-\frac{n}{2}, 4-\frac{n}{2}, 5-\frac{n}{2}, \frac{1}{2}(1-ax)\right]}{a(8-n)}$$

Result (type 5, 272 leaves):

$$\frac{1}{5040a} c^3 e^{n \text{ArcTanh}[ax]} \left(-912n + 58n^3 - n^5 - 5040ax + 912a^2n^2x - 58a^4n^4x + a^6n^6x + 1368a^2n^2x^2 - 64a^2n^3x^2 + a^2n^5x^2 + 5040a^3x^3 - 152a^3n^2x^3 + 2a^3n^4x^3 - \right. \\ \left. 576a^4nx^4 + 6a^4n^3x^4 - 3024a^5x^5 + 24a^5n^2x^5 + 120a^6nx^6 + 720a^7x^7 - e^{2 \text{ArcTanh}[ax]} n (-1152 + 576n + 104n^2 - 52n^3 - 2n^4 + n^5) \right. \\ \left. \text{Hypergeometric2F1}\left[1, 1 + \frac{n}{2}, 2 + \frac{n}{2}, -e^{2 \text{ArcTanh}[ax]}\right] + (-2304 + 784n^2 - 56n^4 + n^6) \text{Hypergeometric2F1}\left[1, \frac{n}{2}, 1 + \frac{n}{2}, -e^{2 \text{ArcTanh}[ax]}\right] \right)$$

Problem 1356: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[ax]} x^m (c - a^2 c x^2)^2 dx$$

Optimal (type 6, 42 leaves, 2 steps):

$$\frac{c^2 x^{1+m} \text{AppellF1}\left[1+m, \frac{1}{2}(-4+n), -2-\frac{n}{2}, 2+m, ax, -ax\right]}{1+m}$$

Result (type 8, 27 leaves):

$$\int e^{n \operatorname{ArcTanh}[a x]} x^m (c - a^2 c x^2)^2 dx$$

Problem 1357: Unable to integrate problem.

$$\int e^{n \operatorname{ArcTanh}[a x]} x^m (c - a^2 c x^2) dx$$

Optimal (type 6, 40 leaves, 2 steps):

$$\frac{c x^{1+m} \operatorname{AppellF1}\left[1+m, \frac{1}{2}(-2+n), -1-\frac{n}{2}, 2+m, a x, -a x\right]}{1+m}$$

Result (type 8, 25 leaves):

$$\int e^{n \operatorname{ArcTanh}[a x]} x^m (c - a^2 c x^2) dx$$

Problem 1358: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]} x^m}{c - a^2 c x^2} dx$$

Optimal (type 6, 42 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, \frac{2+n}{2}, 1-\frac{n}{2}, 2+m, a x, -a x\right]}{c(1+m)}$$

Result (type 6, 106 leaves):

$$\frac{1}{a c n} e^{n \operatorname{ArcTanh}[a x]} \left(-1 + e^{-2 \operatorname{ArcTanh}[a x]}\right)^m \left(1 + e^{-2 \operatorname{ArcTanh}[a x]}\right)^m \\ \left(-e^{-4 \operatorname{ArcTanh}[a x]} \left(-1 + e^{2 \operatorname{ArcTanh}[a x]}\right)^2\right)^{-m} x^m \operatorname{AppellF1}\left[-\frac{n}{2}, m, -m, 1-\frac{n}{2}, -e^{-2 \operatorname{ArcTanh}[a x]}, e^{-2 \operatorname{ArcTanh}[a x]}\right]$$

Problem 1359: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^2} dx$$

Optimal (type 6, 42 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, \frac{4+n}{2}, 2-\frac{n}{2}, 2+m, a x, -a x\right]}{c^2(1+m)}$$

Result (type 8, 27 leaves):

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^2} dx$$

Problem 1360: Unable to integrate problem.

$$\int e^{n \operatorname{ArcTanh}[a x]} x^m (c - a^2 c x^2)^p dx$$

Optimal (type 6, 70 leaves, 3 steps):

$$\frac{x^{1+m} (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \operatorname{AppellF1}\left[1 + m, \frac{1}{2} (n - 2 p), -\frac{n}{2} - p, 2 + m, a x, -a x\right]}{1 + m}$$

Result (type 8, 27 leaves):

$$\int e^{n \operatorname{ArcTanh}[a x]} x^m (c - a^2 c x^2)^p dx$$

Problem 1361: Unable to integrate problem.

$$\int e^{n \operatorname{ArcTanh}[a x]} x (c - a^2 c x^2)^p dx$$

Optimal (type 5, 177 leaves, 4 steps):

$$-\frac{(1 - a x)^{1 - \frac{n}{2} + p} (1 + a x)^{1 + \frac{n}{2} + p} (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p}{2 a^2 (1 + p)} - \frac{1}{a^2 (1 + p) (2 - n + 2 p)}$$

$$2^{\frac{n}{2} + p} n (1 - a x)^{1 - \frac{n}{2} + p} (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[-\frac{n}{2} - p, 1 - \frac{n}{2} + p, 2 - \frac{n}{2} + p, \frac{1}{2} (1 - a x)\right]$$

Result (type 8, 25 leaves):

$$\int e^{n \operatorname{ArcTanh}[a x]} x (c - a^2 c x^2)^p dx$$

Problem 1362: Unable to integrate problem.

$$\int e^{n \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p dx$$

Optimal (type 5, 103 leaves, 3 steps):

$$-\frac{1}{a(2-n+2p)} 2^{1+\frac{n}{2}+p} (1-ax)^{1-\frac{n}{2}+p} (1-a^2x^2)^{-p} (c-a^2cx^2)^p \text{Hypergeometric2F1}\left[-\frac{n}{2}-p, 1-\frac{n}{2}+p, 2-\frac{n}{2}+p, \frac{1}{2}(1-ax)\right]$$

Result (type 8, 24 leaves):

$$\int e^{n \text{ArcTanh}[ax]} (c-a^2cx^2)^p dx$$

Problem 1363: Unable to integrate problem.

$$\int e^{2(1+p) \text{ArcTanh}[ax]} (1-a^2x^2)^{-p} dx$$

Optimal (type 3, 41 leaves, 3 steps):

$$\frac{(1-ax)^{1-2p}}{a(1-2p)} + \frac{(1-ax)^{-2p}}{ap}$$

Result (type 8, 28 leaves):

$$\int e^{2(1+p) \text{ArcTanh}[ax]} (1-a^2x^2)^{-p} dx$$

Problem 1364: Unable to integrate problem.

$$\int e^{2(1+p) \text{ArcTanh}[ax]} (c-a^2cx^2)^{-p} dx$$

Optimal (type 3, 95 leaves, 4 steps):

$$\frac{(1-ax)^{1-2p} (1-a^2x^2)^p (c-a^2cx^2)^{-p}}{a(1-2p)} + \frac{(1-ax)^{-2p} (1-a^2x^2)^p (c-a^2cx^2)^{-p}}{ap}$$

Result (type 8, 29 leaves):

$$\int e^{2(1+p) \text{ArcTanh}[ax]} (c-a^2cx^2)^{-p} dx$$

Test results for the 361 problems in "7.3.7 Inverse hyperbolic tangent functions.m"

Problem 16: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{9/2} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] dx$$

Optimal (type 4, 196 leaves, 6 steps):

$$-\frac{60 d^2 \sqrt{x} \sqrt{d+e x^2}}{847 e^{5/2}} + \frac{36 d x^{5/2} \sqrt{d+e x^2}}{847 e^{3/2}} - \frac{4 x^{9/2} \sqrt{d+e x^2}}{121 \sqrt{e}} + \frac{2}{11} x^{11/2} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] + \frac{30 d^{11/4} (\sqrt{d} + \sqrt{e} x) \sqrt{\frac{d+e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{847 e^{11/4} \sqrt{d+e x^2}}$$

Result (type 4, 161 leaves):

$$\frac{2}{847} \sqrt{x} \left(-\frac{2 \sqrt{d+e x^2} (15 d^2 - 9 d e x^2 + 7 e^2 x^4)}{e^{5/2}} + 77 x^5 \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] \right) + \frac{60 d^{5/2} \sqrt{\frac{i \sqrt{d}}{\sqrt{e}}} \sqrt{1 + \frac{d}{e x^2}} x \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right], -1\right]}{847 e^2 \sqrt{d+e x^2}}$$

Problem 17: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] dx$$

Optimal (type 4, 168 leaves, 5 steps):

$$\frac{20 d \sqrt{x} \sqrt{d+e x^2}}{147 e^{3/2}} - \frac{4 x^{5/2} \sqrt{d+e x^2}}{49 \sqrt{e}} + \frac{2}{7} x^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] - \frac{10 d^{7/4} (\sqrt{d} + \sqrt{e} x) \sqrt{\frac{d+e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{147 e^{7/4} \sqrt{d+e x^2}}$$

Result (type 4, 147 leaves):

$$\frac{2}{147} \sqrt{x} \left(\frac{2 (5 d - 3 e x^2) \sqrt{d+e x^2}}{e^{3/2}} + 21 x^3 \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] \right) + \frac{20 \sqrt{d} \left(\frac{i \sqrt{d}}{\sqrt{e}}\right)^{5/2} \sqrt{1 + \frac{d}{e x^2}} x \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right], -1\right]}{147 \sqrt{d+e x^2}}$$

Problem 18: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{x} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] dx$$

Optimal (type 4, 142 leaves, 4 steps):

$$-\frac{4\sqrt{x}\sqrt{d+e x^2}}{9\sqrt{e}} + \frac{2}{3}x^{3/2}\operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] + \frac{2d^{3/4}(\sqrt{d} + \sqrt{e} x)\sqrt{\frac{d+e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{9e^{3/4}\sqrt{d+e x^2}}$$

Result (type 4, 135 leaves):

$$\frac{2}{9}\sqrt{x}\left(-\frac{2\sqrt{d+e x^2}}{\sqrt{e}} + 3x\operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]\right) + \frac{4\sqrt{d}\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{1+\frac{d}{e x^2}}x \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right], -1\right]}{9\sqrt{d+e x^2}}$$

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{x^{3/2}} dx$$

Optimal (type 4, 113 leaves, 3 steps):

$$-\frac{2\operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{\sqrt{x}} + \frac{2e^{1/4}(\sqrt{d} + \sqrt{e} x)\sqrt{\frac{d+e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{d^{1/4}\sqrt{d+e x^2}}$$

Result (type 4, 111 leaves):

$$-\frac{2\operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{\sqrt{x}} + \frac{4i\sqrt{e}\sqrt{1+\frac{d}{e x^2}}x \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+e x^2}}$$

Problem 20: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{x^{7/2}} dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$-\frac{4\sqrt{e}\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\text{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{5x^{5/2}} - \frac{2e^{5/4}(\sqrt{d}+\sqrt{e}x)\sqrt{\frac{d+ex^2}{(\sqrt{d}+\sqrt{e}x)^2}}\text{EllipticF}\left[2\text{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{15d^{5/4}\sqrt{d+ex^2}}$$

Result (type 4, 142 leaves):

$$-\frac{2\left(2\sqrt{e}x\sqrt{d+ex^2}+3d\text{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]\right)}{15dx^{5/2}} - \frac{4\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{1+\frac{d}{ex^2}}\text{EllipticF}\left[i\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right], -1\right]}{15d^{3/2}\sqrt{d+ex^2}}$$

Problem 21: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{x^{11/2}} dx$$

Optimal (type 4, 173 leaves, 5 steps):

$$-\frac{4\sqrt{e}\sqrt{d+ex^2}}{63d^2x^{7/2}} + \frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\text{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{9x^{9/2}} + \frac{10e^{9/4}(\sqrt{d}+\sqrt{e}x)\sqrt{\frac{d+ex^2}{(\sqrt{d}+\sqrt{e}x)^2}}\text{EllipticF}\left[2\text{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{189d^{9/4}\sqrt{d+ex^2}}$$

Result (type 4, 154 leaves):

$$\frac{4\sqrt{e}x\sqrt{d+ex^2}(-3d+5ex^2)-42d^2\text{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{189d^2x^{9/2}} + \frac{20\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{1+\frac{d}{ex^2}}\text{EllipticF}\left[i\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right], -1\right]}{189d^{5/2}\sqrt{d+ex^2}}$$

Problem 22: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{x^{15/2}} dx$$

Optimal (type 4, 201 leaves, 6 steps):

$$\begin{aligned} & -\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}} + \frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} \\ & \frac{2\text{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{13x^{13/2}} - \frac{30e^{13/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left[2\text{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{1001d^{13/4}\sqrt{d+ex^2}} \end{aligned}$$

Result (type 4, 163 leaves):

$$\frac{1}{1001x^{13/2}} \left(\frac{2\sqrt{e}x\sqrt{d+ex^2}(7d^2-9dex^2+15e^2x^4)}{d^3} - 77\text{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right] - \frac{30\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}e^4\sqrt{1+\frac{d}{ex^2}}x^{15/2}\text{EllipticF}\left[i\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right], -1\right]}{d^{7/2}\sqrt{d+ex^2}} \right)$$

Problem 23: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{7/2} \text{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right] dx$$

Optimal (type 4, 297 leaves, 7 steps):

$$\begin{aligned} & \frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}} - \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} - \frac{28d^2\sqrt{x}\sqrt{d+ex^2}}{135e^2\left(\sqrt{d}+\sqrt{e}x\right)} + \frac{2}{9}x^{9/2}\text{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right] + \\ & \frac{28d^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticE}\left[2\text{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{135e^{9/4}\sqrt{d+ex^2}} - \frac{14d^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left[2\text{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{135e^{9/4}\sqrt{d+ex^2}} \end{aligned}$$

Result (type 4, 224 leaves):

$$\frac{1}{405 e^2 \sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}}} \frac{2 \sqrt{x}}{\sqrt{d+e x^2}} \left(\sqrt{e} x \sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}} \left(14 d^2 + 4 d e x^2 - 10 e^2 x^4 + 45 e^{3/2} x^3 \sqrt{d+e x^2} \operatorname{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right] \right) - \right. \\ \left. 42 d^{5/2} \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}} \right], -1 \right] + 42 d^{5/2} \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}} \right], -1 \right] \right)$$

Problem 24: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{3/2} \operatorname{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right] dx$$

Optimal (type 4, 269 leaves, 6 steps):

$$-\frac{4 x^{3/2} \sqrt{d+e x^2}}{25 \sqrt{e}} + \frac{12 d \sqrt{x} \sqrt{d+e x^2}}{25 e (\sqrt{d} + \sqrt{e} x)} + \frac{2}{5} x^{5/2} \operatorname{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right] - \\ \frac{12 d^{5/4} (\sqrt{d} + \sqrt{e} x) \sqrt{\frac{d+e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticE} \left[2 \operatorname{ArcTan} \left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}} \right], \frac{1}{2} \right] + 6 d^{5/4} (\sqrt{d} + \sqrt{e} x) \sqrt{\frac{d+e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}} \right], \frac{1}{2} \right]}{25 e^{5/4} \sqrt{d+e x^2}}$$

Result (type 4, 211 leaves):

$$-\frac{1}{25 e \sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}}} \frac{2 \sqrt{x}}{\sqrt{d+e x^2}} \left(\sqrt{e} x \sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}} \left(2 d + 2 e x^2 - 5 \sqrt{e} x \sqrt{d+e x^2} \operatorname{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right] \right) - \right. \\ \left. 6 d^{3/2} \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}} \right], -1 \right] + 6 d^{3/2} \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}} \right], -1 \right] \right)$$

Problem 25: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh} \left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right]}{\sqrt{x}} dx$$

Optimal (type 4, 232 leaves, 5 steps):

$$-\frac{4\sqrt{x}\sqrt{d+ex^2}}{\sqrt{d}+\sqrt{ex}} + 2\sqrt{x}\operatorname{ArcTanh}\left[\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right] + \frac{4d^{1/4}(\sqrt{d}+\sqrt{ex})\sqrt{\frac{d+ex^2}{(\sqrt{d}+\sqrt{ex})^2}}\operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{e^{1/4}\sqrt{d+ex^2}} -$$

$$\frac{2d^{1/4}(\sqrt{d}+\sqrt{ex})\sqrt{\frac{d+ex^2}{(\sqrt{d}+\sqrt{ex})^2}}\operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{e^{1/4}\sqrt{d+ex^2}}$$

Result (type 4, 182 leaves):

$$\frac{1}{\sqrt{\frac{i\sqrt{ex}}{\sqrt{d}}}\sqrt{d+ex^2}} - 2\sqrt{x}\left(\sqrt{\frac{i\sqrt{ex}}{\sqrt{d}}}\sqrt{d+ex^2}\operatorname{ArcTanh}\left[\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right] -\right.$$

$$\left.2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{ex}}{\sqrt{d}}}\right], -1\right] + 2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{ex}}{\sqrt{d}}}\right], -1\right]\right)$$

Problem 26: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right]}{x^{5/2}} dx$$

Optimal (type 4, 272 leaves, 6 steps):

$$-\frac{4\sqrt{e}\sqrt{d+ex^2}}{3d\sqrt{x}} + \frac{4e\sqrt{x}\sqrt{d+ex^2}}{3d(\sqrt{d}+\sqrt{ex})} - \frac{2\operatorname{ArcTanh}\left[\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right]}{3x^{3/2}} -$$

$$\frac{4e^{3/4}(\sqrt{d}+\sqrt{ex})\sqrt{\frac{d+ex^2}{(\sqrt{d}+\sqrt{ex})^2}}\operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{3d^{3/4}\sqrt{d+ex^2}} + \frac{2e^{3/4}(\sqrt{d}+\sqrt{ex})\sqrt{\frac{d+ex^2}{(\sqrt{d}+\sqrt{ex})^2}}\operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{3d^{3/4}\sqrt{d+ex^2}}$$

Result (type 4, 214 leaves):

$$\left(-2 \sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}} \left(2 \sqrt{e} x (d + e x^2) + d \sqrt{d + e x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d + e x^2}}\right] \right) + 4 \sqrt{d} e x^2 \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}}\right], -1\right] - \right. \\ \left. 4 \sqrt{d} e x^2 \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}}\right], -1\right] \right) / \left(3 d x^{3/2} \sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}} \sqrt{d + e x^2} \right)$$

Problem 27: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d + e x^2}}\right]}{x^{9/2}} dx$$

Optimal (type 4, 302 leaves, 7 steps):

$$\frac{-\frac{4 \sqrt{e} \sqrt{d + e x^2}}{35 d x^{5/2}} + \frac{12 e^{3/2} \sqrt{d + e x^2}}{35 d^2 \sqrt{x}} - \frac{12 e^2 \sqrt{x} \sqrt{d + e x^2}}{35 d^2 (\sqrt{d} + \sqrt{e} x)} - \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d + e x^2}}\right]}{7 x^{7/2}} + \frac{12 e^{7/4} (\sqrt{d} + \sqrt{e} x) \sqrt{\frac{d + e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right] - 6 e^{7/4} (\sqrt{d} + \sqrt{e} x) \sqrt{\frac{d + e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{35 d^{7/4} \sqrt{d + e x^2}}$$

Result (type 4, 234 leaves):

$$\left(2 \left(\sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}} \left(2 \sqrt{e} x (-d^2 + 2 d e x^2 + 3 e^2 x^4) - 5 d^2 \sqrt{d + e x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d + e x^2}}\right] \right) - \right. \right. \\ \left. \left. 6 \sqrt{d} e^2 x^4 \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}}\right], -1\right] + \right. \right. \\ \left. \left. 6 \sqrt{d} e^2 x^4 \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}}\right], -1\right] \right) \right) / \left(35 d^2 x^{7/2} \sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}} \sqrt{d + e x^2} \right)$$

Problem 31: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3}{1 - c^2 x^2} dx$$

Optimal (type 4, 409 leaves, 9 steps):

$$\begin{aligned} & \frac{2 \left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{c} + \frac{3 b \left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{2 c} \\ & - \frac{3 b \left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{2 c} - \frac{3 b^2 \left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{2 c} \\ & + \frac{3 b^2 \left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{2 c} + \frac{3 b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{4 c} - \frac{3 b^3 \operatorname{PolyLog}\left[4, -1 + \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{4 c} \end{aligned}$$

Result (type 8, 42 leaves):

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3}{1 - c^2 x^2} dx$$

Problem 32: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}{1 - c^2 x^2} dx$$

Optimal (type 4, 268 leaves, 7 steps):

$$\begin{aligned} & \frac{2 \left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{c} + \frac{b \left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{c} \\ & - \frac{b \left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{c} - \frac{b^2 \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{2 c} + \frac{b^2 \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{2 c} \end{aligned}$$

Result (type 8, 42 leaves):

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}{1 - c^2 x^2} dx$$

Problem 47: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{ArcTanh}[\operatorname{Tanh}[a + bx]]^2 dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{x \operatorname{ArcTanh}[\operatorname{Tanh}[a + bx]]^3}{3b} - \frac{\operatorname{ArcTanh}[\operatorname{Tanh}[a + bx]]^4}{12b^2}$$

Result (type 3, 74 leaves):

$$\frac{1}{12b^2} (a + bx) \left(-(3a - bx)(a + bx)^2 + 4(2a^2 + abx - b^2x^2) \operatorname{ArcTanh}[\operatorname{Tanh}[a + bx]] - 6(a - bx) \operatorname{ArcTanh}[\operatorname{Tanh}[a + bx]]^2 \right)$$

Problem 57: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{ArcTanh}[\operatorname{Tanh}[a + bx]]^3 dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{x \operatorname{ArcTanh}[\operatorname{Tanh}[a + bx]]^4}{4b} - \frac{\operatorname{ArcTanh}[\operatorname{Tanh}[a + bx]]^5}{20b^2}$$

Result (type 3, 99 leaves):

$$\frac{1}{20b^2} (a + bx) \left((4a - bx)(a + bx)^3 - 5(3a - bx)(a + bx)^2 \operatorname{ArcTanh}[\operatorname{Tanh}[a + bx]] + 10(2a^2 + abx - b^2x^2) \operatorname{ArcTanh}[\operatorname{Tanh}[a + bx]]^2 - 10(a - bx) \operatorname{ArcTanh}[\operatorname{Tanh}[a + bx]]^3 \right)$$

Problem 71: Result more than twice size of optimal antiderivative.

$$\int x \operatorname{ArcTanh}[\operatorname{Tanh}[a + bx]]^4 dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{x \operatorname{ArcTanh}[\operatorname{Tanh}[a + bx]]^5}{5b} - \frac{\operatorname{ArcTanh}[\operatorname{Tanh}[a + bx]]^6}{30b^2}$$

Result (type 3, 125 leaves):

$$-\frac{1}{30b^2}(a+bx)\left((5a-bx)(a+bx)^4-6(4a-bx)(a+bx)^3\text{ArcTanh}[\text{Tanh}[a+bx]]+\right. \\ \left.15(3a-bx)(a+bx)^2\text{ArcTanh}[\text{Tanh}[a+bx]]^2-20(2a^2+abx-b^2x^2)\text{ArcTanh}[\text{Tanh}[a+bx]]^3+15(a-bx)\text{ArcTanh}[\text{Tanh}[a+bx]]^4\right)$$

Problem 78: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcTanh}[\text{Tanh}[a+bx]]^4}{x^6} dx$$

Optimal (type 3, 31 leaves, 1 step):

$$\frac{\text{ArcTanh}[\text{Tanh}[a+bx]]^5}{5x^5(bx-\text{ArcTanh}[\text{Tanh}[a+bx]])}$$

Result (type 3, 66 leaves):

$$-\frac{1}{5x^5}(b^4x^4+b^3x^3\text{ArcTanh}[\text{Tanh}[a+bx]]+b^2x^2\text{ArcTanh}[\text{Tanh}[a+bx]]^2+bx\text{ArcTanh}[\text{Tanh}[a+bx]]^3+\text{ArcTanh}[\text{Tanh}[a+bx]]^4)$$

Problem 84: Result more than twice size of optimal antiderivative.

$$\int x \text{ArcTanh}[\text{Tanh}[a+bx]]^6 dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{x \text{ArcTanh}[\text{Tanh}[a+bx]]^7}{7b} - \frac{\text{ArcTanh}[\text{Tanh}[a+bx]]^8}{56b^2}$$

Result (type 3, 177 leaves):

$$-\frac{1}{56b^2}(a+bx)\left((7a-bx)(a+bx)^6-8(6a-bx)(a+bx)^5\text{ArcTanh}[\text{Tanh}[a+bx]]+\right. \\ \left.28(5a-bx)(a+bx)^4\text{ArcTanh}[\text{Tanh}[a+bx]]^2-56(4a-bx)(a+bx)^3\text{ArcTanh}[\text{Tanh}[a+bx]]^3+\right. \\ \left.70(3a-bx)(a+bx)^2\text{ArcTanh}[\text{Tanh}[a+bx]]^4-56(2a^2+abx-b^2x^2)\text{ArcTanh}[\text{Tanh}[a+bx]]^5+28(a-bx)\text{ArcTanh}[\text{Tanh}[a+bx]]^6\right)$$

Problem 286: Result more than twice size of optimal antiderivative.

$$\int \text{ArcTanh}[c+d \text{Tanh}[a+bx]] dx$$

Optimal (type 4, 150 leaves, 7 steps):

$$x \operatorname{ArcTanh}[c + d \operatorname{Tanh}[a + b x]] + \frac{1}{2} x \operatorname{Log}\left[1 + \frac{(1 - c - d) e^{2a+2bx}}{1 - c + d}\right] -$$

$$\frac{1}{2} x \operatorname{Log}\left[1 + \frac{(1 + c + d) e^{2a+2bx}}{1 + c - d}\right] + \frac{\operatorname{PolyLog}\left[2, -\frac{(1-c-d) e^{2a+2bx}}{1-c+d}\right]}{4b} - \frac{\operatorname{PolyLog}\left[2, -\frac{(1+c+d) e^{2a+2bx}}{1+c-d}\right]}{4b}$$

Result (type 4, 366 leaves):

$$x \operatorname{ArcTanh}[c + d \operatorname{Tanh}[a + b x]] + \frac{1}{2b} \left((a + b x) \operatorname{Log}\left[1 - \frac{\sqrt{-1 + c + d} e^{a+bx}}{\sqrt{1 - c + d}}\right] + (a + b x) \operatorname{Log}\left[1 + \frac{\sqrt{-1 + c + d} e^{a+bx}}{\sqrt{1 - c + d}}\right] - (a + b x) \operatorname{Log}\left[1 - \frac{\sqrt{1 + c + d} e^{a+bx}}{\sqrt{-1 - c + d}}\right] - \right.$$

$$\left. (a + b x) \operatorname{Log}\left[1 + \frac{\sqrt{1 + c + d} e^{a+bx}}{\sqrt{-1 - c + d}}\right] + a \operatorname{Log}\left[1 + c - d + e^{2(a+bx)} + c e^{2(a+bx)} + d e^{2(a+bx)}\right] - a \operatorname{Log}\left[1 + d + e^{2(a+bx)} - d e^{2(a+bx)} - c \left(1 + e^{2(a+bx)}\right)\right] + \right.$$

$$\left. \operatorname{PolyLog}\left[2, -\frac{\sqrt{-1 + c + d} e^{a+bx}}{\sqrt{1 - c + d}}\right] + \operatorname{PolyLog}\left[2, \frac{\sqrt{-1 + c + d} e^{a+bx}}{\sqrt{1 - c + d}}\right] - \operatorname{PolyLog}\left[2, -\frac{\sqrt{1 + c + d} e^{a+bx}}{\sqrt{-1 - c + d}}\right] - \operatorname{PolyLog}\left[2, \frac{\sqrt{1 + c + d} e^{a+bx}}{\sqrt{-1 - c + d}}\right] \right)$$

Problem 291: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcTanh}[1 + d + d \operatorname{Tanh}[a + b x]] dx$$

Optimal (type 4, 69 leaves, 5 steps):

$$\frac{b x^2}{2} + x \operatorname{ArcTanh}[1 + d + d \operatorname{Tanh}[a + b x]] - \frac{1}{2} x \operatorname{Log}\left[1 + (1 + d) e^{2a+2bx}\right] - \frac{\operatorname{PolyLog}\left[2, -\frac{(1 + d) e^{2a+2bx}}{1 + d}\right]}{4b}$$

Result (type 4, 168 leaves):

$$x \operatorname{ArcTanh}[1 + d + d \operatorname{Tanh}[a + b x]] - \frac{1}{2b} \left(b x \left(-b x - \operatorname{Log}\left[e^{-a-bx} + (1 + d) e^{a+bx}\right] + \operatorname{Log}\left[1 - e^{bx} \sqrt{-(1 + d) e^{2a}}\right] + \operatorname{Log}\left[1 + e^{bx} \sqrt{-(1 + d) e^{2a}}\right] + \operatorname{Log}\left[(2 + d) \operatorname{Cosh}[a + b x] + d \operatorname{Sinh}[a + b x]\right] \right) + \right.$$

$$\left. \operatorname{PolyLog}\left[2, -e^{bx} \sqrt{-(1 + d) e^{2a}}\right] + \operatorname{PolyLog}\left[2, e^{bx} \sqrt{-(1 + d) e^{2a}}\right] \right)$$

Problem 296: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcTanh}[1 - d - d \operatorname{Tanh}[a + b x]] dx$$

Optimal (type 4, 76 leaves, 5 steps):

$$\frac{b x^2}{2} + x \operatorname{ArcTanh}[1 - d - d \operatorname{Tanh}[a + b x]] - \frac{1}{2} x \operatorname{Log}\left[1 + (1 - d) e^{2a+2bx}\right] - \frac{\operatorname{PolyLog}\left[2, -\frac{(1 - d) e^{2a+2bx}}{1 - d}\right]}{4b}$$

Result (type 4, 171 leaves):

$$x \operatorname{ArcTanh}[1 - d - d \operatorname{Tanh}[a + b x]] - \frac{1}{2b} \\ \left(b x \left(-b x - \operatorname{Log}\left[e^{-a-bx} (-1 + (-1+d) e^{2(a+bx)}) \right] \right) + \operatorname{Log}\left[1 - e^{bx} \sqrt{(-1+d) e^{2a}} \right] + \operatorname{Log}\left[1 + e^{bx} \sqrt{(-1+d) e^{2a}} \right] + \operatorname{Log}\left[\right. \right. \\ \left. \left. (-2+d) \operatorname{Cosh}[a + b x] + d \operatorname{Sinh}[a + b x] \right] \right) + \operatorname{PolyLog}\left[2, -e^{bx} \sqrt{(-1+d) e^{2a}} \right] + \operatorname{PolyLog}\left[2, e^{bx} \sqrt{(-1+d) e^{2a}} \right] \right)$$

Problem 300: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcTanh}[c + d \operatorname{Coth}[a + b x]] dx$$

Optimal (type 4, 150 leaves, 7 steps):

$$x \operatorname{ArcTanh}[c + d \operatorname{Coth}[a + b x]] + \frac{1}{2} x \operatorname{Log}\left[1 - \frac{(1-c-d) e^{2a+2bx}}{1-c+d} \right] - \\ \frac{1}{2} x \operatorname{Log}\left[1 - \frac{(1+c+d) e^{2a+2bx}}{1+c-d} \right] + \frac{\operatorname{PolyLog}\left[2, \frac{(1-c-d) e^{2a+2bx}}{1-c+d} \right]}{4b} - \frac{\operatorname{PolyLog}\left[2, \frac{(1+c+d) e^{2a+2bx}}{1+c-d} \right]}{4b}$$

Result (type 4, 369 leaves):

$$x \operatorname{ArcTanh}[c + d \operatorname{Coth}[a + b x]] - \\ \frac{1}{2b} \left(- (a + b x) \operatorname{Log}\left[1 - \frac{\sqrt{-1+c+d} e^{a+bx}}{\sqrt{-1+c-d}} \right] - (a + b x) \operatorname{Log}\left[1 + \frac{\sqrt{-1+c+d} e^{a+bx}}{\sqrt{-1+c-d}} \right] + (a + b x) \operatorname{Log}\left[1 - \frac{\sqrt{1+c+d} e^{a+bx}}{\sqrt{1+c-d}} \right] + \right. \\ \left. (a + b x) \operatorname{Log}\left[1 + \frac{\sqrt{1+c+d} e^{a+bx}}{\sqrt{1+c-d}} \right] + a \operatorname{Log}\left[1 + d - e^{2(a+bx)} + d e^{2(a+bx)} + c (-1 + e^{2(a+bx)}) \right] - a \operatorname{Log}\left[1 + c - e^{2(a+bx)} - c e^{2(a+bx)} - d (1 + e^{2(a+bx)}) \right] \right) - \\ \operatorname{PolyLog}\left[2, -\frac{\sqrt{-1+c+d} e^{a+bx}}{\sqrt{-1+c-d}} \right] - \operatorname{PolyLog}\left[2, \frac{\sqrt{-1+c+d} e^{a+bx}}{\sqrt{-1+c-d}} \right] + \operatorname{PolyLog}\left[2, -\frac{\sqrt{1+c+d} e^{a+bx}}{\sqrt{1+c-d}} \right] + \operatorname{PolyLog}\left[2, \frac{\sqrt{1+c+d} e^{a+bx}}{\sqrt{1+c-d}} \right] \right)$$

Problem 305: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcTanh}[1 + d + d \operatorname{Coth}[a + b x]] dx$$

Optimal (type 4, 69 leaves, 5 steps):

$$\frac{b x^2}{2} + x \operatorname{ArcTanh}[1 + d + d \operatorname{Coth}[a + b x]] - \frac{1}{2} x \operatorname{Log}\left[1 - (1+d) e^{2a+2bx} \right] - \frac{\operatorname{PolyLog}\left[2, (1+d) e^{2a+2bx} \right]}{4b}$$

Result (type 4, 168 leaves):

$$x \operatorname{ArcTanh}[1 + d + d \operatorname{Coth}[a + b x]] - \frac{1}{2b} \left(b x \left(-b x - \operatorname{Log}\left[e^{-a-bx} (-1 + (1+d) e^{2(a+bx)}) \right] \right) + \operatorname{Log}\left[1 - e^{bx} \sqrt{(1+d) e^{2a}} \right] + \operatorname{Log}\left[1 + e^{bx} \sqrt{(1+d) e^{2a}} \right] + \operatorname{Log}\left[d \operatorname{Cosh}[a + b x] + (2+d) \operatorname{Sinh}[a + b x] \right] \right) + \operatorname{PolyLog}\left[2, -e^{bx} \sqrt{(1+d) e^{2a}} \right] + \operatorname{PolyLog}\left[2, e^{bx} \sqrt{(1+d) e^{2a}} \right]$$

Problem 310: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcTanh}[1 - d - d \operatorname{Coth}[a + b x]] dx$$

Optimal (type 4, 76 leaves, 5 steps):

$$\frac{b x^2}{2} + x \operatorname{ArcTanh}[1 - d - d \operatorname{Coth}[a + b x]] - \frac{1}{2} x \operatorname{Log}\left[1 - (1-d) e^{2a+2bx} \right] - \frac{\operatorname{PolyLog}\left[2, (1-d) e^{2a+2bx} \right]}{4b}$$

Result (type 4, 175 leaves):

$$x \operatorname{ArcTanh}[1 - d - d \operatorname{Coth}[a + b x]] - \frac{1}{2b} \left(b x \left(-b x - \operatorname{Log}\left[e^{-a-bx} (1 + (-1+d) e^{2(a+bx)}) \right] \right) + \operatorname{Log}\left[1 - e^{bx} \sqrt{-(-1+d) e^{2a}} \right] + \operatorname{Log}\left[1 + e^{bx} \sqrt{-(-1+d) e^{2a}} \right] + \operatorname{Log}\left[d \operatorname{Cosh}[a + b x] + (-2+d) \operatorname{Sinh}[a + b x] \right] \right) + \operatorname{PolyLog}\left[2, -e^{bx} \sqrt{-(-1+d) e^{2a}} \right] + \operatorname{PolyLog}\left[2, e^{bx} \sqrt{-(-1+d) e^{2a}} \right]$$

Problem 312: Result more than twice size of optimal antiderivative.

$$\int (e + f x)^3 \operatorname{ArcTanh}[\operatorname{Tan}[a + b x]] dx$$

Optimal (type 4, 302 leaves, 12 steps):

$$\frac{i (e + f x)^4 \operatorname{ArcTan}\left[e^{2i(a+bx)} \right]}{4f} + \frac{(e + f x)^4 \operatorname{ArcTanh}[\operatorname{Tan}[a + b x]]}{4f} - \frac{i (e + f x)^3 \operatorname{PolyLog}\left[2, -i e^{2i(a+bx)} \right]}{4b} + \frac{i (e + f x)^3 \operatorname{PolyLog}\left[2, i e^{2i(a+bx)} \right]}{4b} + \frac{3f (e + f x)^2 \operatorname{PolyLog}\left[3, -i e^{2i(a+bx)} \right]}{8b^2} - \frac{3f (e + f x)^2 \operatorname{PolyLog}\left[3, i e^{2i(a+bx)} \right]}{8b^2} + \frac{3i f^2 (e + f x) \operatorname{PolyLog}\left[4, -i e^{2i(a+bx)} \right]}{8b^3} - \frac{3i f^2 (e + f x) \operatorname{PolyLog}\left[4, i e^{2i(a+bx)} \right]}{8b^3} - \frac{3f^3 \operatorname{PolyLog}\left[5, -i e^{2i(a+bx)} \right]}{16b^4} + \frac{3f^3 \operatorname{PolyLog}\left[5, i e^{2i(a+bx)} \right]}{16b^4}$$

Result (type 4, 654 leaves):

$$\frac{1}{4} x (4 e^3 + 6 e^2 f x + 4 e f^2 x^2 + f^3 x^3) \operatorname{ArcTanh}[\operatorname{Tan}[a + b x]] +$$

$$\frac{1}{16 b^4} \left(-8 b^4 e^3 x \operatorname{Log}[1 - i e^{2i(a+bx)}] - 12 b^4 e^2 f x^2 \operatorname{Log}[1 - i e^{2i(a+bx)}] - 8 b^4 e f^2 x^3 \operatorname{Log}[1 - i e^{2i(a+bx)}] - \right.$$

$$2 b^4 f^3 x^4 \operatorname{Log}[1 - i e^{2i(a+bx)}] + 8 b^4 e^3 x \operatorname{Log}[1 + i e^{2i(a+bx)}] + 12 b^4 e^2 f x^2 \operatorname{Log}[1 + i e^{2i(a+bx)}] + 8 b^4 e f^2 x^3 \operatorname{Log}[1 + i e^{2i(a+bx)}] +$$

$$2 b^4 f^3 x^4 \operatorname{Log}[1 + i e^{2i(a+bx)}] - 4 i b^3 (e + f x)^3 \operatorname{PolyLog}[2, -i e^{2i(a+bx)}] + 4 i b^3 (e + f x)^3 \operatorname{PolyLog}[2, i e^{2i(a+bx)}] +$$

$$6 b^2 e^2 f \operatorname{PolyLog}[3, -i e^{2i(a+bx)}] + 12 b^2 e f^2 x \operatorname{PolyLog}[3, -i e^{2i(a+bx)}] + 6 b^2 f^3 x^2 \operatorname{PolyLog}[3, -i e^{2i(a+bx)}] -$$

$$6 b^2 e^2 f \operatorname{PolyLog}[3, i e^{2i(a+bx)}] - 12 b^2 e f^2 x \operatorname{PolyLog}[3, i e^{2i(a+bx)}] - 6 b^2 f^3 x^2 \operatorname{PolyLog}[3, i e^{2i(a+bx)}] +$$

$$6 i b e f^2 \operatorname{PolyLog}[4, -i e^{2i(a+bx)}] + 6 i b f^3 x \operatorname{PolyLog}[4, -i e^{2i(a+bx)}] - 6 i b e f^2 \operatorname{PolyLog}[4, i e^{2i(a+bx)}] -$$

$$6 i b f^3 x \operatorname{PolyLog}[4, i e^{2i(a+bx)}] - 3 f^3 \operatorname{PolyLog}[5, -i e^{2i(a+bx)}] + 3 f^3 \operatorname{PolyLog}[5, i e^{2i(a+bx)}] \left. \right)$$

Problem 319: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcTanh}[c + d \operatorname{Tan}[a + b x]] dx$$

Optimal (type 4, 194 leaves, 7 steps):

$$x \operatorname{ArcTanh}[c + d \operatorname{Tan}[a + b x]] + \frac{1}{2} x \operatorname{Log}\left[1 + \frac{(1 - c + i d) e^{2i a + 2i b x}}{1 - c - i d}\right] -$$

$$\frac{1}{2} x \operatorname{Log}\left[1 + \frac{(1 + c - i d) e^{2i a + 2i b x}}{1 + c + i d}\right] - \frac{i \operatorname{PolyLog}\left[2, -\frac{(1 - c + i d) e^{2i a + 2i b x}}{1 - c - i d}\right]}{4 b} + \frac{i \operatorname{PolyLog}\left[2, -\frac{(1 + c - i d) e^{2i a + 2i b x}}{1 + c + i d}\right]}{4 b}$$

Result (type 4, 4654 leaves):

$$x \operatorname{ArcTanh}[c + d \operatorname{Tan}[a + b x]] +$$

$$\left(d \left(-a \operatorname{Log}\left[-\operatorname{Sec}\left[\frac{1}{2}(a + b x)\right]^2 \left((-1 + c) \operatorname{Cos}[a + b x] + d \operatorname{Sin}[a + b x] \right)\right] + a \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(a + b x)\right]^2 \left(\operatorname{Cos}[a + b x] + c \operatorname{Cos}[a + b x] + d \operatorname{Sin}[a + b x] \right)\right] + \right.$$

$$(a + b x) \operatorname{Log}\left[\frac{-d + \sqrt{1 - 2c + c^2 + d^2}}{-1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] +$$

$$i \operatorname{Log}\left[\frac{(-1 + c) \left(1 + i \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{-1 + c + i d - i \sqrt{1 - 2c + c^2 + d^2}}\right] \operatorname{Log}\left[\frac{-d + \sqrt{1 - 2c + c^2 + d^2}}{-1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - i \operatorname{Log}\left[-\frac{(-1 + c) \left(i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{i - i c - d + \sqrt{1 - 2c + c^2 + d^2}}\right] \right.$$

$$\operatorname{Log}\left[\frac{-d + \sqrt{1 - 2c + c^2 + d^2}}{-1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] + (a + b x) \operatorname{Log}\left[\frac{d + \sqrt{1 - 2c + c^2 + d^2}}{1 - c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] +$$

$$i \operatorname{Log}\left[\frac{(-1 + c) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{i - i c + d + \sqrt{1 - 2c + c^2 + d^2}}\right] \operatorname{Log}\left[\frac{d + \sqrt{1 - 2c + c^2 + d^2}}{1 - c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - i \operatorname{Log}\left[\frac{(-1 + c) \left(i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{-i + i c + d + \sqrt{1 - 2c + c^2 + d^2}}\right] \right.$$

$$\left. \operatorname{Log}\left[\frac{d + \sqrt{1 - 2c + c^2 + d^2}}{1 - c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - (a + b x) \operatorname{Log}\left[-\frac{d + \sqrt{1 + 2c + c^2 + d^2}}{1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - \right)$$

$$\begin{aligned}
& i \operatorname{Log} \left[\frac{(1+c) \left(-i + \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right] \right)}{-i - ic + d + \sqrt{1+2c+c^2+d^2}} \right] \operatorname{Log} \left[-\frac{d + \sqrt{1+2c+c^2+d^2}}{1+c} + \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right] \right] + i \operatorname{Log} \left[\frac{(1+c) \left(i + \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right] \right)}{i + ic + d + \sqrt{1+2c+c^2+d^2}} \right] \\
& \operatorname{Log} \left[-\frac{d + \sqrt{1+2c+c^2+d^2}}{1+c} + \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right] \right] - (a+bx) \operatorname{Log} \left[\frac{-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right]}{1+c} \right] + \\
& i \operatorname{Log} \left[\frac{(1+c) \left(1 - i \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right] \right)}{1+c - id + i\sqrt{1+2c+c^2+d^2}} \right] \operatorname{Log} \left[\frac{-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right]}{1+c} \right] - \\
& i \operatorname{Log} \left[\frac{(1+c) \left(1 + i \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right] \right)}{1+c + id - i\sqrt{1+2c+c^2+d^2}} \right] \operatorname{Log} \left[\frac{-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right]}{1+c} \right] + \\
& i \operatorname{PolyLog} \left[2, \frac{d + \sqrt{1-2c+c^2+d^2} - (-1+c) \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right]}{i - ic + d + \sqrt{1-2c+c^2+d^2}} \right] - i \operatorname{PolyLog} \left[2, \frac{d + \sqrt{1-2c+c^2+d^2} - (-1+c) \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right]}{-i + ic + d + \sqrt{1-2c+c^2+d^2}} \right] - \\
& i \operatorname{PolyLog} \left[2, \frac{-d + \sqrt{1-2c+c^2+d^2} + (-1+c) \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right]}{i - ic - d + \sqrt{1-2c+c^2+d^2}} \right] + i \operatorname{PolyLog} \left[2, \frac{-d + \sqrt{1-2c+c^2+d^2} + (-1+c) \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right]}{-i + ic - d + \sqrt{1-2c+c^2+d^2}} \right] - \\
& i \operatorname{PolyLog} \left[2, \frac{d + \sqrt{1+2c+c^2+d^2} - (1+c) \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right]}{-i - ic + d + \sqrt{1+2c+c^2+d^2}} \right] + i \operatorname{PolyLog} \left[2, \frac{d + \sqrt{1+2c+c^2+d^2} - (1+c) \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right]}{i + ic + d + \sqrt{1+2c+c^2+d^2}} \right] + \\
& i \operatorname{PolyLog} \left[2, \frac{-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right]}{-i - ic - d + \sqrt{1+2c+c^2+d^2}} \right] - i \operatorname{PolyLog} \left[2, \frac{-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right]}{i + ic - d + \sqrt{1+2c+c^2+d^2}} \right] \Big) \\
& \left(- \left((2a) / (b(-1+c^2+d^2 - \operatorname{Cos}[2(a+bx)] + c^2 \operatorname{Cos}[2(a+bx)] - d^2 \operatorname{Cos}[2(a+bx)] + 2cd \operatorname{Sin}[2(a+bx)])) \right) + \right. \\
& \left. (2(a+bx)) / (b(-1+c^2+d^2 - \operatorname{Cos}[2(a+bx)] + c^2 \operatorname{Cos}[2(a+bx)] - d^2 \operatorname{Cos}[2(a+bx)] + 2cd \operatorname{Sin}[2(a+bx)])) \right) \Big) / \\
& \left(\operatorname{Log} \left[\frac{-d + \sqrt{1-2c+c^2+d^2}}{-1+c} + \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right] \right] + \operatorname{Log} \left[\frac{d + \sqrt{1-2c+c^2+d^2}}{1-c} + \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right] \right] - \right. \\
& \operatorname{Log} \left[-\frac{d + \sqrt{1+2c+c^2+d^2}}{1+c} + \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right] \right] - \operatorname{Log} \left[\frac{-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right]}{1+c} \right] + \\
& \left. \frac{\operatorname{Log} \left[\frac{-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right]}{1+c} \right] \operatorname{Sec} \left[\frac{1}{2} (a+bx) \right]^2}{2 \left(1 - i \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right] \right)} - \frac{\operatorname{Log} \left[\frac{-d + \sqrt{1-2c+c^2+d^2}}{-1+c} + \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right] \right] \operatorname{Sec} \left[\frac{1}{2} (a+bx) \right]^2}{2 \left(1 + i \operatorname{Tan} \left[\frac{1}{2} (a+bx) \right] \right)} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{\operatorname{Log}\left[\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c)\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1+c}\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(1+i\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \frac{i\operatorname{Log}\left[\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
& \frac{i\operatorname{Log}\left[-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \frac{i\operatorname{Log}\left[\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
& \frac{i\operatorname{Log}\left[\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \frac{i\operatorname{Log}\left[-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
& \frac{(a+bx)\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \frac{i\operatorname{Log}\left[\frac{(-1+c)\left(1+i\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{-1+c+i d-i\sqrt{1-2c+c^2+d^2}}\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
& \frac{i\operatorname{Log}\left[-\frac{(-1+c)\left(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{i-i c-d+\sqrt{1-2c+c^2+d^2}}\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \frac{(a+bx)\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
& \frac{i\operatorname{Log}\left[\frac{(-1+c)\left(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{i-i c+d+\sqrt{1-2c+c^2+d^2}}\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \frac{i\operatorname{Log}\left[\frac{(-1+c)\left(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{-i+i c+d+\sqrt{1-2c+c^2+d^2}}\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
& \frac{(a+bx)\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \frac{i\operatorname{Log}\left[\frac{(1+c)\left(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{-i-i c+d+\sqrt{1+2c+c^2+d^2}}\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
& \frac{i\operatorname{Log}\left[\frac{(1+c)\left(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{i+i c+d+\sqrt{1+2c+c^2+d^2}}\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \frac{i(-1+c)\operatorname{Log}\left[1-\frac{d+\sqrt{1-2c+c^2+d^2}-(-1+c)\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i-i c+d+\sqrt{1-2c+c^2+d^2}}\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(d+\sqrt{1-2c+c^2+d^2}-(-1+c)\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} -
\end{aligned}$$

$$\begin{aligned}
& \frac{i(-1+c) \operatorname{Log}\left[1 - \frac{d + \sqrt{1-2c+c^2+d^2} - (-1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i+i c+d+\sqrt{1-2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(d + \sqrt{1-2c+c^2+d^2} - (-1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \frac{i(-1+c) \operatorname{Log}\left[1 - \frac{-d + \sqrt{1-2c+c^2+d^2} + (-1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i-i c-d+\sqrt{1-2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1-2c+c^2+d^2} + (-1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} \\
& \frac{i(-1+c) \operatorname{Log}\left[1 - \frac{-d + \sqrt{1-2c+c^2+d^2} + (-1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i+i c-d+\sqrt{1-2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1-2c+c^2+d^2} + (-1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
& \frac{i(1+c) \operatorname{Log}\left[1 - \frac{d + \sqrt{1+2c+c^2+d^2} - (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i-i c+d+\sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(d + \sqrt{1+2c+c^2+d^2} - (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \frac{i(1+c) \operatorname{Log}\left[1 - \frac{d + \sqrt{1+2c+c^2+d^2} - (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i+i c+d+\sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(d + \sqrt{1+2c+c^2+d^2} - (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} \\
& \frac{(1+c)(a+bx) \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \frac{i(1+c) \operatorname{Log}\left[\frac{(1+c)(1-i \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{1+c-i d+i \sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} \\
& \frac{i(1+c) \operatorname{Log}\left[\frac{(1+c)(1+i \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{1+c+i d-i \sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \frac{i(1+c) \operatorname{Log}\left[1 - \frac{-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i-i c-d+\sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
& \frac{i(1+c) \operatorname{Log}\left[1 - \frac{-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i+i c-d+\sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \left(a \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right)^2 \\
& \left(-\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 (d \operatorname{Cos}[a+bx] - (-1+c) \operatorname{Sin}[a+bx]) - \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 ((-1+c) \operatorname{Cos}[a+bx] + d \operatorname{Sin}[a+bx]) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right] \right) / \\
& \left((-1+c) \operatorname{Cos}[a+bx] + d \operatorname{Sin}[a+bx] \right) + \left(a \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right] \right)^2 \left(\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 (d \operatorname{Cos}[a+bx] - \operatorname{Sin}[a+bx] - c \operatorname{Sin}[a+bx]) + \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 (\operatorname{Cos}[a+bx] + c \operatorname{Cos}[a+bx] + d \operatorname{Sin}[a+bx]) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right] \right) / \left(\operatorname{Cos}[a+bx] + c \operatorname{Cos}[a+bx] + d \operatorname{Sin}[a+bx] \right)
\end{aligned}$$

Problem 329: Result more than twice size of optimal antiderivative.

$$\int (e + f x)^3 \operatorname{ArcTanh}[\operatorname{Cot}[a + b x]] dx$$

Optimal (type 4, 302 leaves, 12 steps):

$$\frac{i (e + f x)^4 \operatorname{ArcTan}[e^{2i(a+bx)}]}{4f} + \frac{(e + f x)^4 \operatorname{ArcTanh}[\operatorname{Cot}[a + b x]]}{4f} - \frac{i (e + f x)^3 \operatorname{PolyLog}[2, -i e^{2i(a+bx)}]}{4b} +$$

$$\frac{i (e + f x)^3 \operatorname{PolyLog}[2, i e^{2i(a+bx)}]}{4b} + \frac{3f (e + f x)^2 \operatorname{PolyLog}[3, -i e^{2i(a+bx)}]}{8b^2} - \frac{3f (e + f x)^2 \operatorname{PolyLog}[3, i e^{2i(a+bx)}]}{8b^2} +$$

$$\frac{3i f^2 (e + f x) \operatorname{PolyLog}[4, -i e^{2i(a+bx)}]}{8b^3} - \frac{3i f^2 (e + f x) \operatorname{PolyLog}[4, i e^{2i(a+bx)}]}{8b^3} - \frac{3f^3 \operatorname{PolyLog}[5, -i e^{2i(a+bx)}]}{16b^4} + \frac{3f^3 \operatorname{PolyLog}[5, i e^{2i(a+bx)}]}{16b^4}$$

Result (type 4, 654 leaves):

$$\frac{1}{4} x (4e^3 + 6e^2 f x + 4e f^2 x^2 + f^3 x^3) \operatorname{ArcTanh}[\operatorname{Cot}[a + b x]] +$$

$$\frac{1}{16b^4} \left(-8b^4 e^3 x \operatorname{Log}[1 - i e^{2i(a+bx)}] - 12b^4 e^2 f x^2 \operatorname{Log}[1 - i e^{2i(a+bx)}] - 8b^4 e f^2 x^3 \operatorname{Log}[1 - i e^{2i(a+bx)}] - \right.$$

$$2b^4 f^3 x^4 \operatorname{Log}[1 - i e^{2i(a+bx)}] + 8b^4 e^3 x \operatorname{Log}[1 + i e^{2i(a+bx)}] + 12b^4 e^2 f x^2 \operatorname{Log}[1 + i e^{2i(a+bx)}] + 8b^4 e f^2 x^3 \operatorname{Log}[1 + i e^{2i(a+bx)}] +$$

$$2b^4 f^3 x^4 \operatorname{Log}[1 + i e^{2i(a+bx)}] - 4i b^3 (e + f x)^3 \operatorname{PolyLog}[2, -i e^{2i(a+bx)}] + 4i b^3 (e + f x)^3 \operatorname{PolyLog}[2, i e^{2i(a+bx)}] +$$

$$6b^2 e^2 f \operatorname{PolyLog}[3, -i e^{2i(a+bx)}] + 12b^2 e f^2 x \operatorname{PolyLog}[3, -i e^{2i(a+bx)}] + 6b^2 f^3 x^2 \operatorname{PolyLog}[3, -i e^{2i(a+bx)}] -$$

$$6b^2 e^2 f \operatorname{PolyLog}[3, i e^{2i(a+bx)}] - 12b^2 e f^2 x \operatorname{PolyLog}[3, i e^{2i(a+bx)}] - 6b^2 f^3 x^2 \operatorname{PolyLog}[3, i e^{2i(a+bx)}] +$$

$$6i b e f^2 \operatorname{PolyLog}[4, -i e^{2i(a+bx)}] + 6i b f^3 x \operatorname{PolyLog}[4, -i e^{2i(a+bx)}] - 6i b e f^2 \operatorname{PolyLog}[4, i e^{2i(a+bx)}] -$$

$$6i b f^3 x \operatorname{PolyLog}[4, i e^{2i(a+bx)}] - 3f^3 \operatorname{PolyLog}[5, -i e^{2i(a+bx)}] + 3f^3 \operatorname{PolyLog}[5, i e^{2i(a+bx)}] \Big)$$

Problem 336: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcTanh}[c + d \operatorname{Cot}[a + b x]] dx$$

Optimal (type 4, 194 leaves, 7 steps):

$$x \operatorname{ArcTanh}[c + d \operatorname{Cot}[a + b x]] + \frac{1}{2} x \operatorname{Log}\left[1 - \frac{(1 - c - i d) e^{2ia+2ibx}}{1 - c + i d}\right] -$$

$$\frac{1}{2} x \operatorname{Log}\left[1 - \frac{(1 + c + i d) e^{2ia+2ibx}}{1 + c - i d}\right] - \frac{i \operatorname{PolyLog}\left[2, \frac{(1 - c - i d) e^{2ia+2ibx}}{1 - c + i d}\right]}{4b} + \frac{i \operatorname{PolyLog}\left[2, \frac{(1 + c + i d) e^{2ia+2ibx}}{1 + c - i d}\right]}{4b}$$

Result (type 4, 4463 leaves):

$$x \operatorname{ArcTanh}[c + d \operatorname{Cot}[a + b x]] -$$

$$\left(d \left(a \operatorname{Log}\left[-\operatorname{Sec}\left[\frac{1}{2}(a + b x)\right]^2 (d \operatorname{Cos}[a + b x] + (-1 + c) \operatorname{Sin}[a + b x])\right] - a \operatorname{Log}\left[-\operatorname{Sec}\left[\frac{1}{2}(a + b x)\right]^2 (d \operatorname{Cos}[a + b x] + \operatorname{Sin}[a + b x] + c \operatorname{Sin}[a + b x])\right] \right) - \right.$$

$$\begin{aligned}
& (a + b x) \operatorname{Log}\left[-\frac{-1 + c + \sqrt{1 - 2 c + c^2 + d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - \\
& i \operatorname{Log}\left[\frac{d\left(-i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{-1 + c - i d + \sqrt{1 - 2 c + c^2 + d^2}}\right] \operatorname{Log}\left[-\frac{-1 + c + \sqrt{1 - 2 c + c^2 + d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] + i \operatorname{Log}\left[\frac{d\left(i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{-1 + c + i d + \sqrt{1 - 2 c + c^2 + d^2}}\right] \\
& \operatorname{Log}\left[-\frac{-1 + c + \sqrt{1 - 2 c + c^2 + d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] + (a + b x) \operatorname{Log}\left[-\frac{1 + c + \sqrt{1 + 2 c + c^2 + d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] + \\
& i \operatorname{Log}\left[\frac{d\left(-i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{1 + c - i d + \sqrt{1 + 2 c + c^2 + d^2}}\right] \operatorname{Log}\left[-\frac{1 + c + \sqrt{1 + 2 c + c^2 + d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - i \operatorname{Log}\left[\frac{d\left(i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{1 + c + i d + \sqrt{1 + 2 c + c^2 + d^2}}\right] \\
& \operatorname{Log}\left[-\frac{1 + c + \sqrt{1 + 2 c + c^2 + d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - (a + b x) \operatorname{Log}\left[\frac{1 - c + \sqrt{1 - 2 c + c^2 + d^2} + d \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{d}\right] - \\
& i \operatorname{Log}\left[\frac{d\left(-i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{1 - c + i d + \sqrt{1 - 2 c + c^2 + d^2}}\right] \operatorname{Log}\left[\frac{1 - c + \sqrt{1 - 2 c + c^2 + d^2} + d \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{d}\right] + i \operatorname{Log}\left[\frac{d\left(i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{1 - c - i d + \sqrt{1 - 2 c + c^2 + d^2}}\right] \\
& \operatorname{Log}\left[\frac{1 - c + \sqrt{1 - 2 c + c^2 + d^2} + d \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{d}\right] + (a + b x) \operatorname{Log}\left[\frac{-1 - c + \sqrt{1 + 2 c + c^2 + d^2} + d \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{d}\right] + \\
& i \operatorname{Log}\left[\frac{d\left(-i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{-1 - c + i d + \sqrt{1 + 2 c + c^2 + d^2}}\right] \operatorname{Log}\left[\frac{-1 - c + \sqrt{1 + 2 c + c^2 + d^2} + d \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{d}\right] - \\
& i \operatorname{Log}\left[\frac{d\left(i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{-1 - c - i d + \sqrt{1 + 2 c + c^2 + d^2}}\right] \operatorname{Log}\left[\frac{-1 - c + \sqrt{1 + 2 c + c^2 + d^2} + d \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{d}\right] - \\
& i \operatorname{PolyLog}\left[2, \frac{-1 + c + \sqrt{1 - 2 c + c^2 + d^2} - d \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{-1 + c - i d + \sqrt{1 - 2 c + c^2 + d^2}}\right] + i \operatorname{PolyLog}\left[2, \frac{-1 + c + \sqrt{1 - 2 c + c^2 + d^2} - d \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{-1 + c + i d + \sqrt{1 - 2 c + c^2 + d^2}}\right] - \\
& i \operatorname{PolyLog}\left[2, \frac{1 + c - \sqrt{1 + 2 c + c^2 + d^2} - d \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{1 + c + i d - \sqrt{1 + 2 c + c^2 + d^2}}\right] + i \operatorname{PolyLog}\left[2, \frac{1 + c + \sqrt{1 + 2 c + c^2 + d^2} - d \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{1 + c - i d + \sqrt{1 + 2 c + c^2 + d^2}}\right] - \\
& i \operatorname{PolyLog}\left[2, \frac{1 + c + \sqrt{1 + 2 c + c^2 + d^2} - d \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{1 + c + i d + \sqrt{1 + 2 c + c^2 + d^2}}\right] + i \operatorname{PolyLog}\left[2, \frac{1 - c + \sqrt{1 - 2 c + c^2 + d^2} + d \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{1 - c - i d + \sqrt{1 - 2 c + c^2 + d^2}}\right] - \\
& i \operatorname{PolyLog}\left[2, \frac{1 - c + \sqrt{1 - 2 c + c^2 + d^2} + d \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{1 - c + i d + \sqrt{1 - 2 c + c^2 + d^2}}\right] + i \operatorname{PolyLog}\left[2, \frac{-1 - c + \sqrt{1 + 2 c + c^2 + d^2} + d \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{-1 - c + i d + \sqrt{1 + 2 c + c^2 + d^2}}\right] \Big) \\
& \left(\frac{2 a}{b\left(1 - c^2 - d^2 - \operatorname{Cos}\left[2(a + b x)\right] + c^2 \operatorname{Cos}\left[2(a + b x)\right] - d^2 \operatorname{Cos}\left[2(a + b x)\right] - 2 c d \operatorname{Sin}\left[2(a + b x)\right]\right)}\right) -
\end{aligned}$$

$$\left. \frac{2(a+bx)}{b(1-c^2-d^2-\cos[2(a+bx)]+c^2\cos[2(a+bx)]-d^2\cos[2(a+bx)]-2cd\sin[2(a+bx)])} \right) \Bigg/$$

$$\left(-\operatorname{Log}\left[-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]+\operatorname{Log}\left[-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]-\right.$$

$$\operatorname{Log}\left[\frac{1-c+\sqrt{1-2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]+\operatorname{Log}\left[\frac{-1-c+\sqrt{1+2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]-$$

$$\frac{i\operatorname{Log}\left[-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}+\frac{i\operatorname{Log}\left[-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}-$$

$$\frac{i\operatorname{Log}\left[\frac{1-c+\sqrt{1-2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}+\frac{i\operatorname{Log}\left[\frac{-1-c+\sqrt{1+2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}+$$

$$\frac{i\operatorname{Log}\left[-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}-\frac{i\operatorname{Log}\left[-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}+$$

$$\frac{i\operatorname{Log}\left[\frac{1-c+\sqrt{1-2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}-\frac{i\operatorname{Log}\left[\frac{-1-c+\sqrt{1+2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}-$$

$$\frac{(a+bx)\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}-\frac{i\operatorname{Log}\left[\frac{d\left(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{-1-c-i d+\sqrt{1-2c+c^2+d^2}}\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}+$$

$$\frac{i\operatorname{Log}\left[\frac{d\left(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{-1+c+i d+\sqrt{1-2c+c^2+d^2}}\right]\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}+\frac{(a+bx)\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}+$$

$$\begin{aligned}
& \frac{i \operatorname{Log} \left[\frac{d(-i + \operatorname{Tan}[\frac{1}{2}(a+bx)])}{1+c-i d + \sqrt{1+2c+c^2+d^2}} \right] \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d} + \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} - \frac{i \operatorname{Log} \left[\frac{d(i + \operatorname{Tan}[\frac{1}{2}(a+bx)])}{1+c+i d + \sqrt{1+2c+c^2+d^2}} \right] \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d} + \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} - \\
& \frac{i d \operatorname{Log} \left[1 - \frac{-1+c+\sqrt{1-2c+c^2+d^2} - d \operatorname{Tan}[\frac{1}{2}(a+bx)]}{-1+c-i d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(-1+c + \sqrt{1-2c+c^2+d^2} - d \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} + \frac{i d \operatorname{Log} \left[1 - \frac{-1+c+\sqrt{1-2c+c^2+d^2} - d \operatorname{Tan}[\frac{1}{2}(a+bx)]}{-1+c+i d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(-1+c + \sqrt{1-2c+c^2+d^2} - d \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} - \\
& \frac{i d \operatorname{Log} \left[1 - \frac{1+c-\sqrt{1+2c+c^2+d^2} - d \operatorname{Tan}[\frac{1}{2}(a+bx)]}{1+c+i d - \sqrt{1+2c+c^2+d^2}} \right] \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(1+c - \sqrt{1+2c+c^2+d^2} - d \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} + \frac{i d \operatorname{Log} \left[1 - \frac{1+c+\sqrt{1+2c+c^2+d^2} - d \operatorname{Tan}[\frac{1}{2}(a+bx)]}{1+c-i d + \sqrt{1+2c+c^2+d^2}} \right] \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(1+c + \sqrt{1+2c+c^2+d^2} - d \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} - \\
& \frac{i d \operatorname{Log} \left[1 - \frac{1+c+\sqrt{1+2c+c^2+d^2} - d \operatorname{Tan}[\frac{1}{2}(a+bx)]}{1+c+i d + \sqrt{1+2c+c^2+d^2}} \right] \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(1+c + \sqrt{1+2c+c^2+d^2} - d \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} - \frac{d(a+bx) \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(1-c + \sqrt{1-2c+c^2+d^2} + d \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} - \\
& \frac{i d \operatorname{Log} \left[-\frac{d(-i + \operatorname{Tan}[\frac{1}{2}(a+bx)])}{1-c+i d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(1-c + \sqrt{1-2c+c^2+d^2} + d \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} + \frac{i d \operatorname{Log} \left[-\frac{d(i + \operatorname{Tan}[\frac{1}{2}(a+bx)])}{1-c-i d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(1-c + \sqrt{1-2c+c^2+d^2} + d \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} - \\
& \frac{i d \operatorname{Log} \left[1 - \frac{1-c+\sqrt{1-2c+c^2+d^2} + d \operatorname{Tan}[\frac{1}{2}(a+bx)]}{1-c-i d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(1-c + \sqrt{1-2c+c^2+d^2} + d \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} + \frac{i d \operatorname{Log} \left[1 - \frac{1-c+\sqrt{1-2c+c^2+d^2} + d \operatorname{Tan}[\frac{1}{2}(a+bx)]}{1-c+i d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(1-c + \sqrt{1-2c+c^2+d^2} + d \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} + \\
& \frac{d(a+bx) \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(-1-c + \sqrt{1+2c+c^2+d^2} + d \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} + \frac{i d \operatorname{Log} \left[-\frac{d(-i + \operatorname{Tan}[\frac{1}{2}(a+bx)])}{-1-c+i d + \sqrt{1+2c+c^2+d^2}} \right] \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(-1-c + \sqrt{1+2c+c^2+d^2} + d \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} - \\
& \frac{i d \operatorname{Log} \left[-\frac{d(i + \operatorname{Tan}[\frac{1}{2}(a+bx)])}{-1-c-i d + \sqrt{1+2c+c^2+d^2}} \right] \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(-1-c + \sqrt{1+2c+c^2+d^2} + d \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} - \frac{i d \operatorname{Log} \left[1 - \frac{-1-c+\sqrt{1+2c+c^2+d^2} + d \operatorname{Tan}[\frac{1}{2}(a+bx)]}{-1-c+i d + \sqrt{1+2c+c^2+d^2}} \right] \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2}{2 \left(-1-c + \sqrt{1+2c+c^2+d^2} + d \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \right)} - \left(a \operatorname{Cos} \left[\frac{1}{2}(a+bx) \right] \right)^2 \\
& \left(-\operatorname{Sec} \left[\frac{1}{2}(a+bx) \right] \right)^2 \left((-1+c) \operatorname{Cos}[a+bx] - d \operatorname{Sin}[a+bx] \right) - \operatorname{Sec} \left[\frac{1}{2}(a+bx) \right]^2 \left(d \operatorname{Cos}[a+bx] + (-1+c) \operatorname{Sin}[a+bx] \right) \operatorname{Tan} \left[\frac{1}{2}(a+bx) \right] \Big) \Big) / \\
& \left(d \operatorname{Cos}[a+bx] + (-1+c) \operatorname{Sin}[a+bx] \right) + \left(a \operatorname{Cos} \left[\frac{1}{2}(a+bx) \right] \right)^2 \left(-\operatorname{Sec} \left[\frac{1}{2}(a+bx) \right] \right)^2 \left(\operatorname{Cos}[a+bx] + c \operatorname{Cos}[a+bx] - d \operatorname{Sin}[a+bx] \right) -
\end{aligned}$$

$$\left. \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2 (d \cos[a+bx] + \sin[a+bx] + c \sin[a+bx]) \tan\left[\frac{1}{2}(a+bx)\right] \right) / (d \cos[a+bx] + \sin[a+bx] + c \sin[a+bx]) \right\}$$

Problem 346: Result more than twice size of optimal antiderivative.

$$\int \text{ArcTanh}[e^x] dx$$

Optimal (type 4, 21 leaves, 2 steps):

$$-\frac{1}{2} \text{PolyLog}[2, -e^x] + \frac{1}{2} \text{PolyLog}[2, e^x]$$

Result (type 4, 46 leaves):

$$x \text{ArcTanh}[e^x] + \frac{1}{2} (-x (-\text{Log}[1 - e^x] + \text{Log}[1 + e^x]) - \text{PolyLog}[2, -e^x] + \text{PolyLog}[2, e^x])$$

Problem 361: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b \text{ArcTanh}[c x^n]) (d + e \text{Log}[f x^m])}{x} dx$$

Optimal (type 4, 136 leaves, 11 steps):

$$a d \text{Log}[x] + \frac{a e \text{Log}[f x^m]^2}{2 m} - \frac{b d \text{PolyLog}[2, -c x^n]}{2 n} - \frac{b e \text{Log}[f x^m] \text{PolyLog}[2, -c x^n]}{2 n} +$$

$$\frac{b d \text{PolyLog}[2, c x^n]}{2 n} + \frac{b e \text{Log}[f x^m] \text{PolyLog}[2, c x^n]}{2 n} + \frac{b e m \text{PolyLog}[3, -c x^n]}{2 n^2} - \frac{b e m \text{PolyLog}[3, c x^n]}{2 n^2}$$

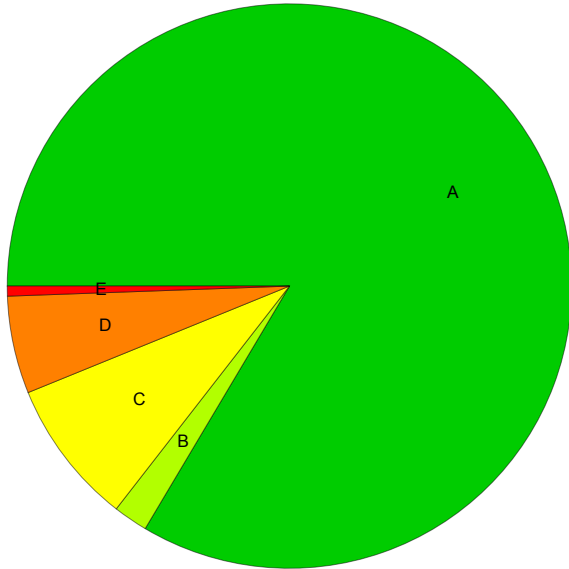
Result (type 5, 114 leaves):

$$-\frac{b c e m x^n \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right\}, c^2 x^{2n}\right]}{n^2} +$$

$$\frac{b c x^n \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, c^2 x^{2n}\right] (d + e \text{Log}[f x^m])}{n} + \frac{1}{2} a \text{Log}[x] (2 d - e m \text{Log}[x] + 2 e \text{Log}[f x^m])$$

Summary of Integration Test Results

2631 integration problems



A - 2198 optimal antiderivatives

B - 52 more than twice size of optimal antiderivatives

C - 219 unnecessarily complex antiderivatives

D - 147 unable to integrate problems

E - 15 integration timeouts